Matrix-Based Visualization of Graphs

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Aviz

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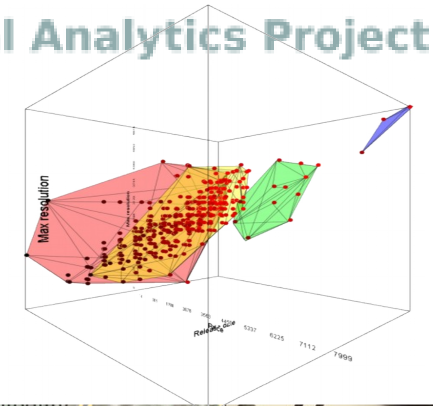
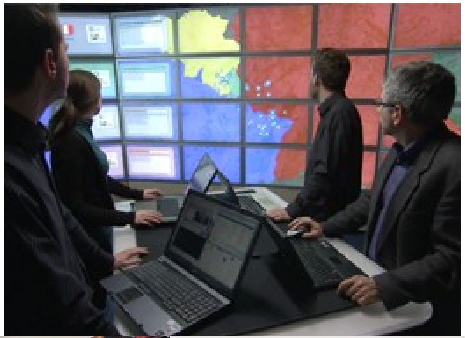
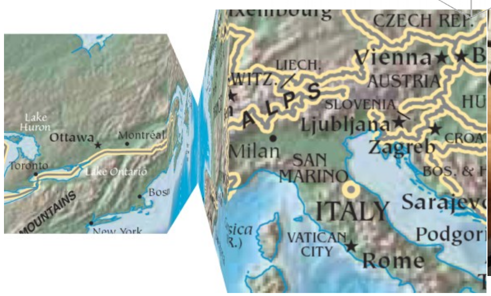
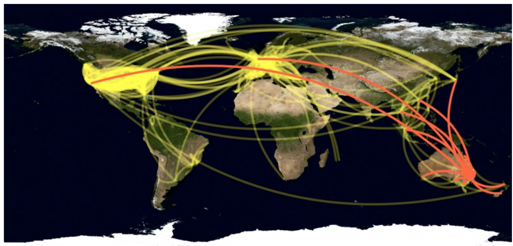
Visual Analytics Project

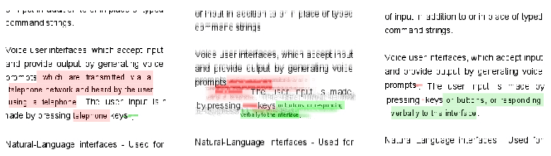




- 4 INRIA Researchers
- 2 Post-docs
- 6 PhD students
- Lots of cool stuff



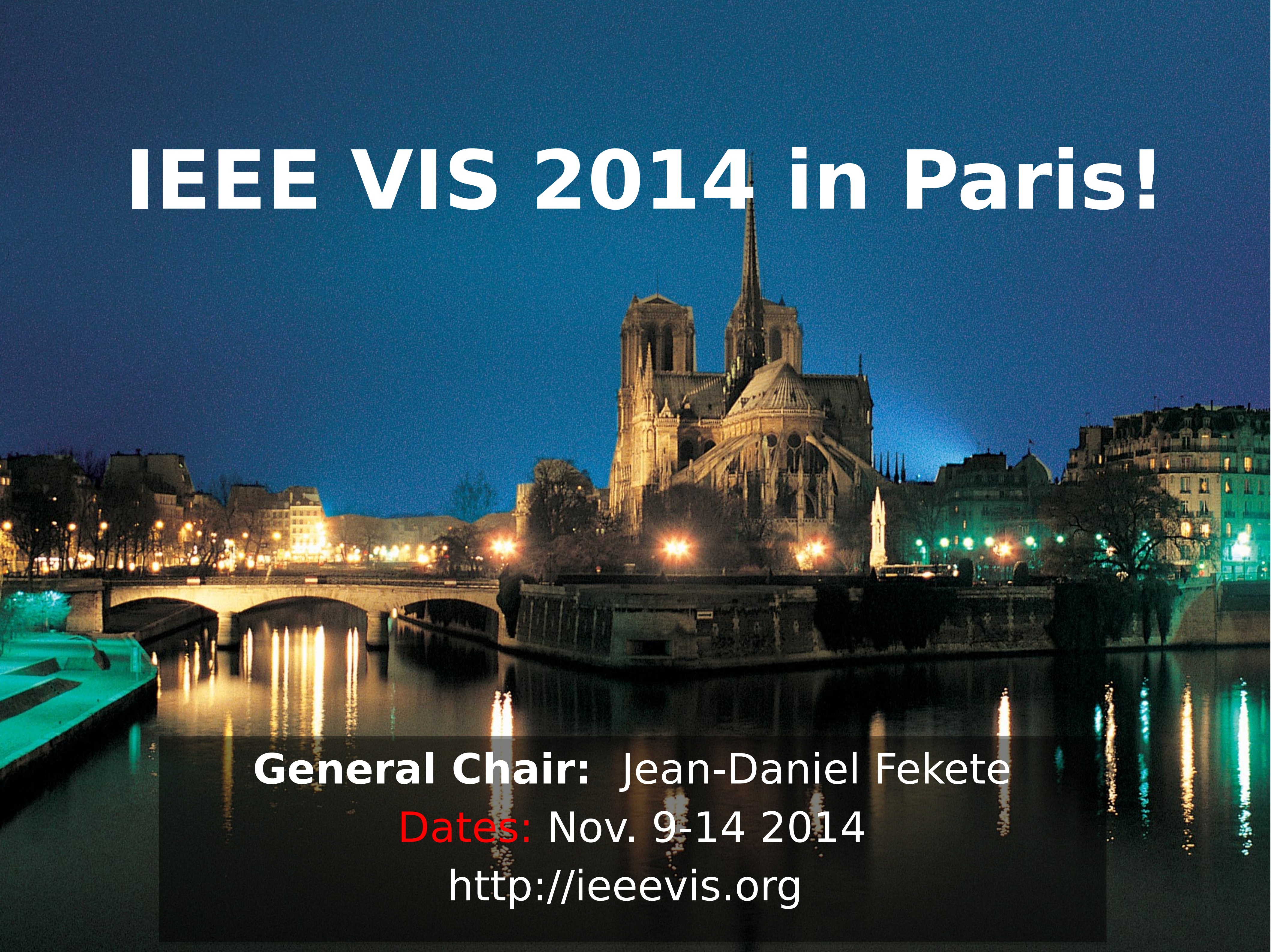




informatics mathematics

inria

IEEE VIS 2014 in Paris!



General Chair: Jean-Daniel Fekete

Dates: Nov. 9-14 2014

http://ieeevis.org

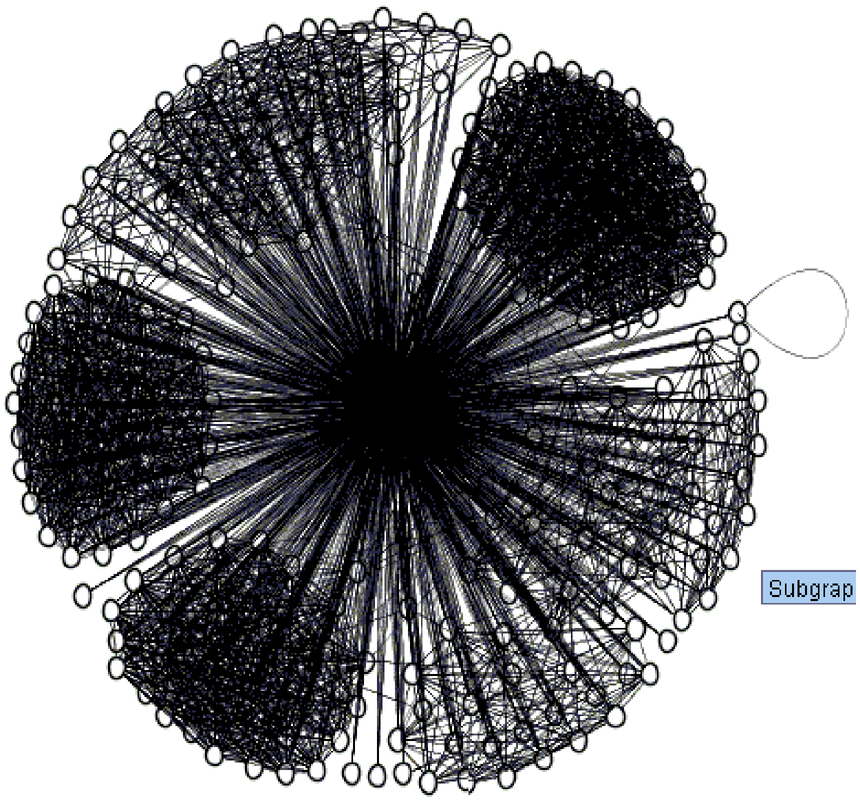
Outline

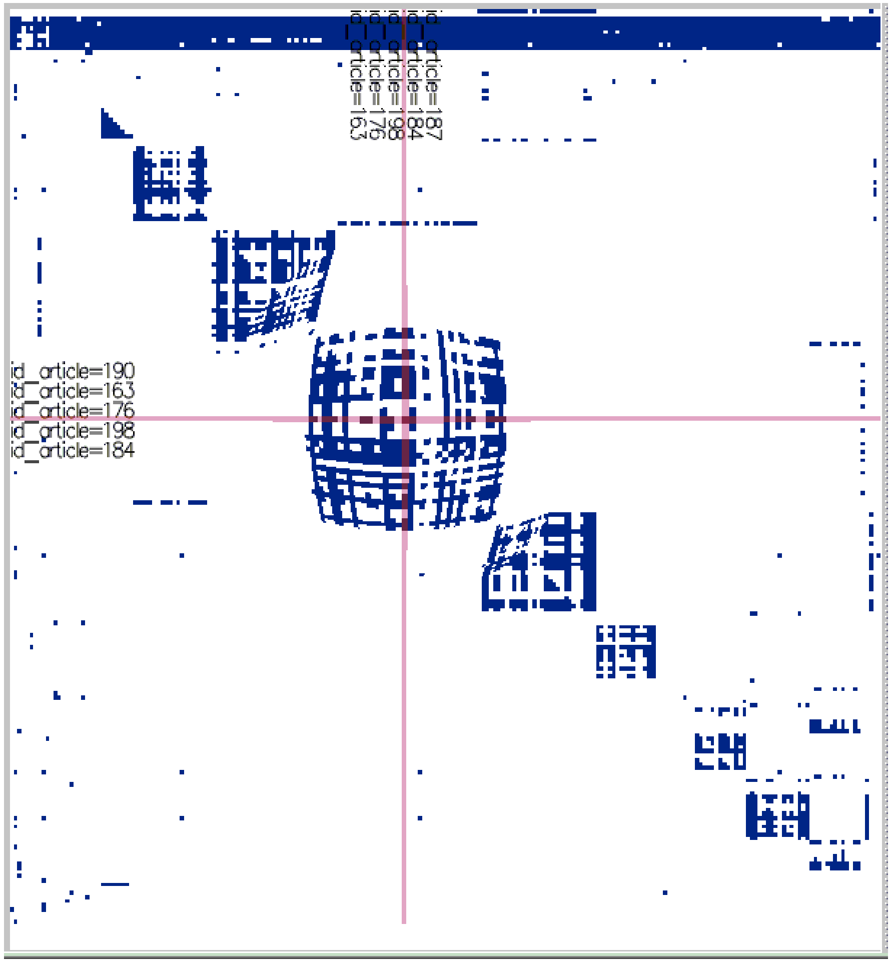
- Background
- Recent Matrix-Based Visualizations
- The Ordering Problem
- Families of Ordering Methods
- Interesting Results
- Assessing Ordering Algorithms Quality
- Challenges

•Background

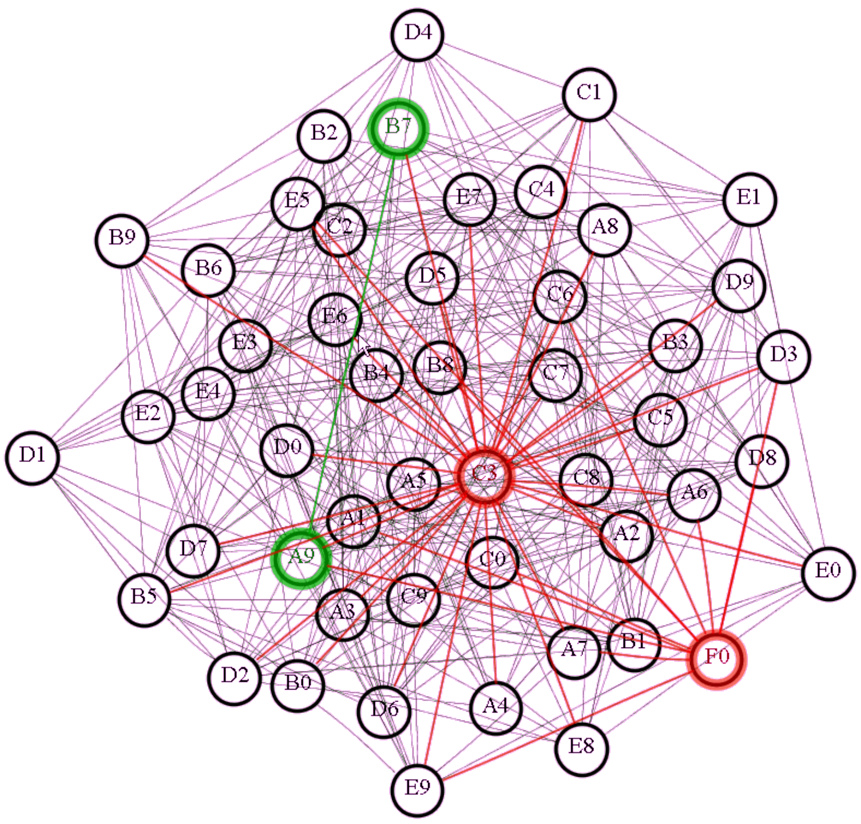
- Motivations
- Early experiments

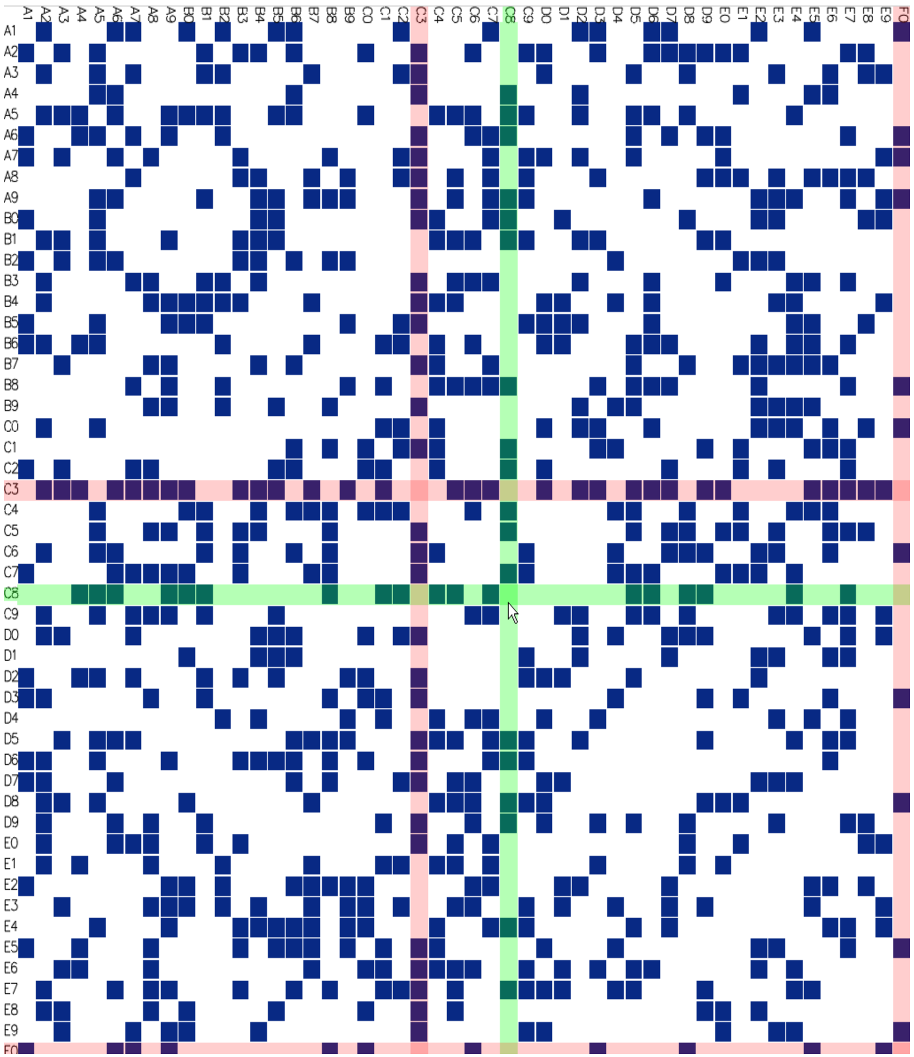
Web Site Example





Readability Experiment





Controlled Experiment:
Node Link Diagrams vs. Adjacency Matrices

The Tasks:

- Tasks related to the overview
  1. Estimate number of vertices
  2. Estimate number of edges
- Tasks related to graph elements
  3. Finding the most connected vertex (a central actor, a pivot, a hub)
  4. Finding a vertex by name
  5. Finding an edge connecting two vertices
  6. Finding a common neighbor
  7. Finding a path
- Random graphs (3 sizes et 3 densities)
- 2 representations: Node-Link + Matrix

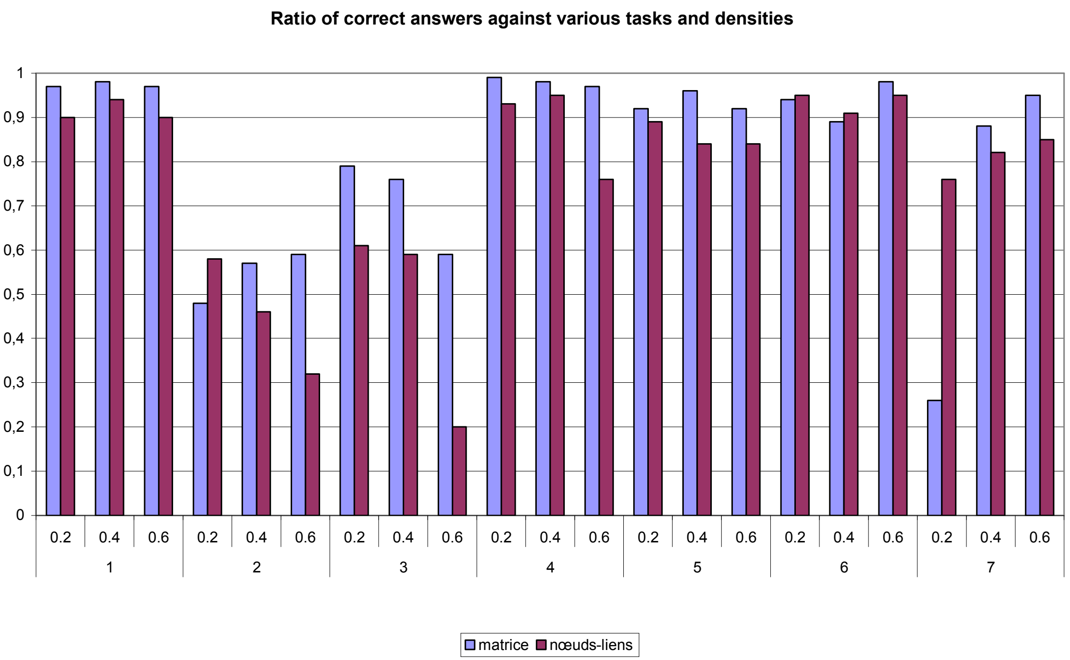
Results:

