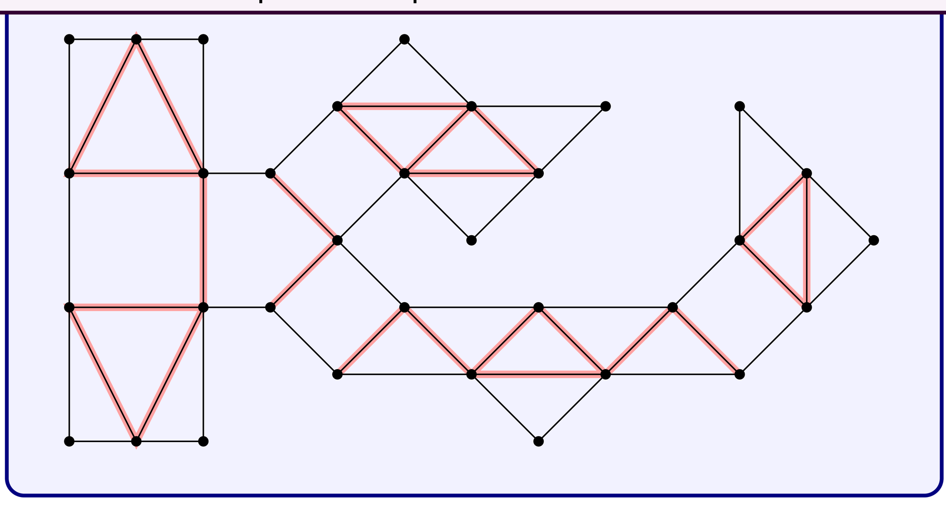
Nieke Aerts
aerts@math.tu-berlin.de

TU Berlin
Diskrete Mathematik

Touching Triangle Representations in a k-gon (kTTR)
of Biconnected Outerplanar Graphs

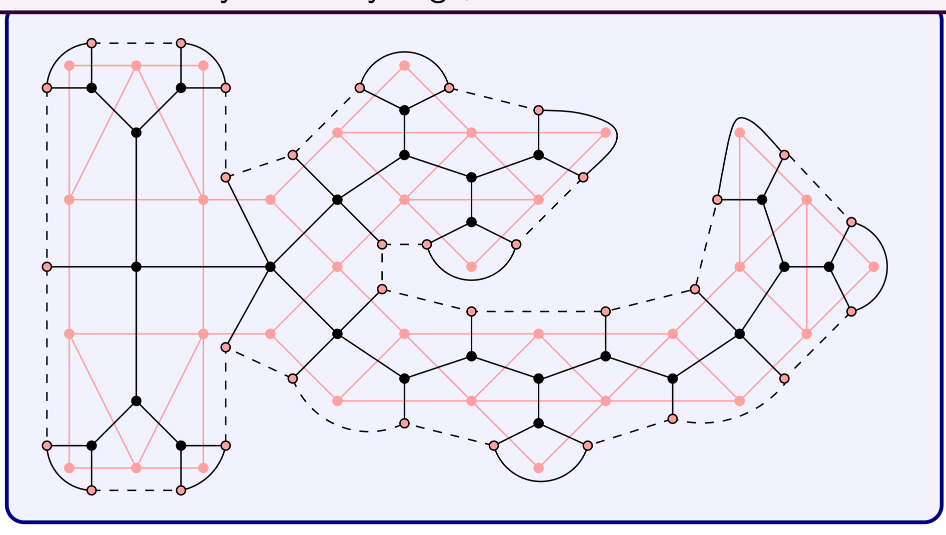
Input

Biconnected Outerplanar Graph G



Auxiliary Graph

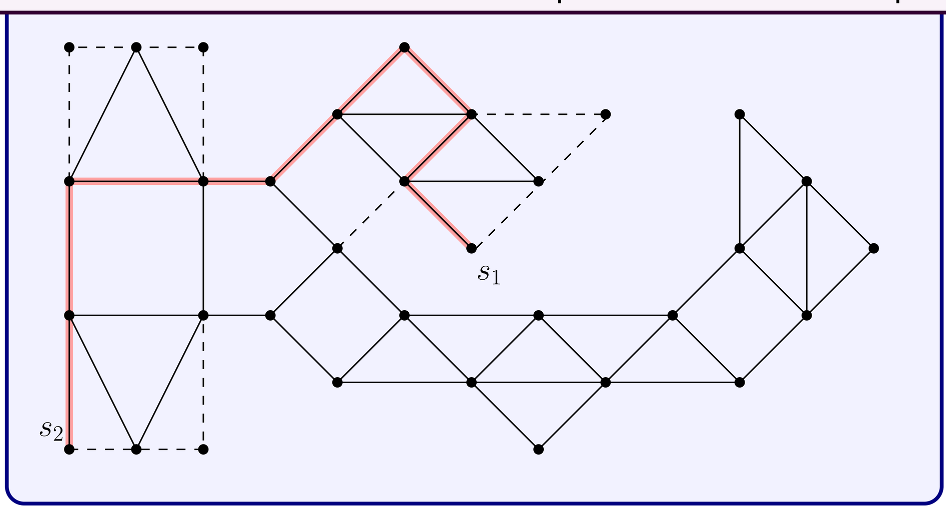
- Start with the weak dual of G
- Add an edge into the outer face for each boundary edge of G
- Cyclically connect the vertices in the outer face
- Contract every boundary edge, whose contraction does not induce a 2-face



Dividing Path

Let c be a component of interior edges with two interior faces, f and f'. An edge on the boundary of G with one end on f, is called a *petiole* of f. A simple path P, between two boundary vertices of a biconnected outerplanar graph is said to be a *dividing path* for c if

- there is at most one edge of c in P, and,
- the interior faces of c are on opposite sides of P, and,
- each of these faces has at least one petiole that is not completely in P.