- Node-link diagrams are preferable for small sparse graphs (20 vertices)

Matrices are more readable *wrt* dense graphs and medium/large graphs (> 20 vertices) *wrt* the selected tasks, except path finding

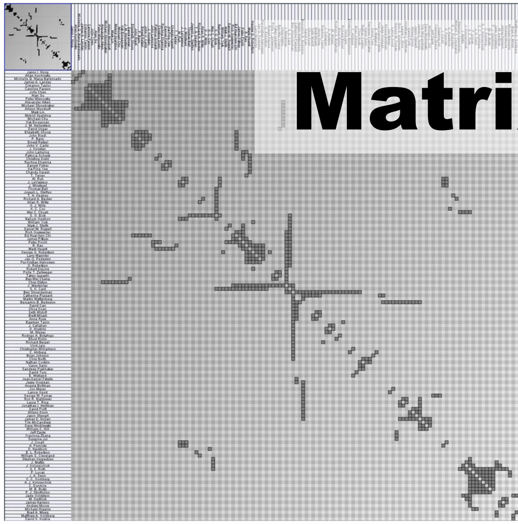
References:

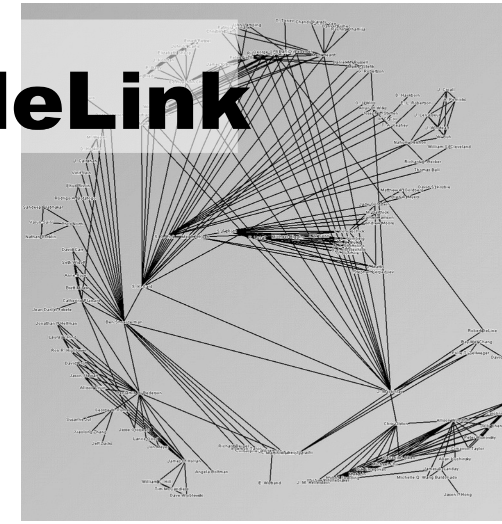
Mohammad Ghoniem, Jean-Daniel Fekete and Philippe Castagliola *Readability of Graphs Using Node-Link and Matrix-Based Representations: Controlled Experiment and Statistical Analysis*, Information Visualization Journal, 4(2), Palgrave Macmillan, Summer 2005, pp. 114-135.



% of correct answers for the 7 tasks, 3 densities and 2 representations (Node-Link in blue, Matrix in purple)

Matrix vs. NodeLink





+

Matrix:

- Usable without reordering
- No node overlapping
  No edge crossing
  ➔ Readable for dense graphs
- Fast navigation
- Fast manipulation
  ➔ Usable interactively
- More readable for some tasks

NodeLink:

- Familiar
- Compact
- More readable for path following
- More effective for small graphs
- More effective for sparse graphs

–

Matrix:

- Less familiar
- Use more space
- Weak for path following tasks

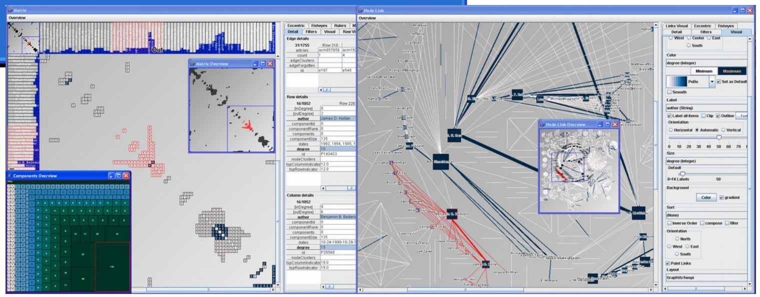
NodeLink:

- Useless without layout
- Node overlapping
  Edge crossing
  ➔ Not readable for dense graphs
- Manipulation requires layout computation

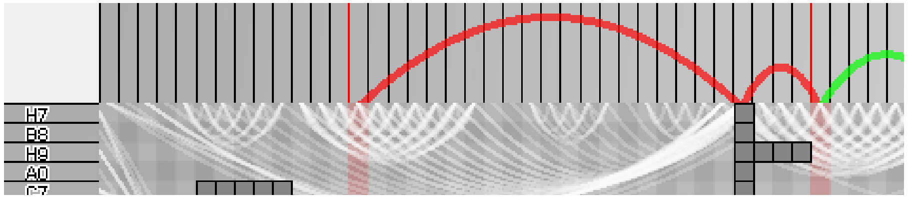
Social Network Visualization: Improving Matrices

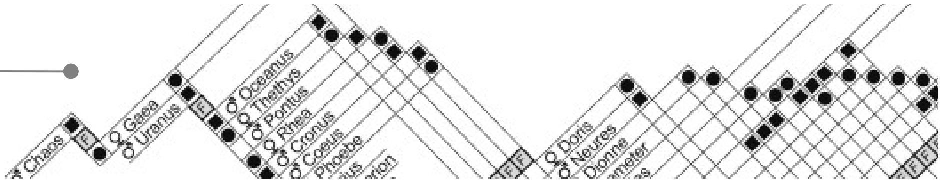
Several representations:

1. **Combined**
   - MatrixExplorer
     (Henry&Fekete InfoVis'06)

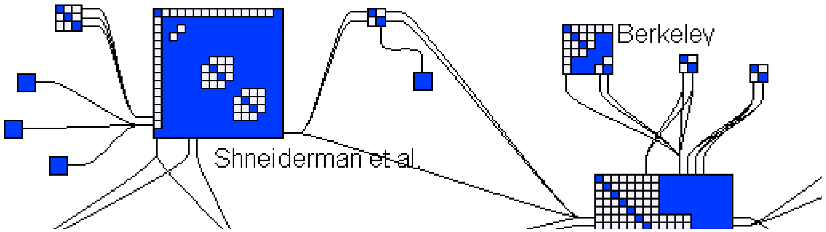


2. **Augmented**
   - MatLink
     (Henry&Fekete Interact'07, **Best Paper**)
   - GeneaQuilts
     (Bezerianos et al. InfoVis'10)

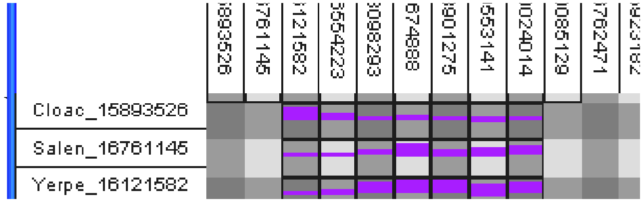




3. **Hybrid**
   - NodeTrix
     (Henry et al. InfoVis'07)
   - CoCoNutTrix
     (Isenberg et al. CG&A'09)



4. **Multiscale**
   - ZAME
     (Elmqvist et al. PacificVis'08)

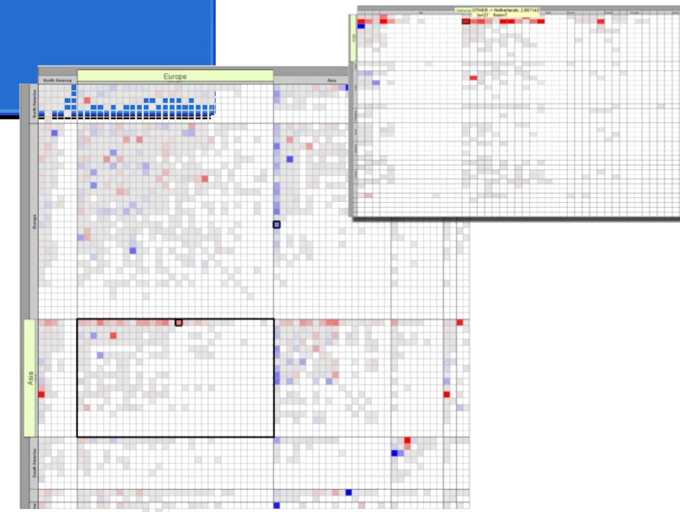


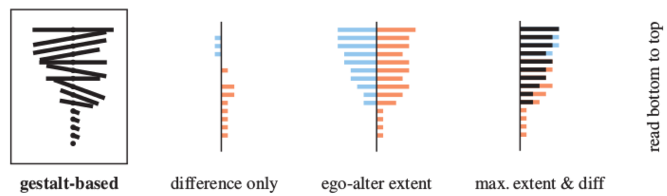
NodeTrix

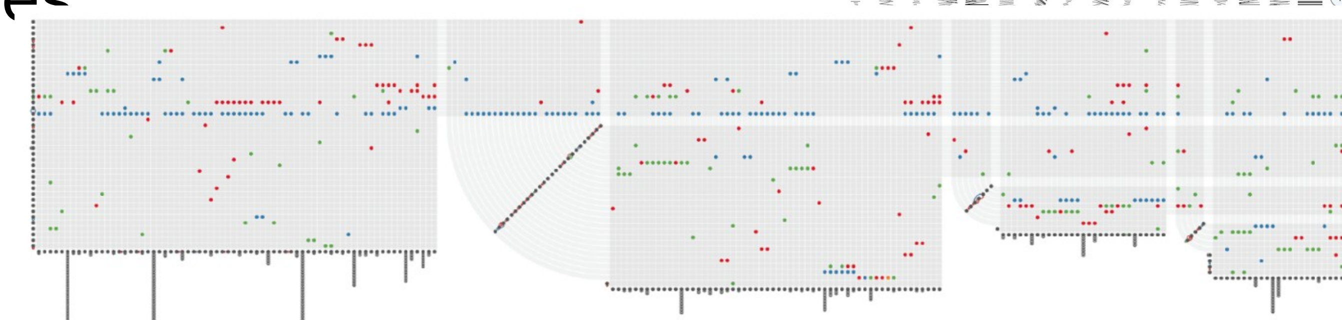
Hybrid Visualization for Analyzing Social Networks

Recent Matrix-Based Visualizations

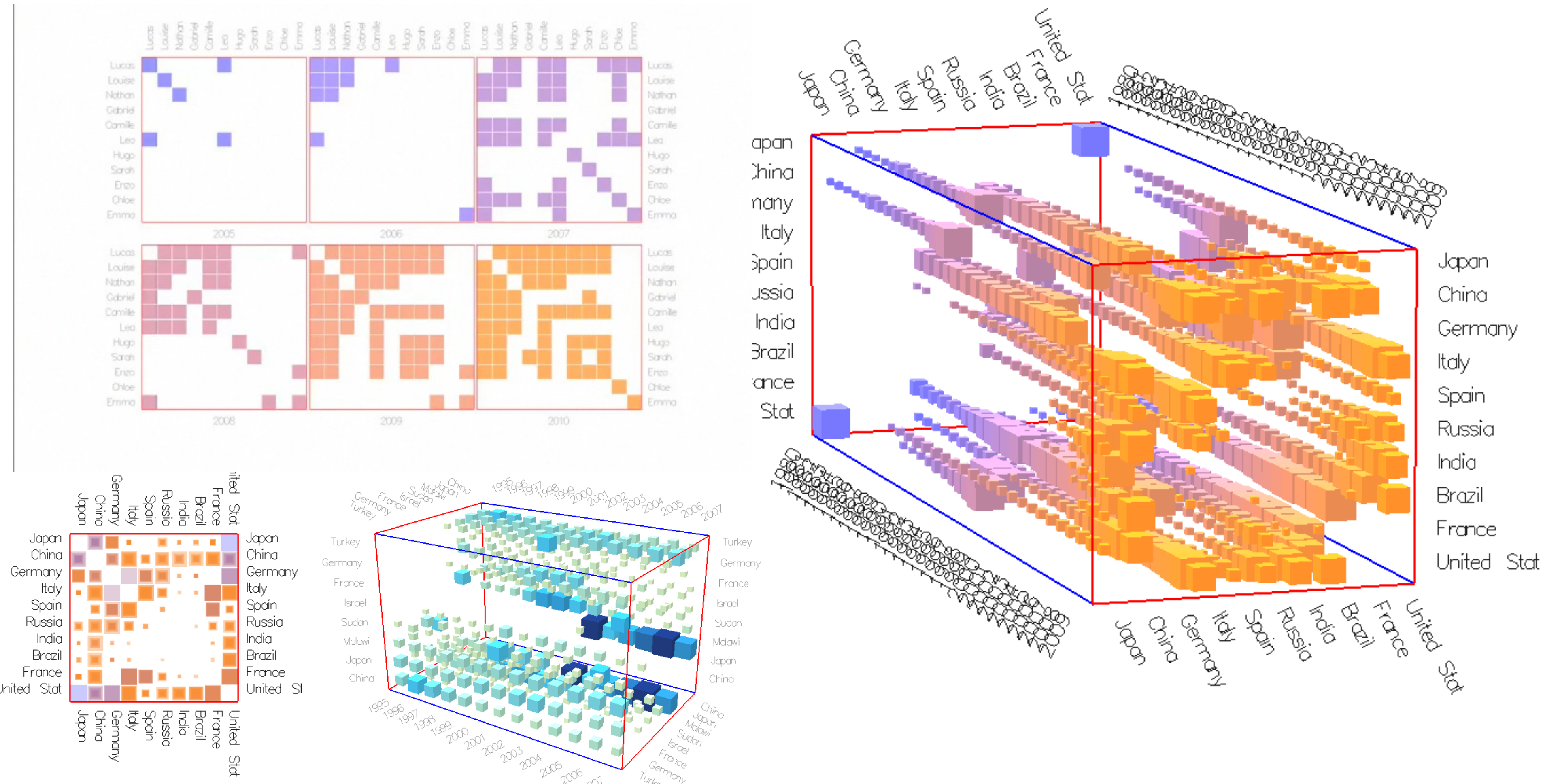
- Van Ham et al. 2004-2005 have shown techniques to navigate in large matrices
- Brandes&Nick 2011 h visualized temporal networks (friendship evolution)
- Dinkla et al. 2012 have introduced Compressed Adjacency Matrices



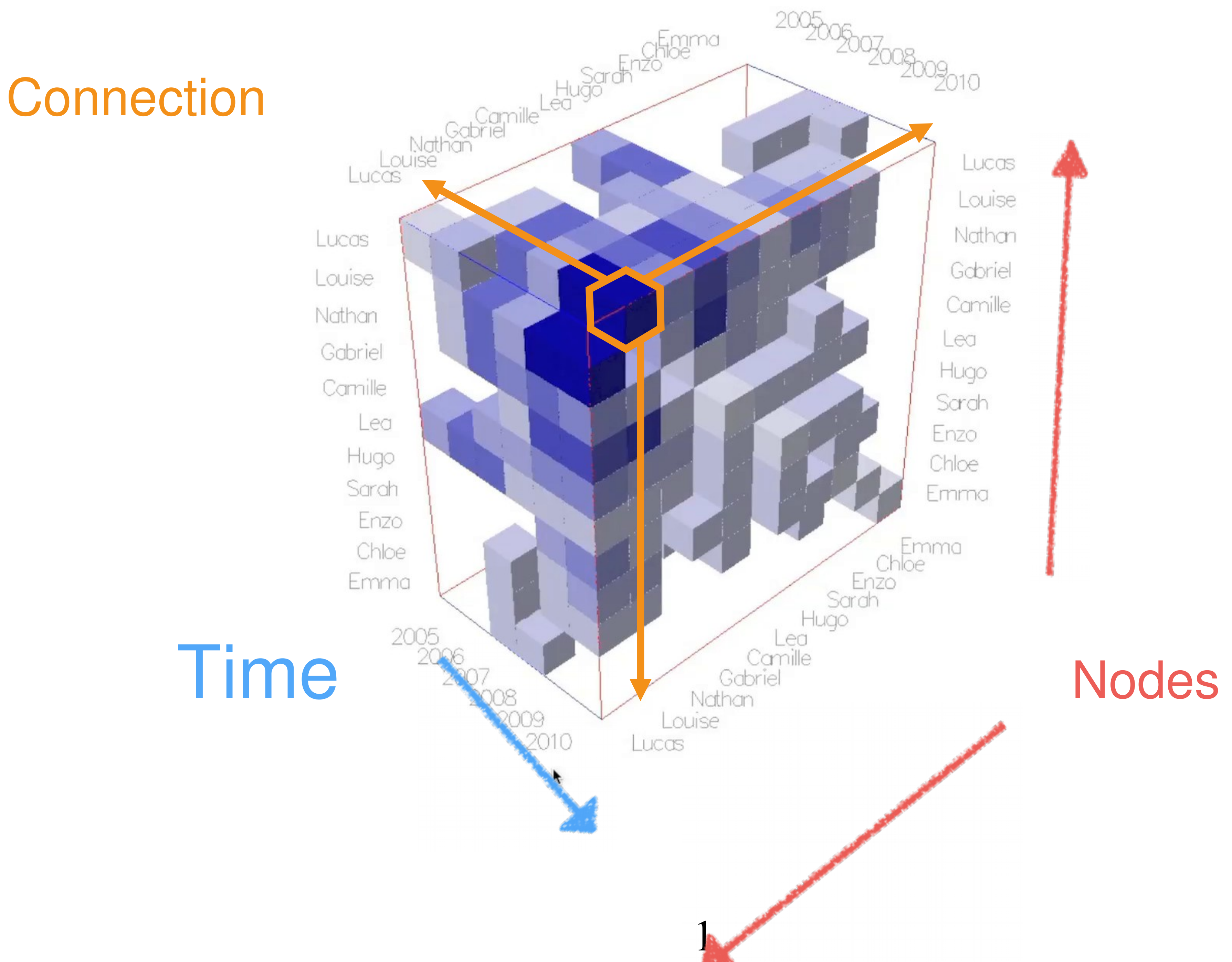




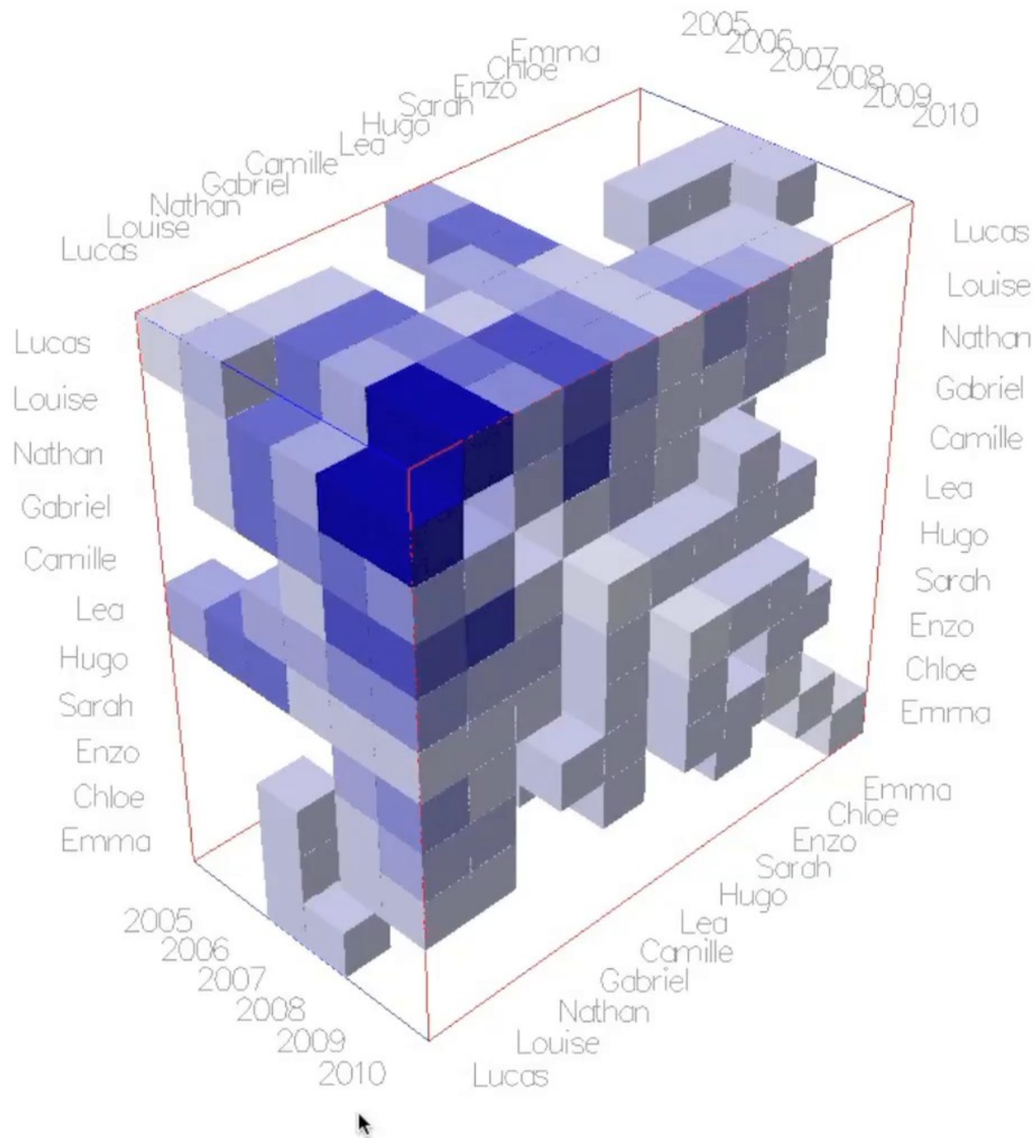
Cubix: Visualizating Dense Dynamic Networks



Benjamin Bach, Emmanuel Pietriga, Jean-Daniel Fekete. Visualizing Dynamic Networks with Matrix Cubes. Proceedings of the 2014 Annual Conference on Human Factors in Computing Systems (CHI 2014), Apr 2014,



Connection
Time
Nodes
Lucas
Louise
Nathan
Gabriel
Camille
Lea
Hugo
Sarah
Enzo
Chloe
Emma
2005
2006
2007
2008
2009
2010



Ceci n'est pas une visualisation 3D

Visualizing Dynamic Networks
with Matrix Cubes

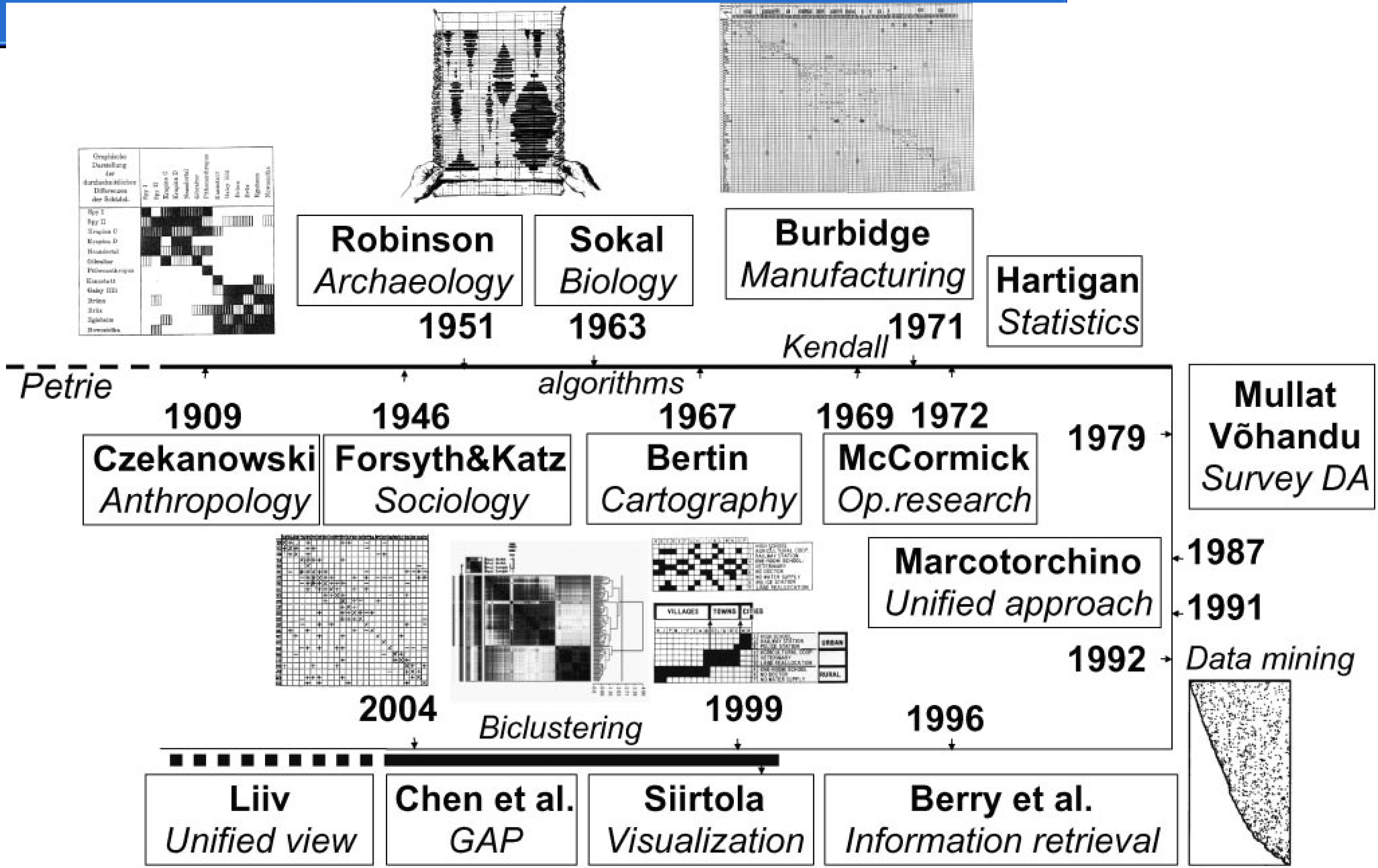
submitted to
CHI2014

Seriation and Matrix Ordering

- Excellent historical perspective by Innar Liiv
  - Seriation and Matrix Reordering Methods: An Historical Overview
    Statistical analysis and data mining, 2010 – Wiley

- First, let's look at ordering tables in general
- See how it applies to adjacency matrices

Liiv: visual abstract of the history of seriation from different disciplines



The Ordering Problem

- Seriation instead of clustering
  - Findind a linear order for rows and columns
  - Postpone the decision of separting into clusters
  - Avoid creating clusters when they don't make sense

Naïve approach:

- Define an objective function (e.g. favor diagonal placement and dense blocks)
- Try all permutations and keep the best wrt the function
- Problem: for a $n \times m$ table, there are $n! \times m!$ permutations
- Problem 2: there is no consensual objective function

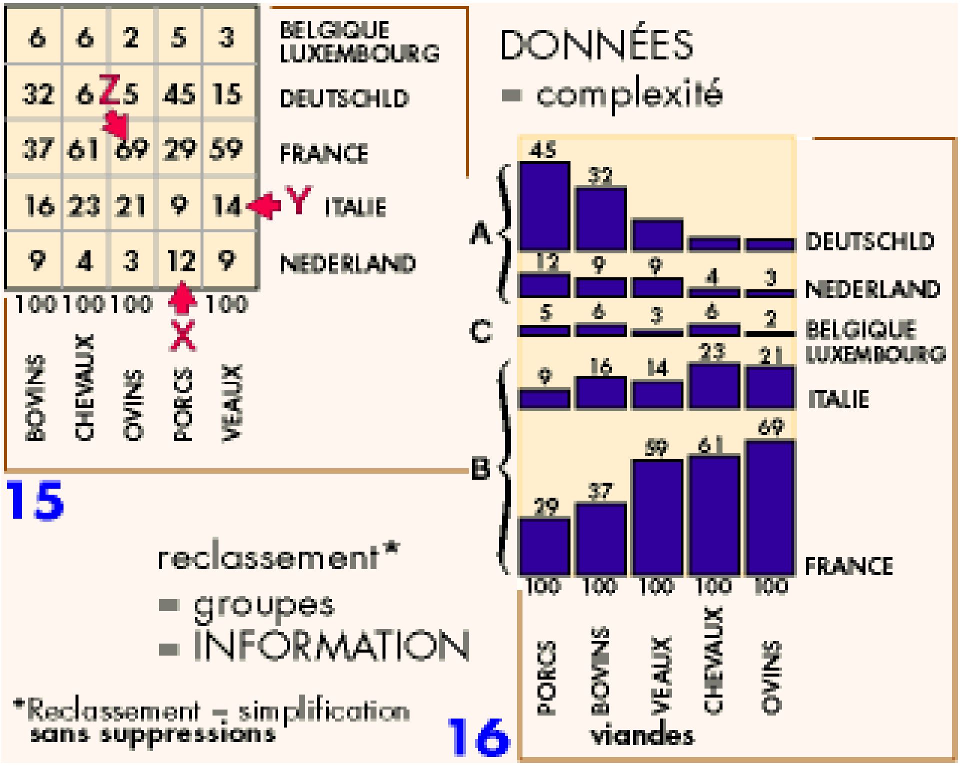
The Reorderable Matrix

Introduced by Bertin 67 as a representation for relational data

- Table or Network
- The value table provides details
- The reordered table provides details AND overall structure in the same representation

Problems:

- how to compute a good ordering?
  - Row and column permutations
- how to assess its quality?

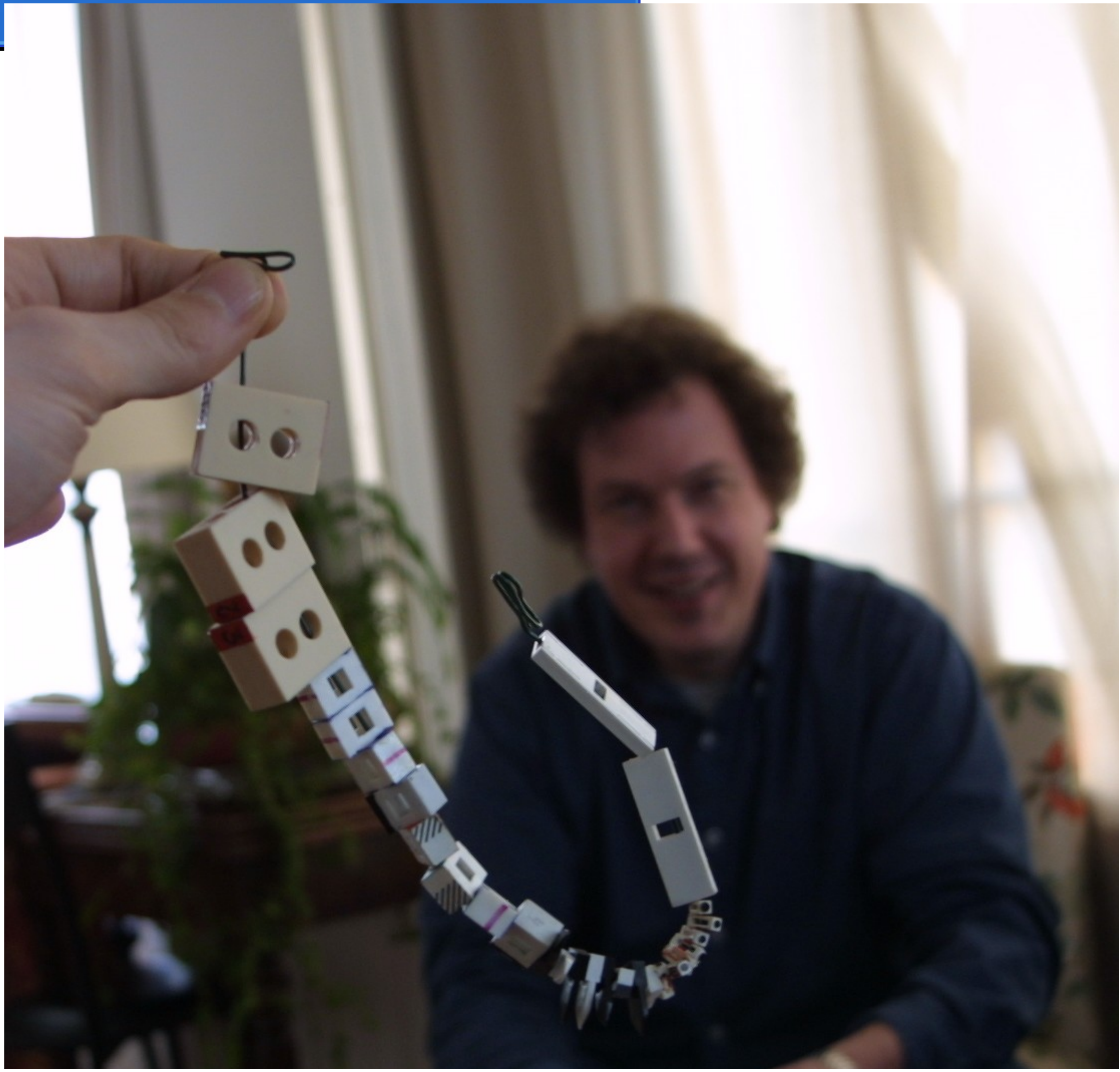


Interactive Reordering

- Manual ordering (Bertin’s Domino)
- TableLens (Rao & Card 94)
- InfoZoom (Spenke et al. 96)
- Bertifier (Perin et al. 2014)

Hand Reordering with the Domino

- Free one dimension
- Reorder by visual similarity
- Reconnect
- Free the other dimension
- Reorder by visual similarity
- Reconnect
- Take a picture



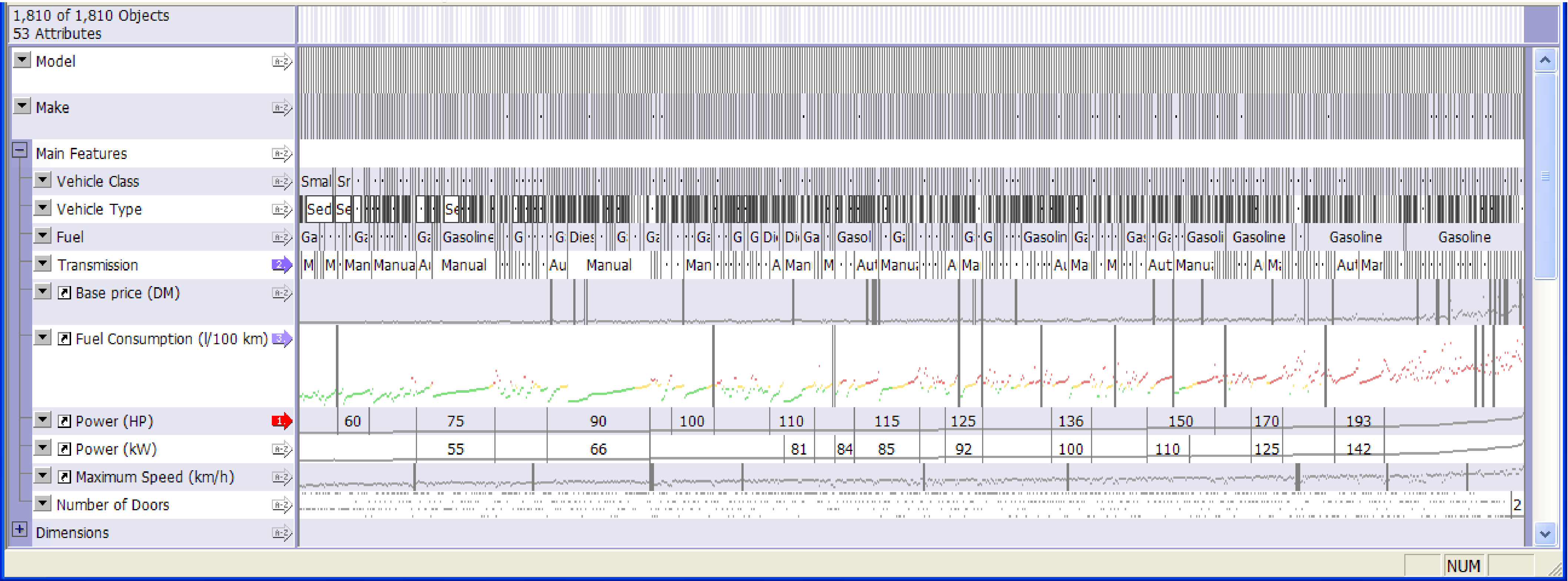
Large dominos

TableLens (Rao & Card 94)

- Interactively reorder based on one attribute
- See the correlation with others
- Fisheye on focus regions
- Only reorder columns, not rows

InfoZoom (Spenke et al 96)

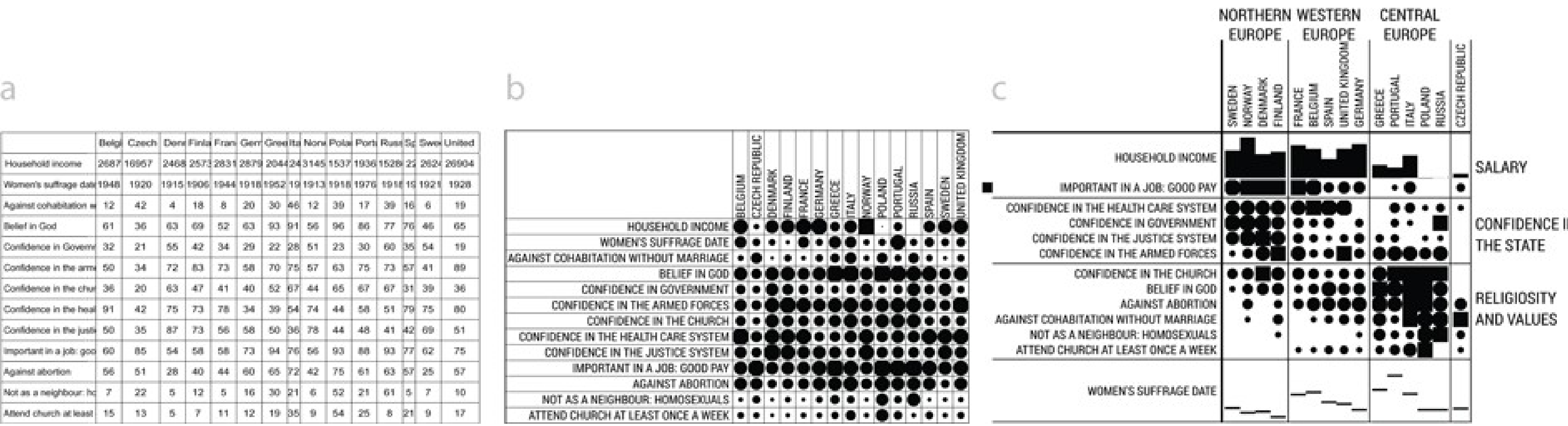
- Interactively reorder based on one attribute
- See the correlation with others
- Collapse identical values
- Only reorder attributes on rows, not columns



Bertifier

- Charles Perin, Pierre Dragicevic, Jean-Daniel Fekete (2014). Revisiting Bertin's Matrices: New Interactions for Crafting Tabular Visualizations. VIS' 2014.

- User-assisted reordering



Families of Ordering Methods

Seven families of methods:

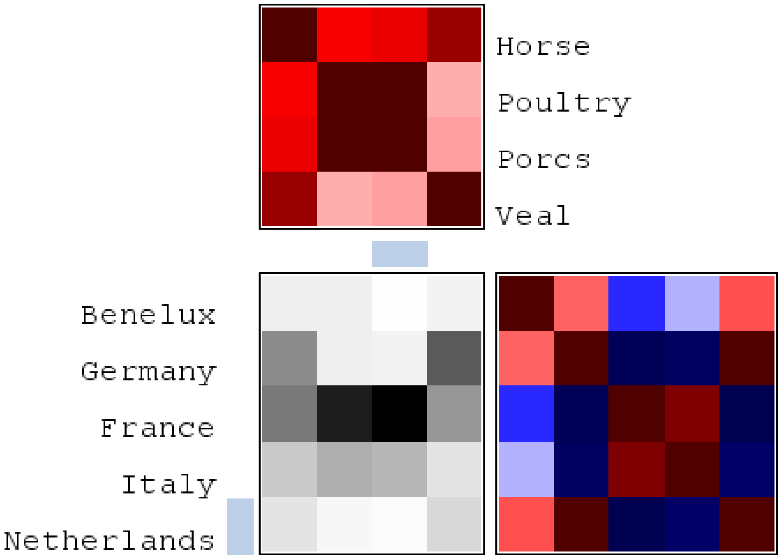
- Robinsonian (statistics)
- Spectral
- Dimension reduction
- Heuristics
- TSP-based
- Bi-clustering
- Graph layout

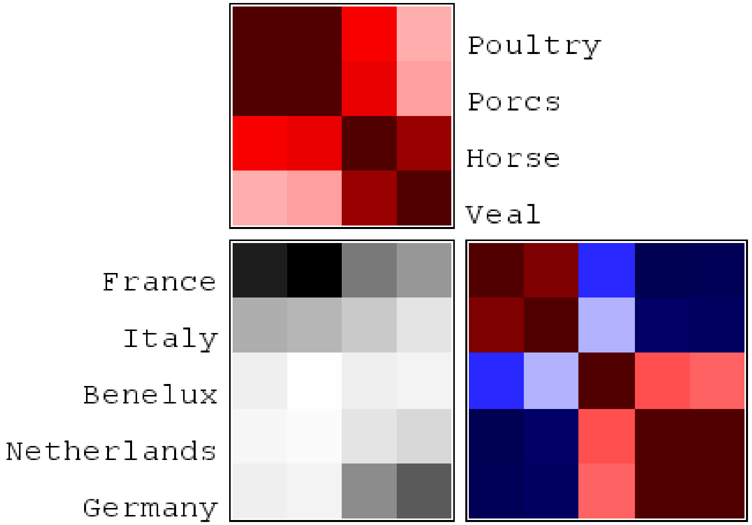
Reordering Tables: Robinsonian

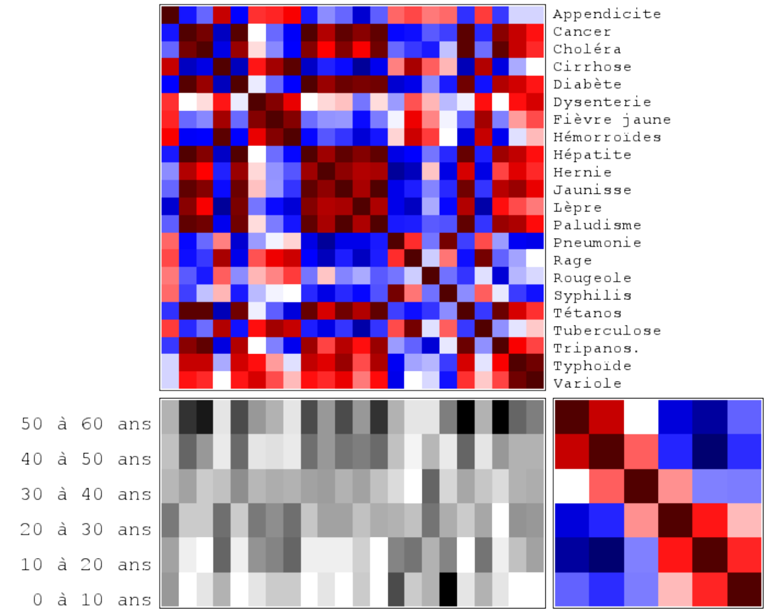
- Goal: Reorder the matrix so that similar rows (resp. columns) are close and dissimilar rows (resp. columns) are farther away

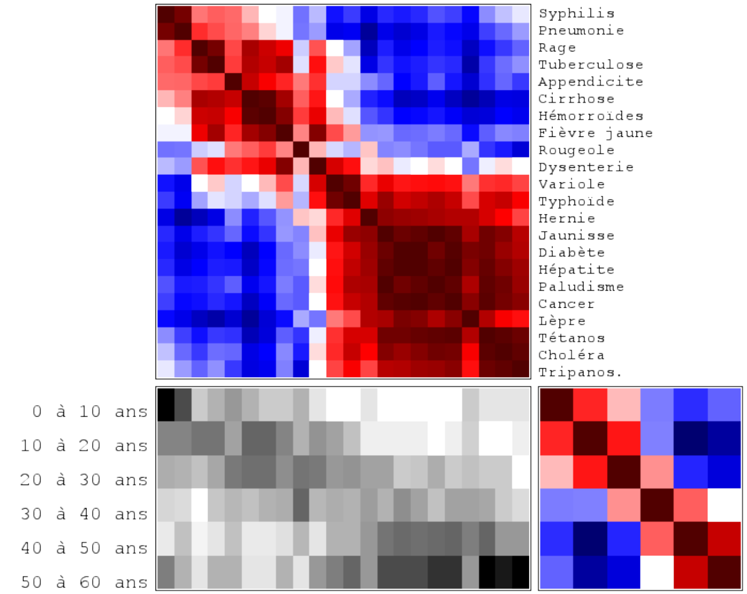
- Load numeric table
- Condition row/columns if necessary
- Choose a distance/dissimilarity for rows & columns
- Compute the distance matrix for rows & for columns
- Compute the row (resp. column) permutation to get a Robinsonian distance matrix
- Apply the permutation to the table row (resp. column)

GAP:http://gap.stat.sinica.edu.tw/Software









Reordering Tables: Conditioning

- Standard operation on tables
- Some tables need conditioning
  - For example, the CIA World Fact Book
    - Area (sq km)
    - Birth rate(births/1000 population)
    - Death rate (deaths/1000 population)
- Values vary widely in range
- Conditioning make them comparable:
  - e.g. center mean to 1 and variance to 1
- Other tables don’t need it
  - For example student grade per class (all between 0 and 5 or 20)

Reordering Tables: Distance / dissimilarity

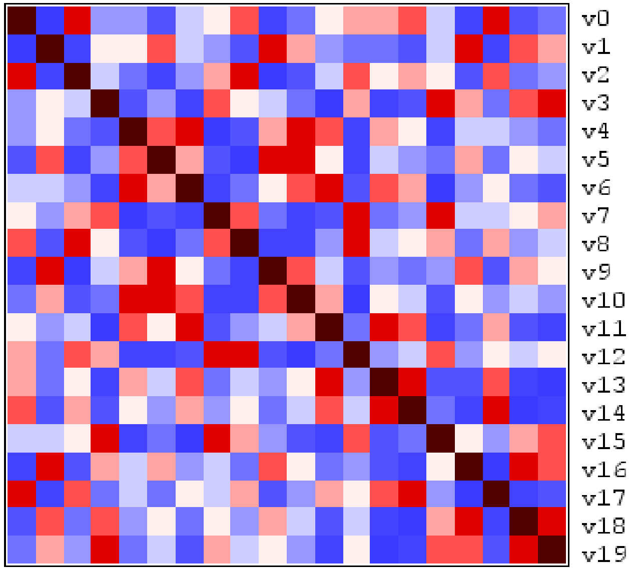
- Compute row/row and column/column distance
- Several popular distances between row x and row y
- L0 : $\sum |x_i - y_i|$
- L1 : $\sqrt{\sum (x_i - y_i)^2}$
- L∞: $\max |x_i - y_i|$
- Pearson’s correlation

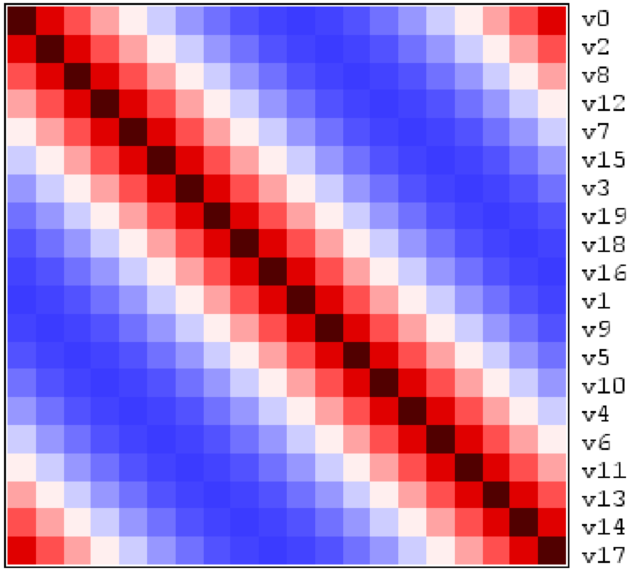
$$\frac{\sum_k (d_{ik} - \bar{d}_{i\cdot})(d_{jk} - \bar{d}_{j\cdot})}{\sqrt{\sum_k (d_{ik} - \bar{d}_{i\cdot})^2}\sqrt{\sum_k (d_{jk} - \bar{d}_{j\cdot})^2}},$$

- Complexity: $\max(n^2, m^2)$

Reordering Tables: Robinsonian Matrix

- The distance matrix is positive and symmetric
- In a perfect world, we would like to find a consistent similarity matrix:
- maximum on the diagonal and decreases off the diagonal
- This is called a Robinsonian Matrix (R-matrix)
- A pre-R matrix is a similarity matrix that can be permuted to become an R-matrix
- If that permutation exists, we want to find it
- Otherwise, we want a good approximation





Reordering Tables: R-matrix computation

- Known methods to solve the simple case:

Chun-Houh Chen uses two methods
  - Hierarchical clustering + seriation
    Several options for clustering
  - His own Ellipse seriation technique

Atkins et al. 98 describe a “Spectral” method that solves the problem when the R-matrix exists

Results

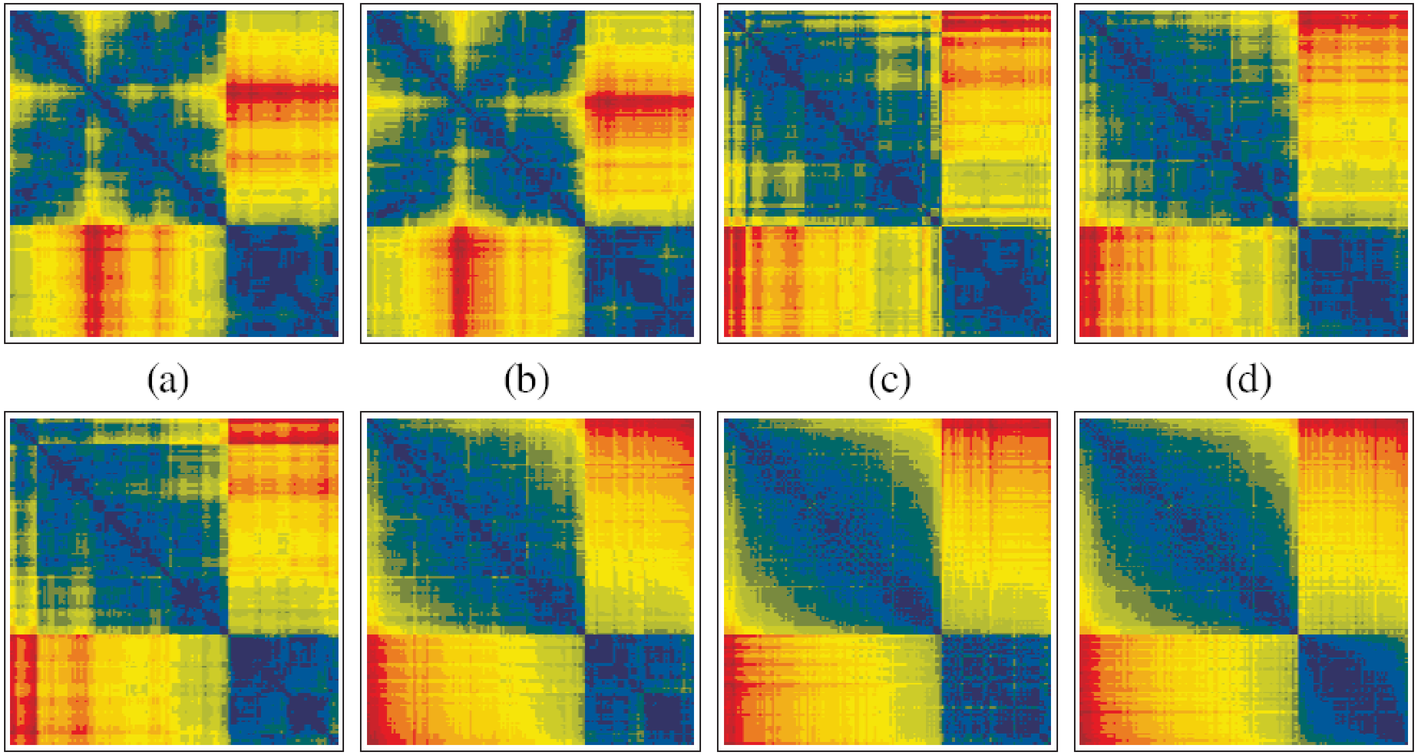
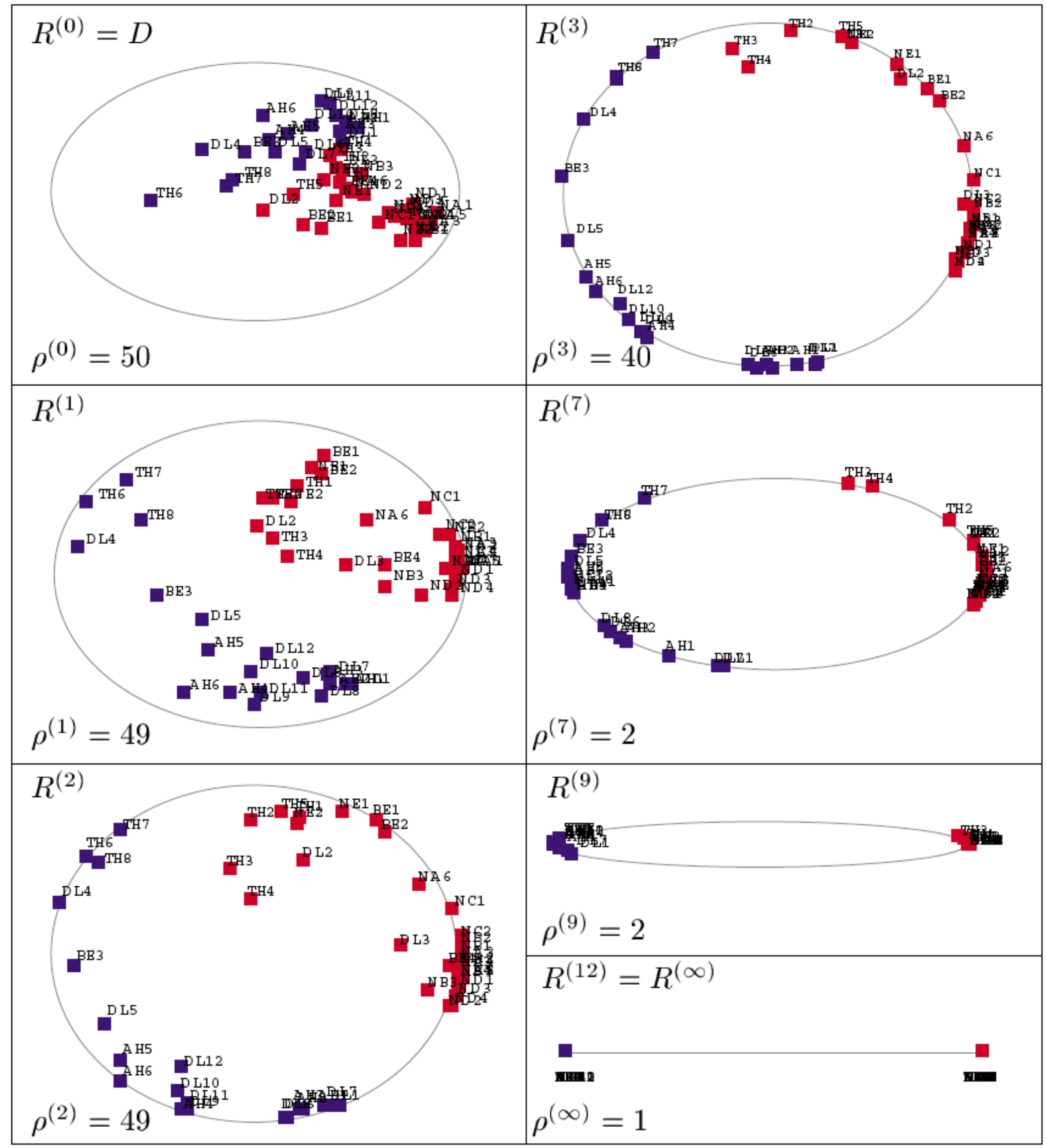

(a) (b) (c) (d)
(e) (f) (g) (h)

Figure 4: Permuted Euclidean distance maps for Iris data with eight seriation algorithms: (a) farthest insertion spanning; (b) nearest insertion spanning; (c) single linkage tree; (d) complete linkage tree; (e) average linkage tree; (f) GAP rank-one tree; (g) GAP rank-two ellipse; (h) GAP double ellipse.

Ellipse seriation

- Method based on computing the correlation matrix D, the distance matrix using Pearson's correlation coefficient
- Iteratively, a second correlation matrix is computed from that
- $\varphi(A)=D$, $R^1= \varphi(D)$, $R^{n+1}= \varphi(R^n)$
- With no theoretical reason, when looking at the projection of the columns of R^n onto the plane of the 2 main eigenvectors, we have an ellipse when $n\rightarrow\infty$
- Unrolling the ellipse, we have a good approximation of an R-matrix



A SPECTRAL ALGORITHM FOR SERIATION AND THE CONSECUTIVE ONES PROBLEM

JONATHAN E. ATKINS, ERIK G. BOMAN, AND BRUCE HENDRICKSON (SIAM 98)

- A distance matrix $A_{i=1..n,j=1..n}=[a_{i,j}]$
- A is positive and symmetric: $a_{i,j} \geq 0$ and $a_{i,j} = a_{j,i}$
- Computing the permutation π to transform a pre-R matrix into A
- BEGIN MAGIC
- π can be computed from the eigenvector x with the smallest non-null eigenvalue of the Laplacian of A: it is the permutation that orders the components of *x*.
- END MAGIC
- An Eigenvector *x* of a matrix square matrix M of size *n* is a vector
- $x=[x_1, x_2, ..., x_n]^T$ such that
- $Mx = \lambda x$ where λ is a real number called the Eigenvalue of *x*
- M has *n* eigenvalues/eigenvectors that can be sorted by eigenvalue
- The Laplacian of A, $L_A = D_A - A$ where $D_A=[d_{i,i}]$, $d_{i,i} = \sum_{i=1..n} a_{i,j}$
- Computing this Eigenvector is sub-quadratic in practice using Power-Iterations (only one should be computed) or improvements

Reordering Table: TSP

- We can compute a seriation by solving the Traveling Salesman Problem (TSP).
- TSP is NP complete but heuristics running in polynomial time exist (Lin&Kernighan 73) and free programs are available:
- LKH http://www.akira.ruc.dk/~keld/research/LKH/
- Concorde http://www.tsp.gatech.edu/concorde.html
- TSP has been used for matrix seriation
- 1 page article (Lenstra 74) “It performs well…”
- (Climer&Zhang 04) applied it to microarray data
- (Henry&Fekete 06) applied it to tables and graphs

Hierarchical clustering + seriation

- Method popular in Bioinformatics: Eisen et al. 98

1. Hierarchical clustering from distance matrix
2. Linear order constrained by the binary tree

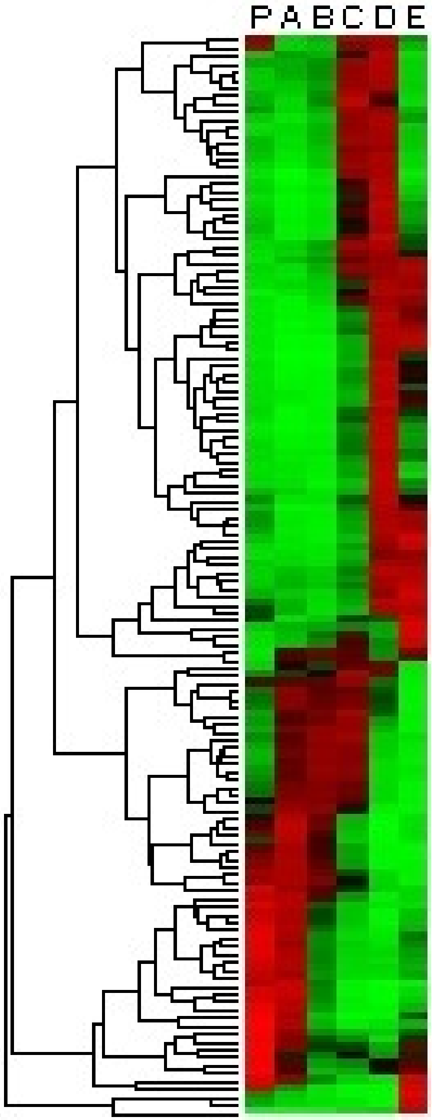
For *n* leaves, 2^{n-1} linear orderings possible

We want an order that minimizes the sum of distances between consecutive items (TSP)

(Bar-Joseph et al. 02) describe a $O(n^3)$ algorithm

(Brandes 07) describes a $O(n^2 \log(n))$ algorithm

(Deĭneko&Tiskin 07) describe a complex $O(n^2)$ algorithm



Reordering Tables: Dimension reduction

- (Chauchat&Risson 95) use 2 dimension reduction techniques to reorder matrices:

1. Principal Component Analysis (PCA)
2. Correspondence Analysis (CA)

- . Their methods are available in the AMADO system

- . Both method assume linear relationships between rows and columns

Correspondence Analysis

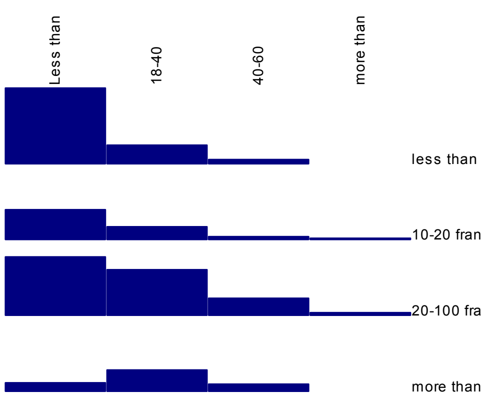
- Designed by (Benzécri 74) for contingency tables
- Compute vectors as difference from the expected values
- Then, same as PCA except compute Eigenvector ONCE
- Very good results for contingency tables!

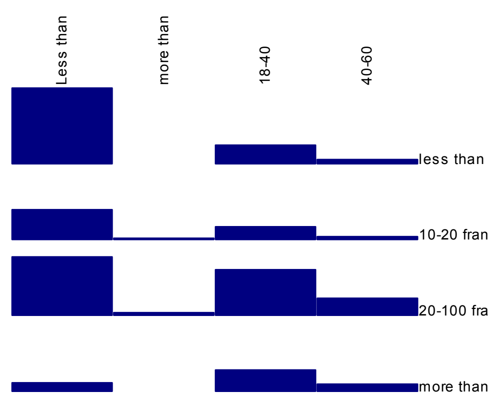
Thefts Value/Age	Less than 18	18-40	40-60	more than 60	Row total	Row percent
less than 10 francs	113	27	5	0	145	32,01%
10 ˆ 20 francs	44	19	4	1	68	15,01%
20 ˆ 100 francs	89	68	26	3	186	41,06%
more than 100 francs	12	32	10	0	54	11,92%
Column total	258	146	45	4	453	100,00%
Column percent	56,95%	32,23%	9,93%	0,88%	100,00 %	

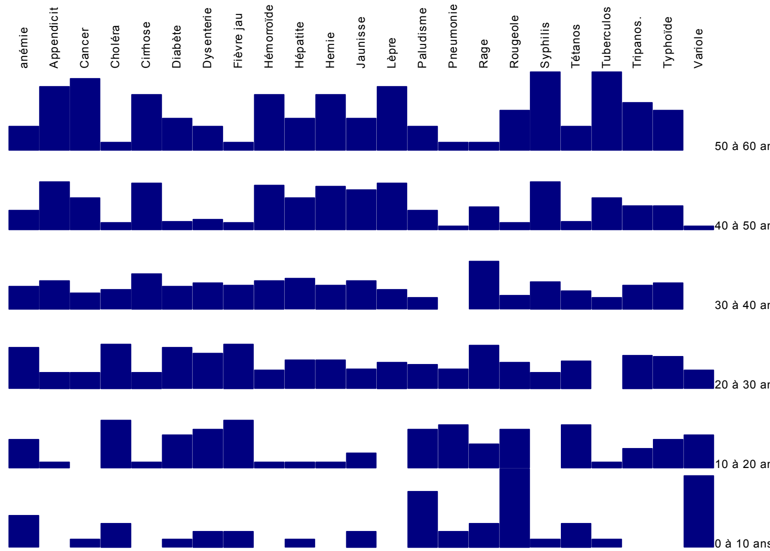
Expected percentages	18,23%	10,32%	3,18%	0,28%		32,01%
Col%*Row%	8,55%	4,84%	1,49%	0,13%		15,01%
	23,38%	13,23%	4,08%	0,36%		41,06%
	6,79%	3,84%	1,18%	0,11%		11,92%
Column percent	56,95%	32,23%	9,93%	0,88%		

Expected value	82,582781	46,732892	14,403974	1,2803532
Expected%*total	38,728477	21,916115	6,7549669	0,6004415
	105,93377	59,94702	18,476821	1,6423841
	30,754967	17,403974	5,3642384	0,4768212

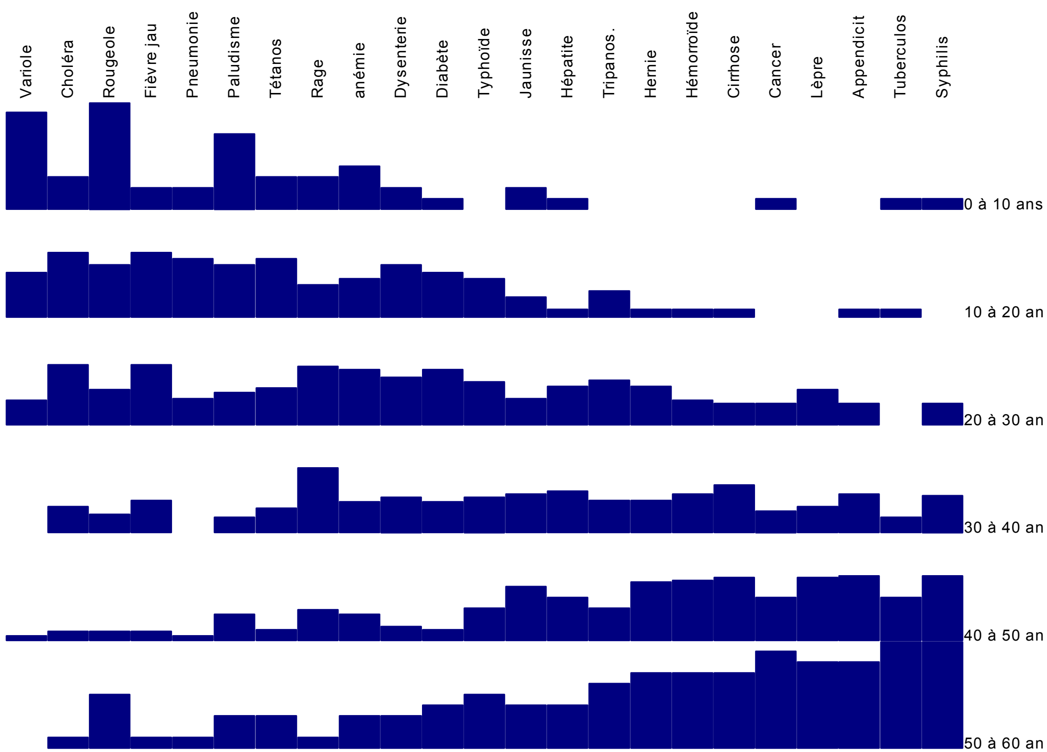
Difference in %	6,71%	-4,36%	-2,08%	-0,28%






anémie
Appendicit
Cancer
Choléra
Cirrhose
Diabète
Dysenterie
Fièvre jau
Hémorroïde
Hépatite
Hernie
Jaunisse
Lèpre
Paludisme
Pneumonie
Rage
Rougeole
Syphilis
Tétanos
Tuberculos
Tripanos.
Typhoïde
Variole
50 à 60 an
40 à 50 an
30 à 40 an
20 à 30 an
10 à 20 an
0 à 10 ans

PCA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Choléra
Rougeole
Fièvre jau
Pneumonie
Paludisme
Tétanos
Rage
anémie
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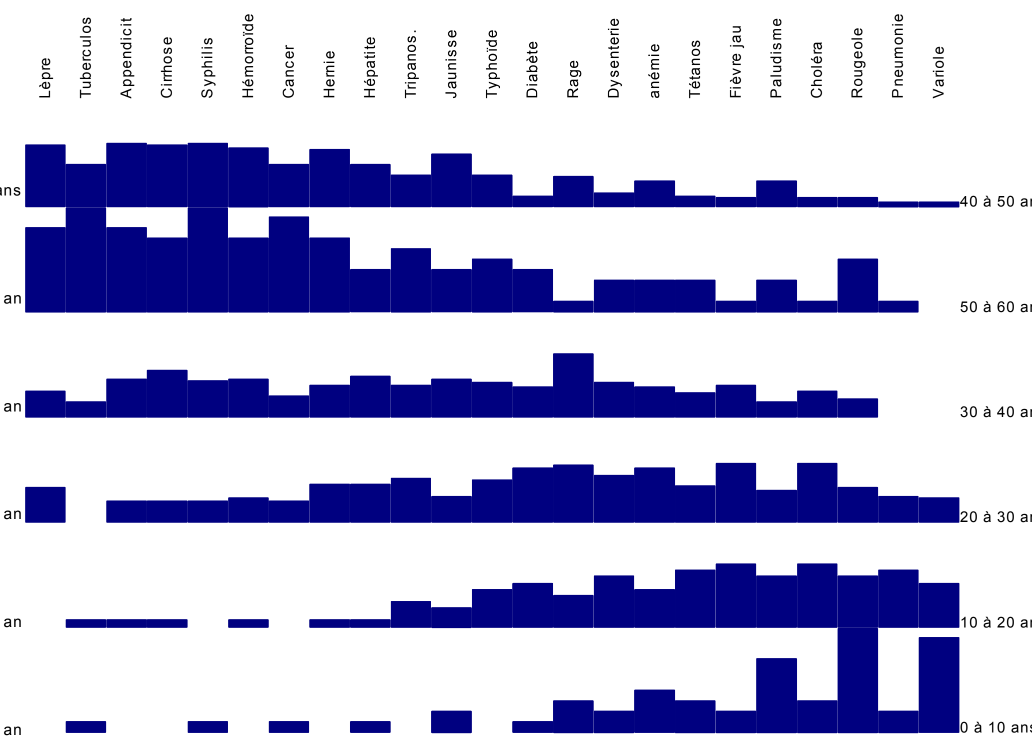
Diabète
Typhoïde
Jaunisse
Hépatite
Tripanos.
Hernie
Hémorroïde
Cirrhose
Cancer
Lèpre
Appendicit
Tuberculos
Syphilis
0 à 10 ans
10 à 20 an
20 à 30 an
30 à 40 an
40 à 50 an
50 à 60 an

CA


Lèpre
Tuberculos
Appendicit
Cirrhose
Syphilis
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Cancer
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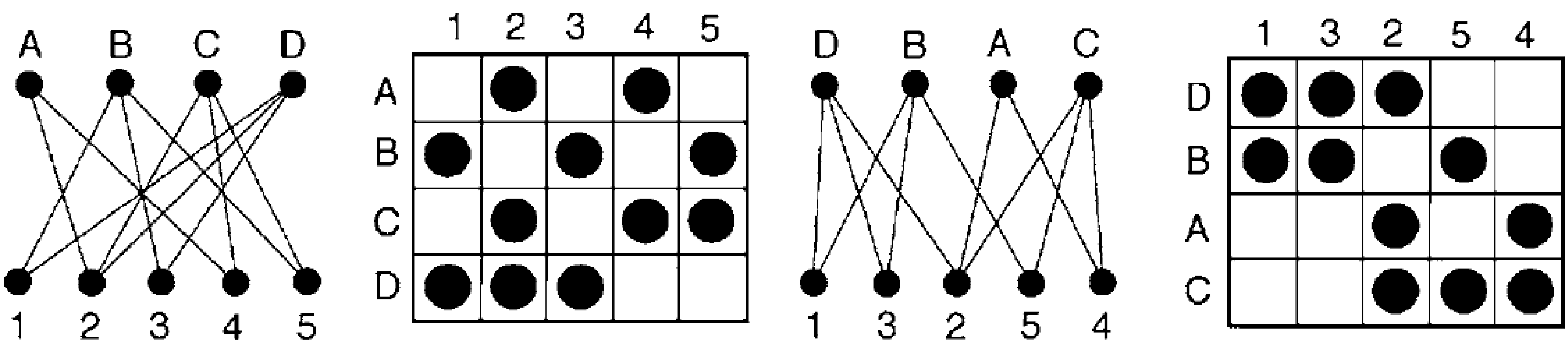
Diabète
Rage
Dysenterie
anémie
Tétanos
Fièvre jau
Paludisme
Choléra
Rougeole
Pneumonie
Variole
40 à 50 an
50 à 60 an
30 à 40 an
20 à 30 an
10 à 20 an
0 à 10 ans

Reordering Tables: Heuristics

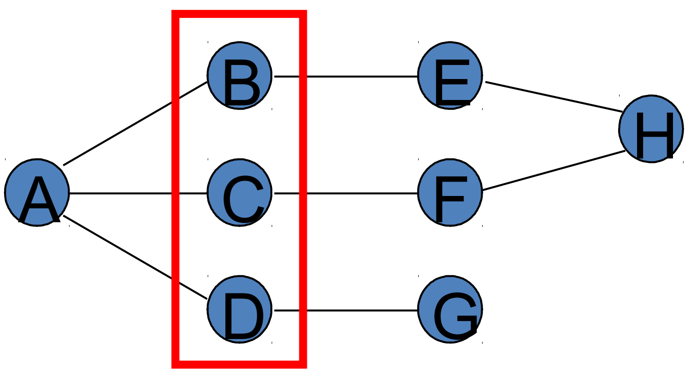
- (Siirtola&Mäkinen 05) Consider a Table as a bipartite graph
- Then use the Barycenter Heuristic to minimize the number of crossings
  - find the best permutation of rows and columns
- Barycenter Heuristic repeatedly reorder the vertices according to the average of their adjacent vertices in the opposite vertex set.
  - Same as Sugyiama



Reordering Graphs: Mixed approach

[Henry and Fekete, InfoVis'06]

- Place actors with similar connection patterns next to each other



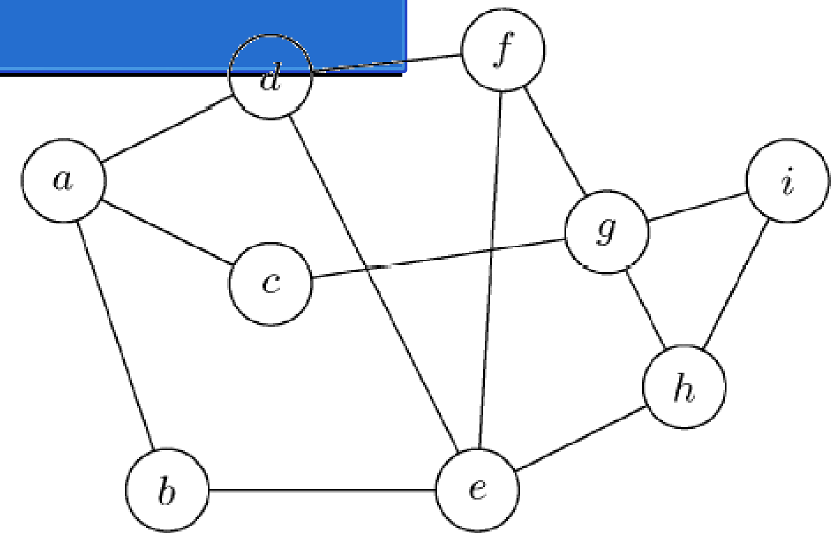
	A	B	C	D	E	F	G	H
A	0	1	1	1	0	0	0	0
B	1	0	0	0	1	0	0	0
C	1	0	0	0	0	1	0	0
D	1	0	0	0	0	0	1	0
E	0	1	0	0	0	0	0	1
F	0	0	1	0	0	0	0	1
G	0	0	0	1	0	0	0	0
H	0	0	0	0	1	1	0	0

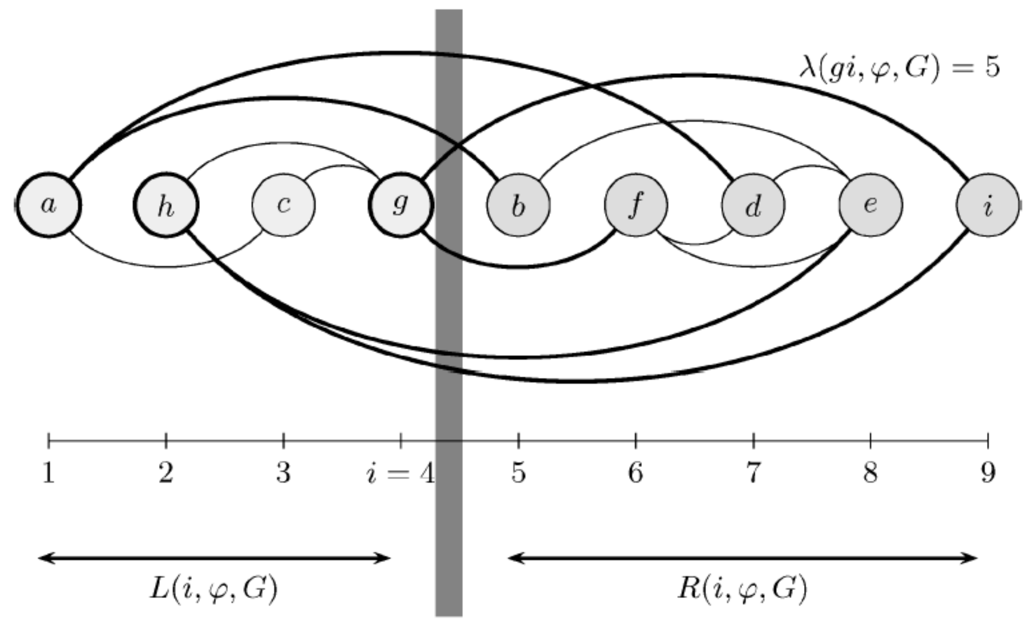
	A	B	C	D	E	F	G	H
A	0	1	1	1	2	2	2	3
B	1	0	2	2	1	3	3	2
C	1	2	0	2	3	1	3	2
D	1	2	2	0	3	3	1	4
E	2	1	3	3	0	2	4	1
F	2	3	1	3	2	0	4	1
G	2	3	3	1	4	4	0	5
H	3	2	2	4	1	1	5	0

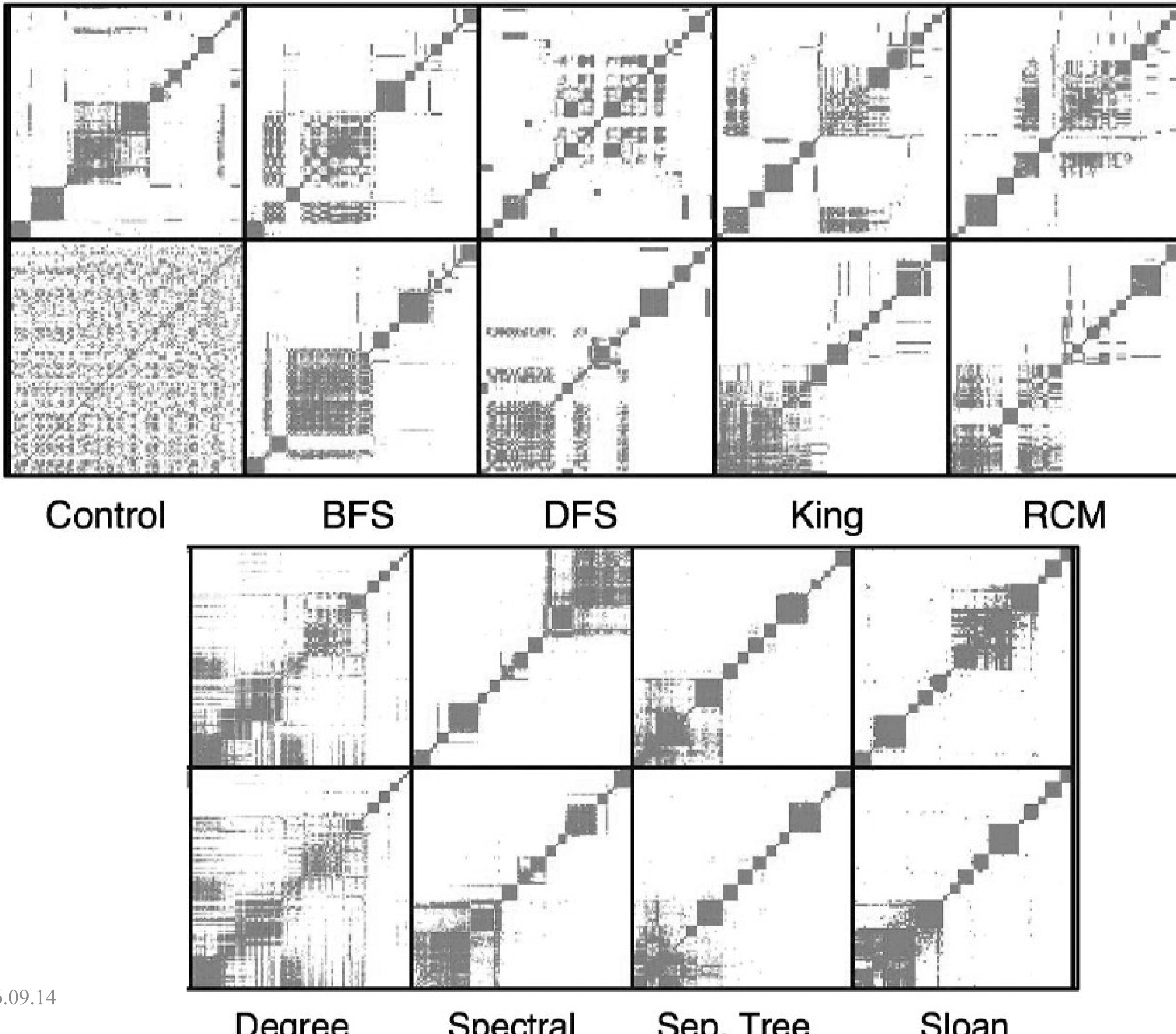
- Add information to the adjacency matrix

Reordering Graphs: Graph Layout

- Main Methods:
- Compute a linear order that optimizes an objective function
- Well-known objective functions (Diaz et al. 02):
  - Bandwidth (widest diagonal)
  - MinLA
  - Cutwidth or Min Cut
  - Modified CutWidth
  - Vertex Separation
  - SumCut
- Simple functions (Mueller et al. 07)
  - DFS, BFS, Degree
  - Bandwidth: Reverse Cuthill-McKee (RCM) and King's
  - Sloan & Spectral






Control
BFS
DFS
King
RCM
Degree
Spectral
Sep. Tree
Sloan

Conclusion

What method is the best for me?

- For Tables

1. Try fastest first: PCA / CA
2. Robust method: Clustering+seriation
3. Others: no guarantees

- For Graphs

1. Fastest: Spectral or Siirtola's Heuristic
2. Robust: Distance Table + Clustering+seriation

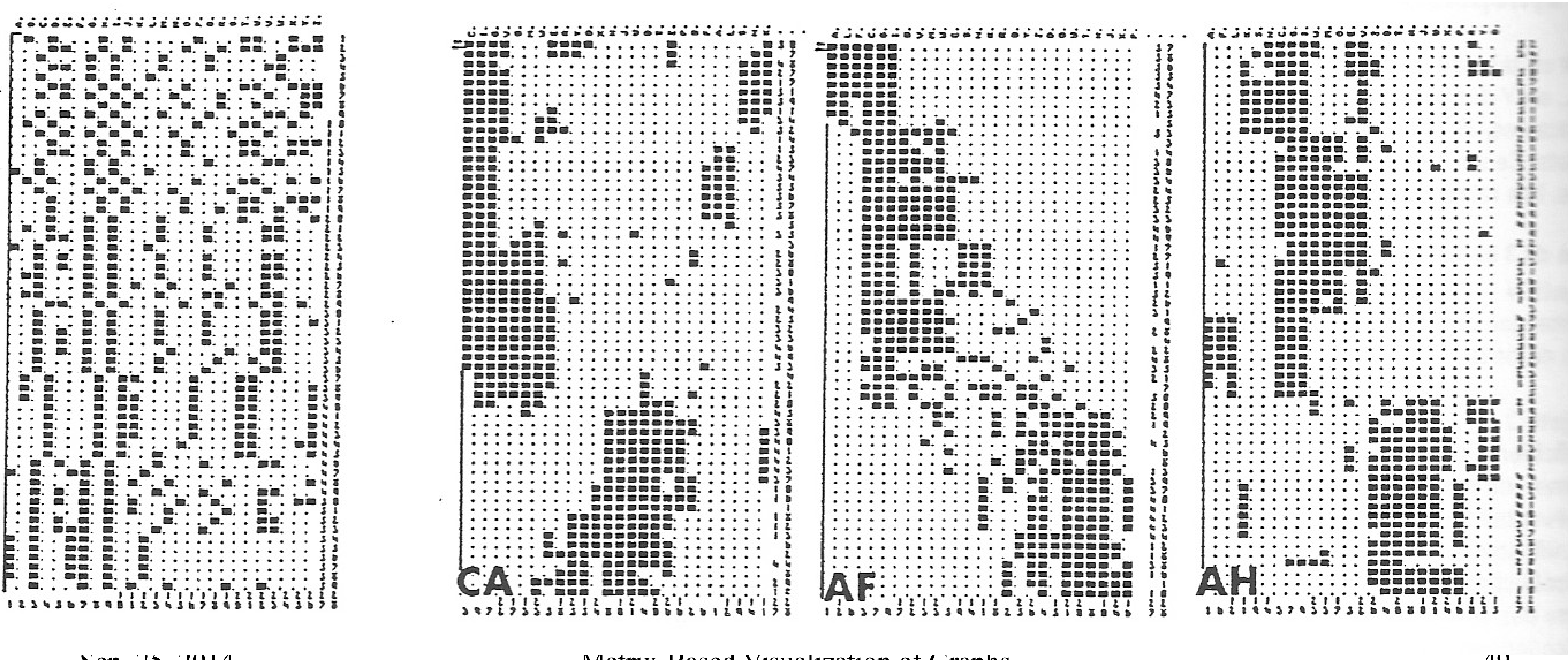
Interesting Results

- Seriation provide 1D ordering
- Node-Link layout provide a 2D embedding
- 2 > 1
- Therefore, 2D embedding is better and 1D

- Well, maybe, but in reality, we see a lot of structures in a matrix

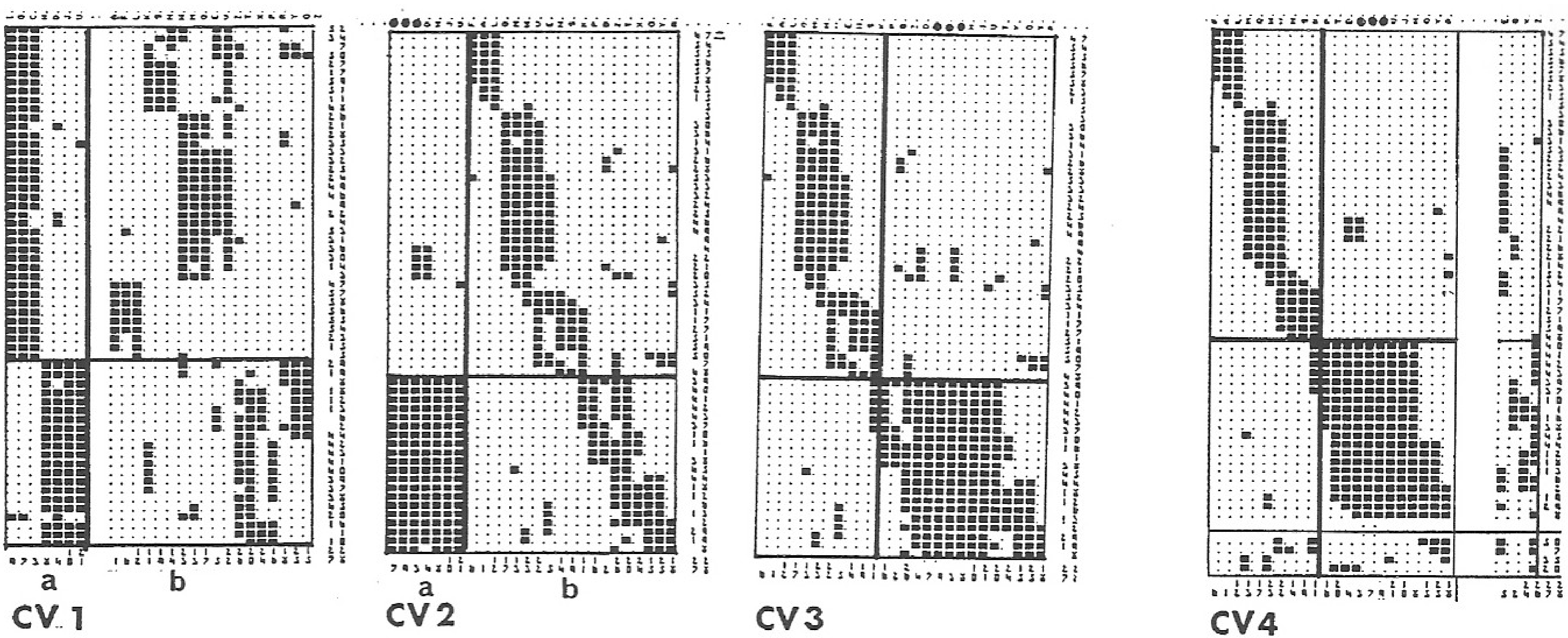
Bertin's Manual Ordering Process

- 59 merovingien objects with 27 traits

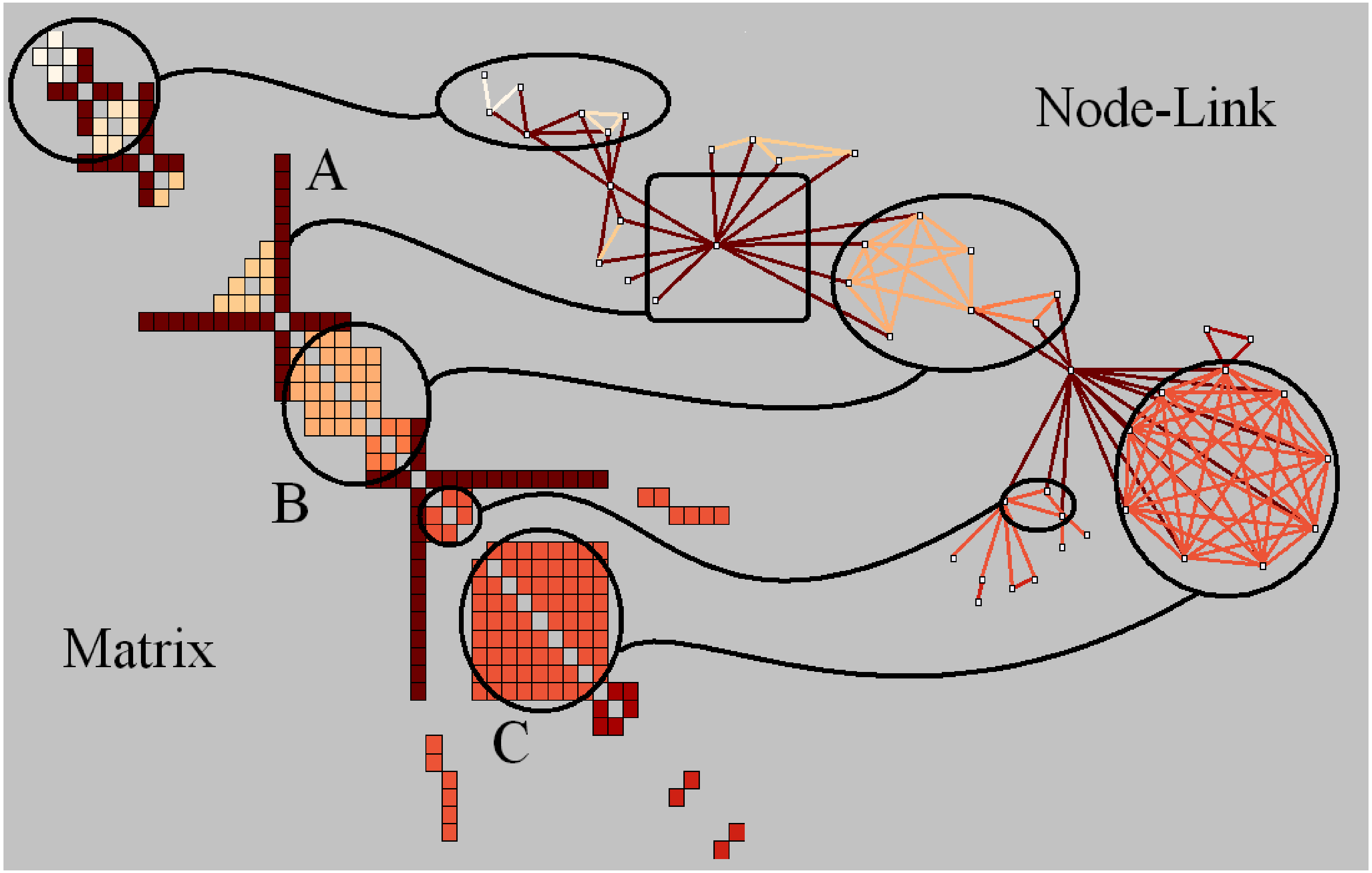


Manual Reordering

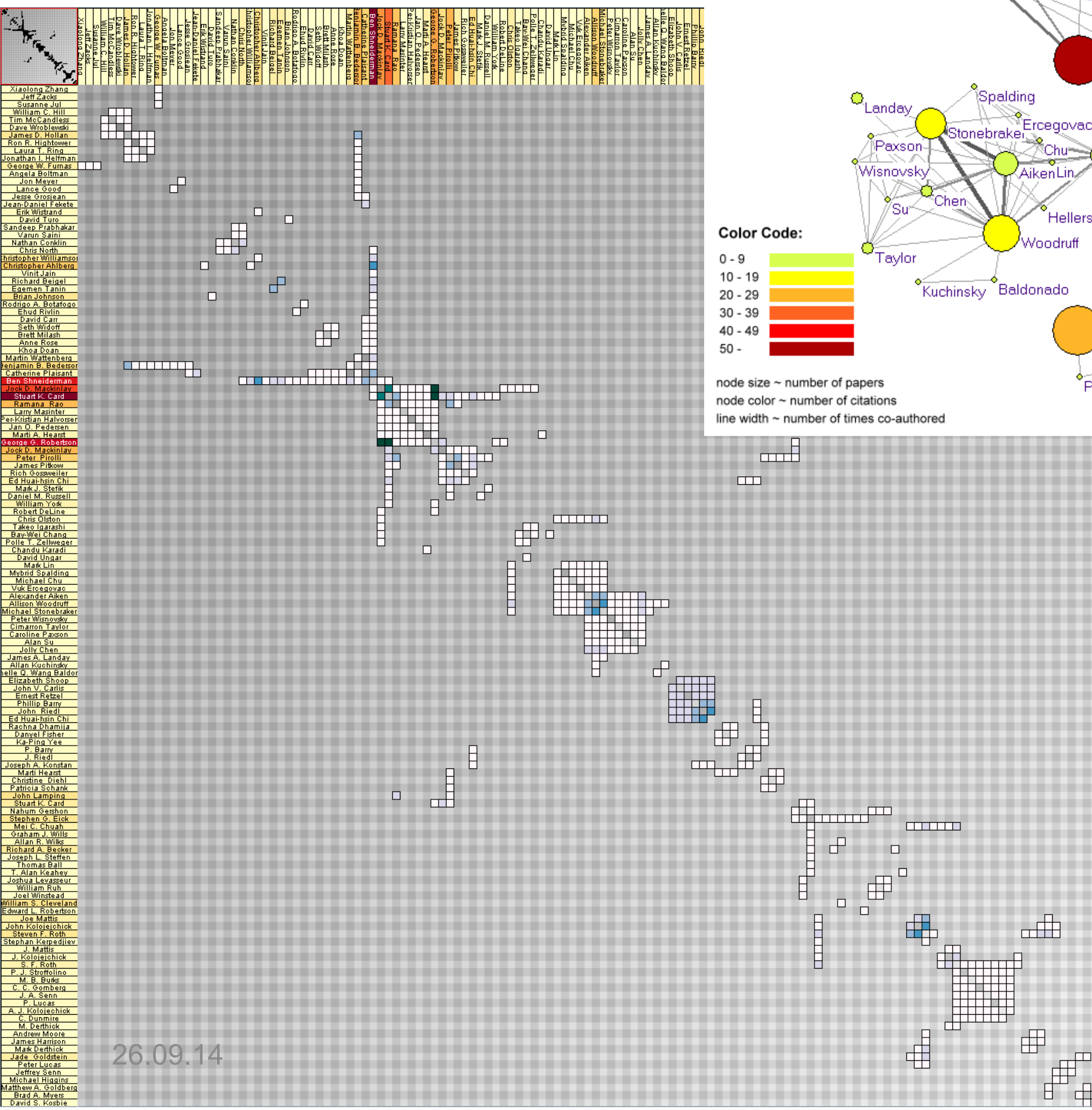
- Starts from Hierarchical clustering
- Create blocks

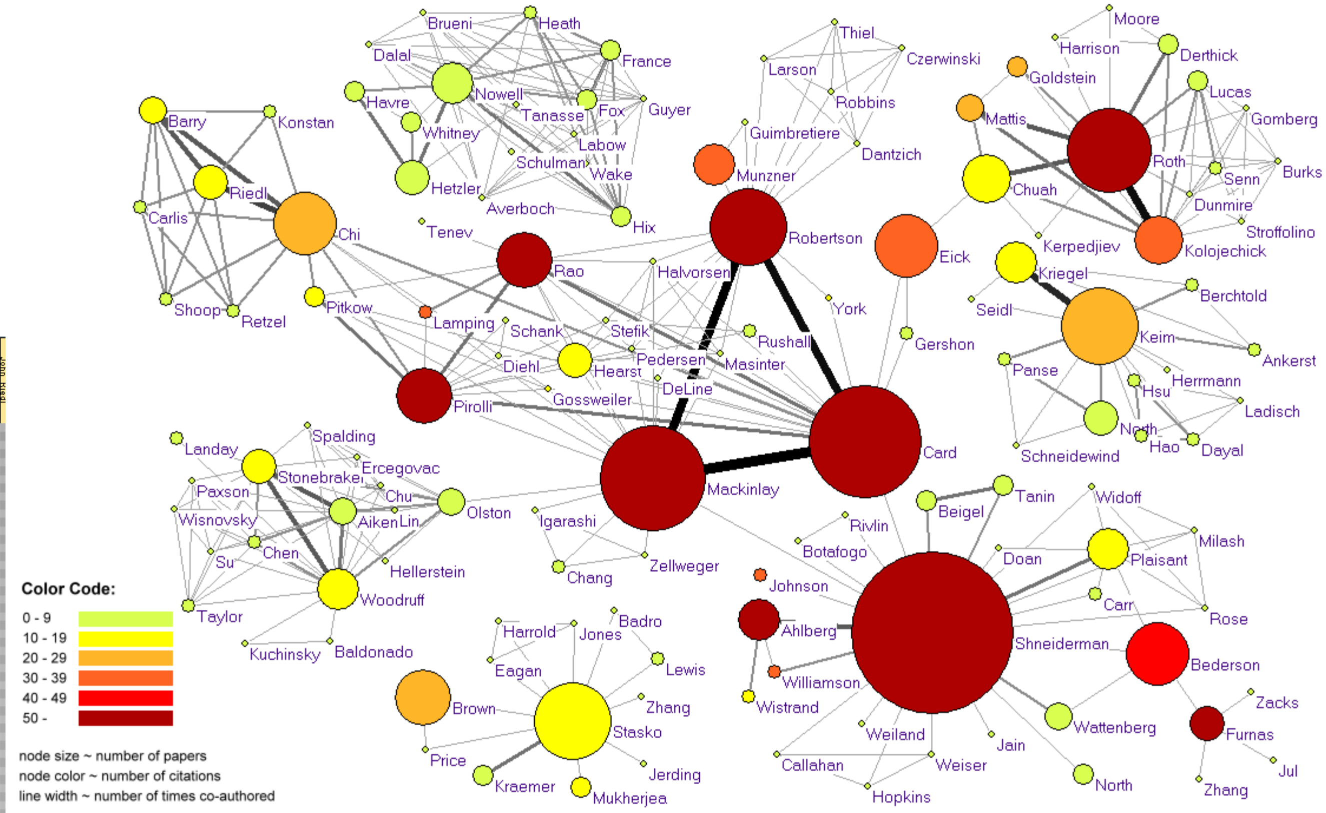


Visual Patterns with Ordered Matrices



Les Misérables Co-occurrence

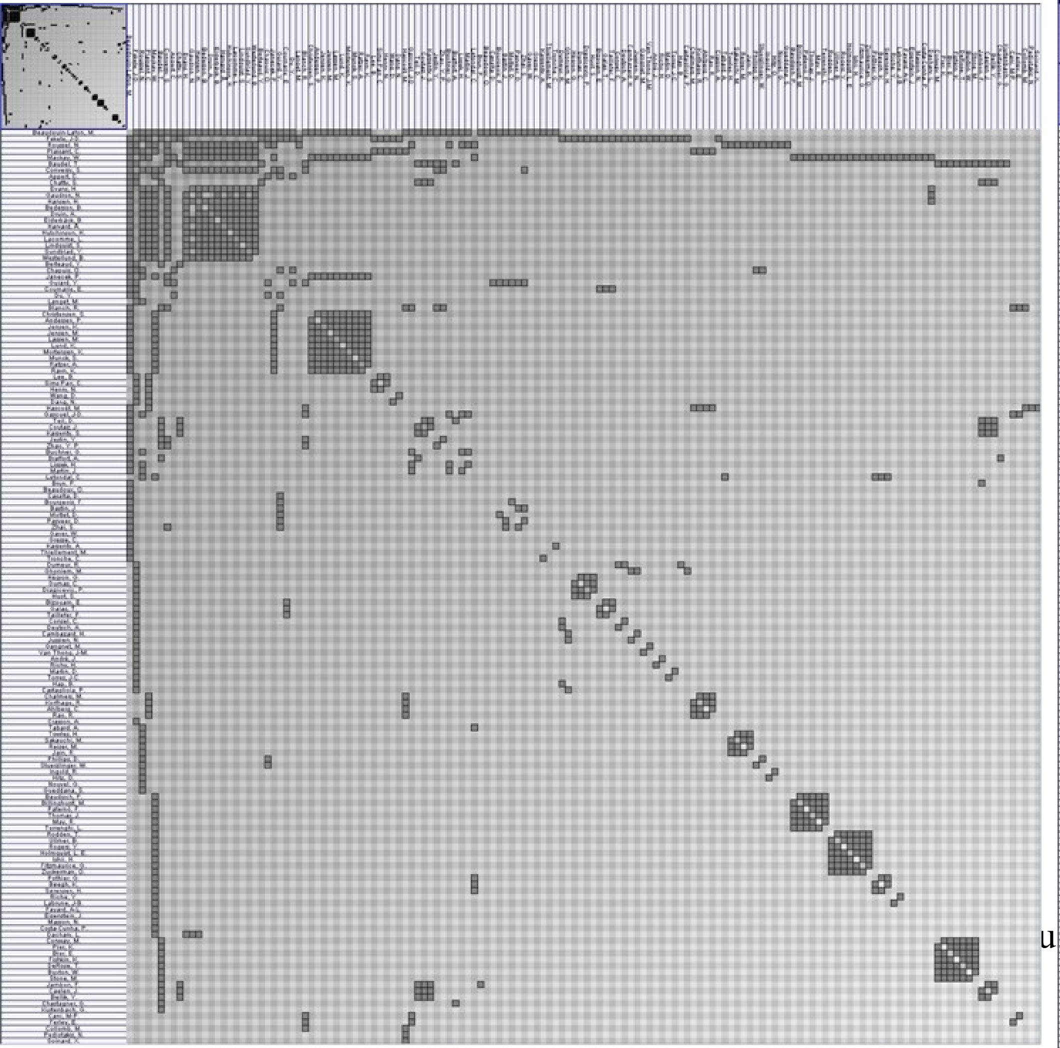


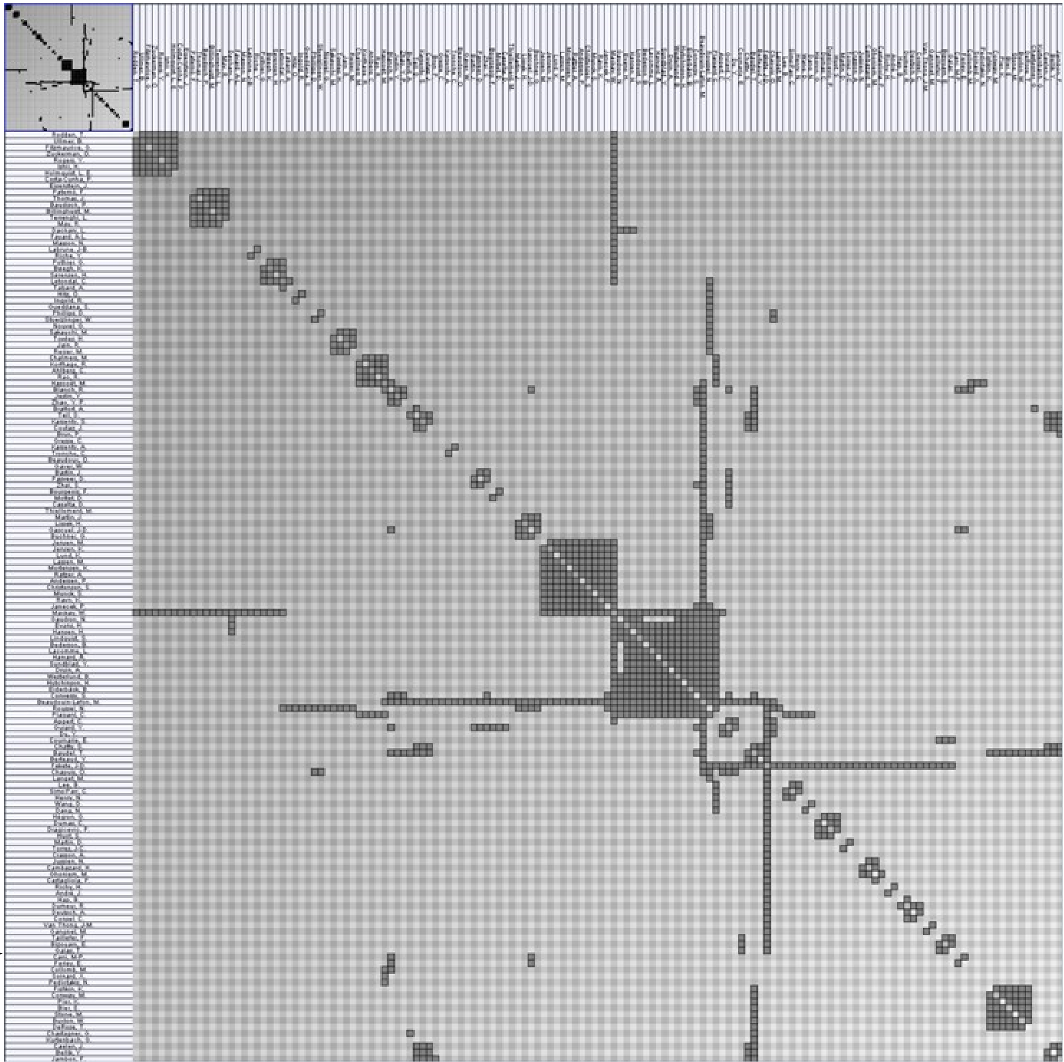


Brueni
Heath
Dalal
France
Nowell
Havre
Tanasse
Fox
Guyer
Whitney
Labow
Schulman
Wake
Hetzler
Averboch
Hix
Barry
Konstan
Riedl
Carlis
Chi
Shoop
Retzel
Pitkow
Tenev
Thiel
Czerwinski
Larson
Robbins
Guimbretiere
Dantzich
Munzner
Robertson
Rao
Halvorsen
Lamping
Schank
Stefik
Diehl
Hearst
Pedersen
Masinter
Rushall
York
Gershon
Eick
Pirolli
Gossweiler
DeLine
Card
Mackinlay
Moore
Harrison
Derthick
Goldstein
Lucas
Mattis
Gomberg
Roth
Senn
Burks
Chuah
Dunmire
Stroffolino
Kerpedjiev
Kolojechick
Kriegel
Seidl
Berchtold
Keim
Ankerst
Panse
Hsu
Herrmann
Ladisch
North
Hao
Dayal
Schneidewind
Landay
Spalding
Ercegovac
Paxson
Stonebraker
Chu
Wisnovsky
Aiken
Lin
Olston
Chen
Su
Igarashi
Hellerstein
Woodruff
Zellweger
Chang
Taylor
Kuchinsky
Baldonado
Harrold
Jones
Badro
Eagan
Lewis
Brown
Zhang
Stasko
Price
Jerding
Kraemer
Mukherjea
Tanin
Beigel
Widoff
Rivlin
Botafogo
Doan
Milash
Plaisant
Johnson
Carr
Rose
Ahlberg
Shneiderman
Bederson
Williamson
Wistrand
Weiland
Wattenberg
Jain
Zacks
Furnas
Callahan
Weiser
North
Jul
Hopkins
Zhang
Color Code:
0 - 9
10 - 19
20 - 29
30 - 39
40 - 49
50 -
node size ~ number of papers
node color ~ number of citations
line width ~ number of times co-authored

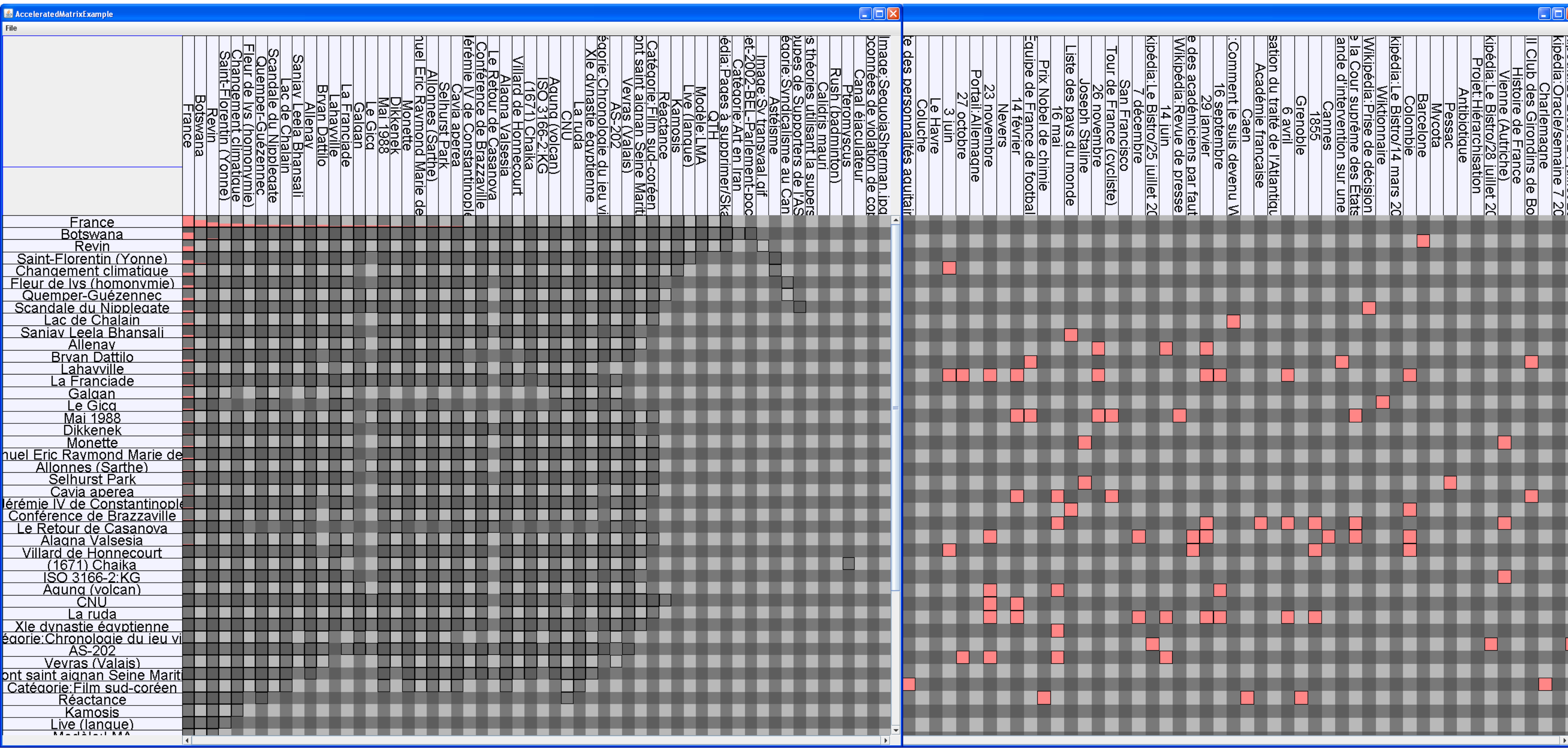
53

HDE vs. TSP based orderings

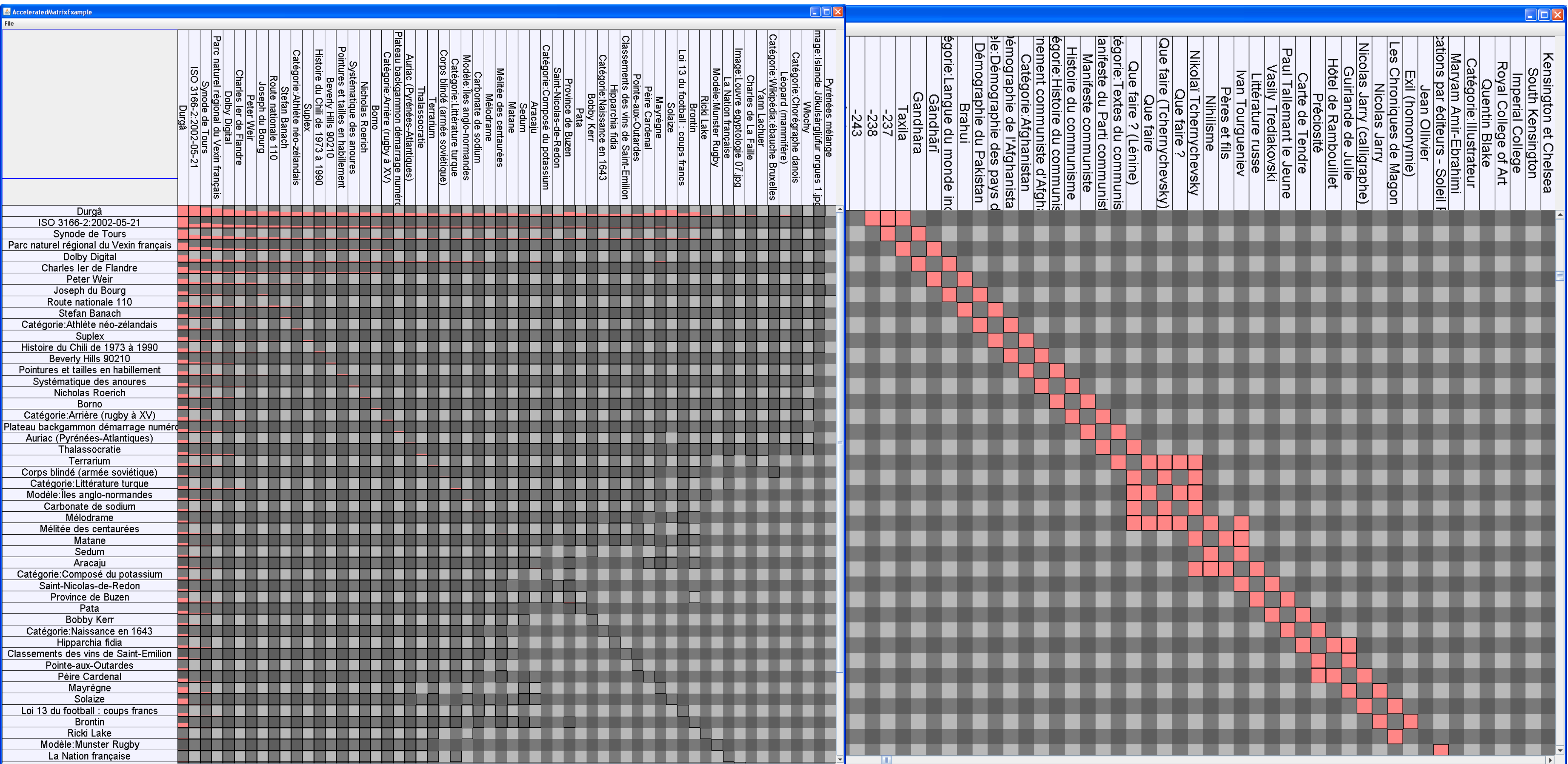




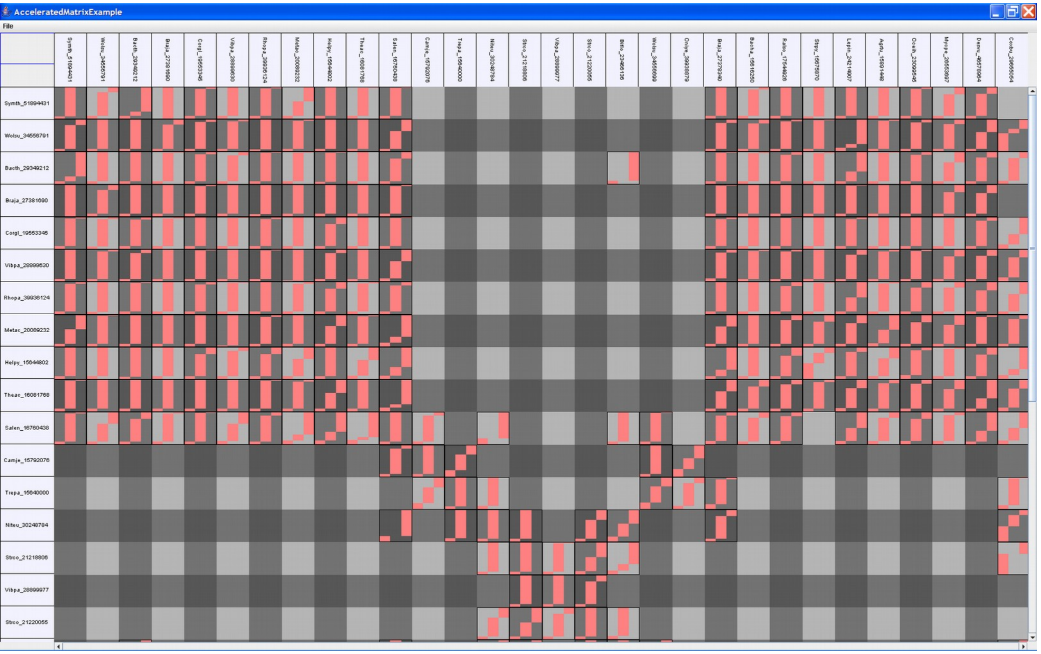
HDE overview and details

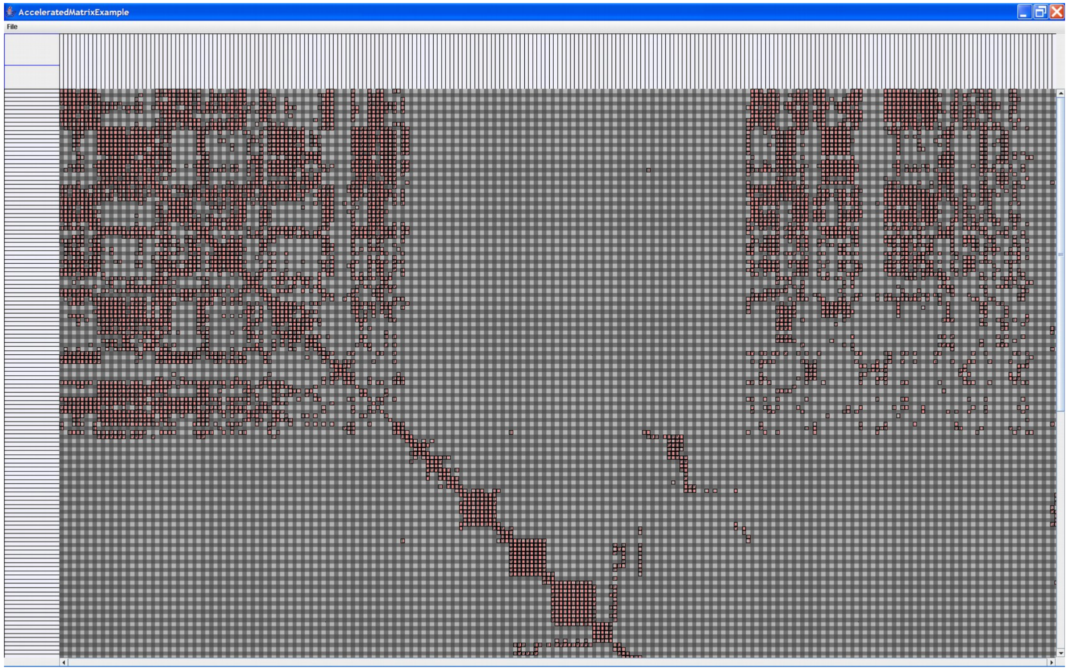


NNTSP overview and details



NNTSP overview of Protein-Protein Network



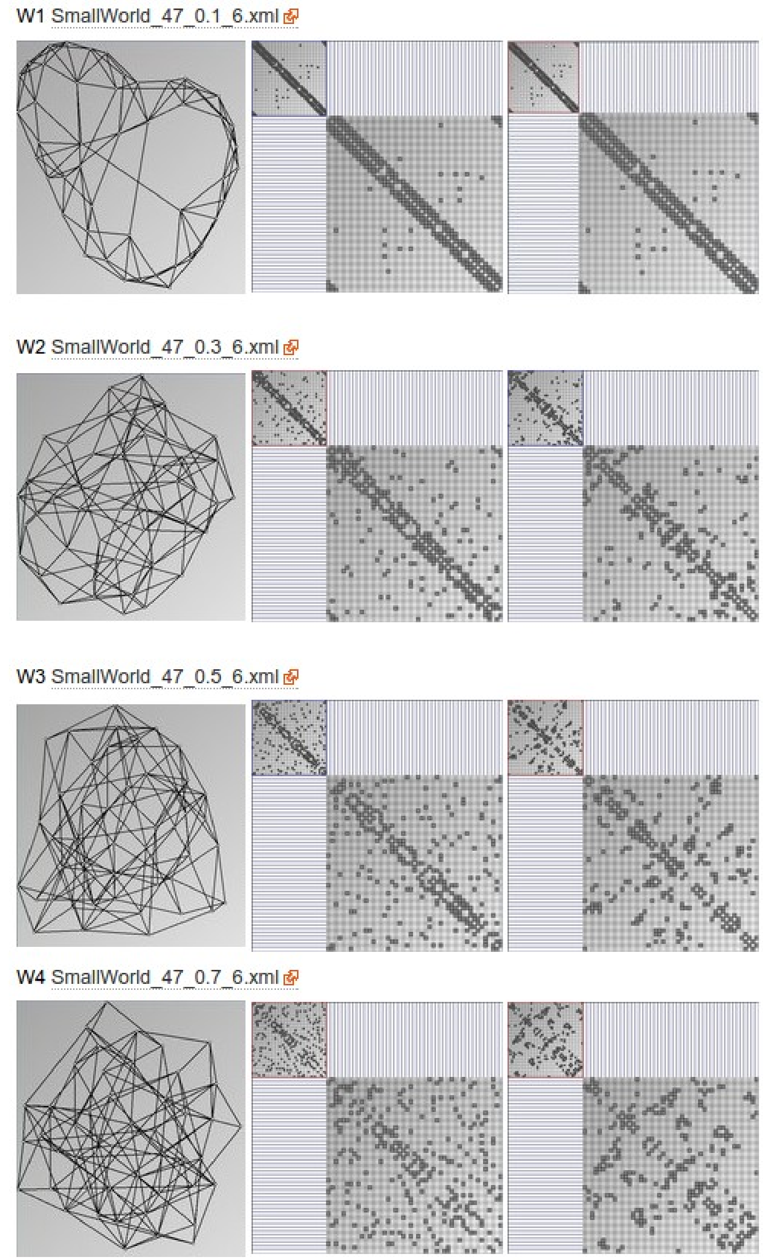


Random Graphs

http://www.infovis-wiki.net/index.php?title=Social_Network_Generation

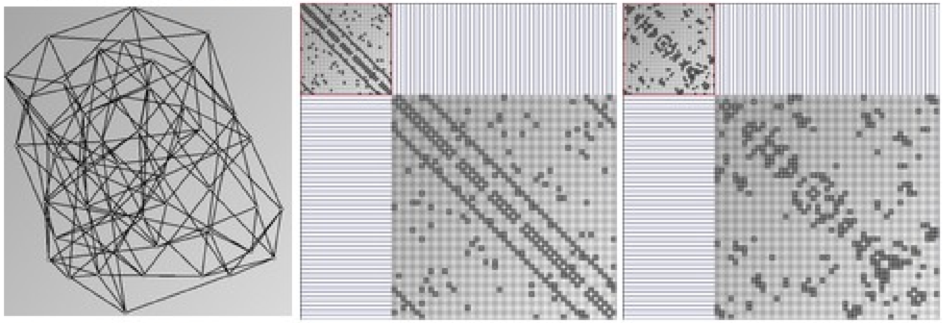
Watts&Strogatz 1998 "Collective dynamics of 'small-world' networks." Nature

Parameters and Resulting Graph characteristics								
graphs	W1	W2	W3	W4	W5	W6	W7	W8
numVertices	47	47	47	47	47	47	47	47
beta	**0.1**	**0.3**	**0.5**	**0.7**	**0.9**	0.3	0.3	0.3
degree	6	6	6	6	6	**2**	**4**	**8**
numVertices	47	47	47	47	47	47	47	47
numEdges	282	282	282	282	282	94	188	376
components	1	1	1	1	1	2	1	1
density	0.36	0.36	0.36	0.36	0.36	0.21	0.29	0.41
clusteringCoefficient	0.51	0.25	0.15	0.09	0.12	0.23	0.25	0.32
diameter	6	4	4	4	4	-	6	4
averageShortestDistance	2.97	2.4	2.32	2.3	2.29	-	3.24	2.15
minDegree	5	4	4	3	4	1	2	5
maxDegree	8	9	9	9	9	4	6	10

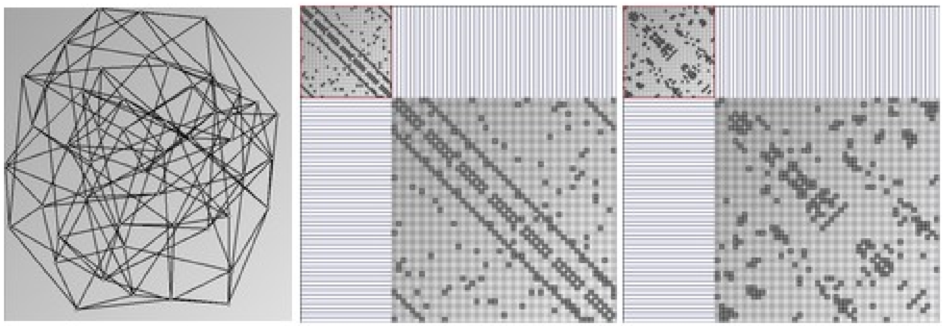


Kleinberg gen.

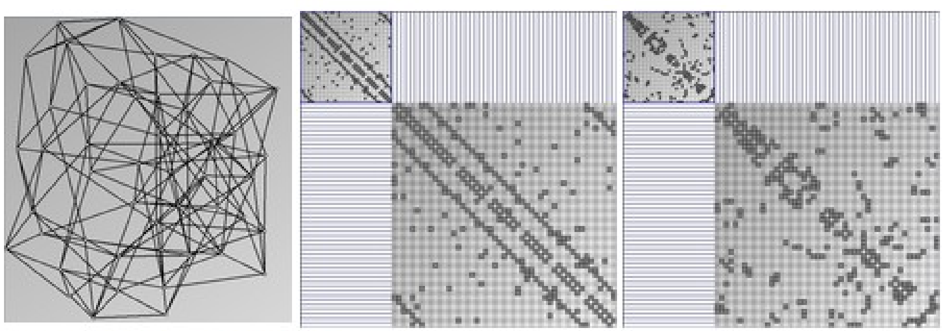
Parameters and Resulting Graph characteristics				
graphs	W1	W2	W3	W4
latticeSize	7	7	7	7
clusteringExponent	**0.1**	**0.5**	**1**	**2**
numVertices	49	49	49	49
numEdges	490	490	490	490
components	1	1	1	1
density	0.45	0.45	0.45	0.45
clusteringCoefficient	0.08	0.09	0.14	0.19
diameter	4	4	4	4
averageShortestDistance	2.38	2.36	2.37	2.44
minDegree	9	9	9	9
maxDegree	14	12	13	12



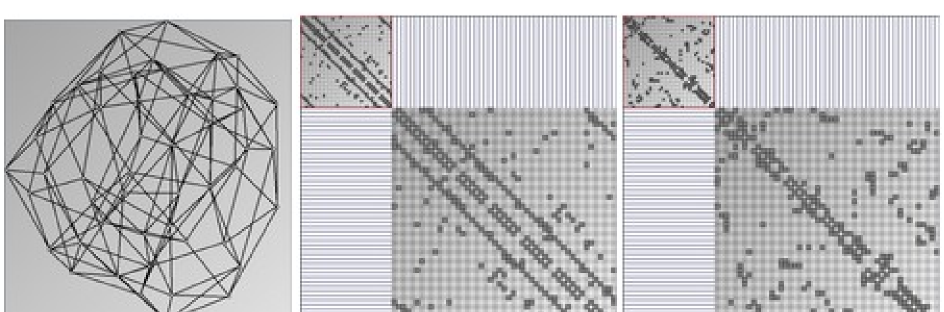
W2 SmallWorld_49_0.5.xml



W3 SmallWorld_49_1.0.xml



W4 SmallWorld_49_2.0.xml



Challenges

- How can we evaluate the effectiveness of seriation algorithms?
  - Liiv cites 171 articles, Díaz et al. Cite 261 articles
  - Hasler et al. 2008 describe the R Seriation package providing 20+ methods
- Find visual features and rank by feature
  - Bandwidth (on the diagonal), Cutwidth (clustering), Symmetry
- Scalability issues
  - Almost always quadratic except Dimension reduction, Barycentric, and Spectral
  - Find sampling / incremental computation
  - Combine Reordering and Clustering

- Reordering is just one (important) step of Matrix Visualization
- Effective interactions are needed to explore the matrix

Assessing the Quality of Ordering Algorithms

- Select networks, e.g. 50
  - Random, real, multiple structures
- Compute multiple orderings for each
- Compute measures
  - image-based and others
- Perform human tests with basic tasks
  - Use Amazon Mechanical Turks with between-subject design
- Report on performance (Time + Accuracy)
- Search for correlations between
  - Network structure, Measures, Performance

Challenges

- For NodeTrix, find a good criteria for using a Matrix
- How to slice an ordered matrix?
  - Eisen et al. 98 have a TSP-based solution
- Find good characterization of reorderings
  - Beyond searching the best algorithm for each instance of a graph
- 2 clear visual patterns (block, cross), diagonal seems important too
  - Can we find more useful motifs?

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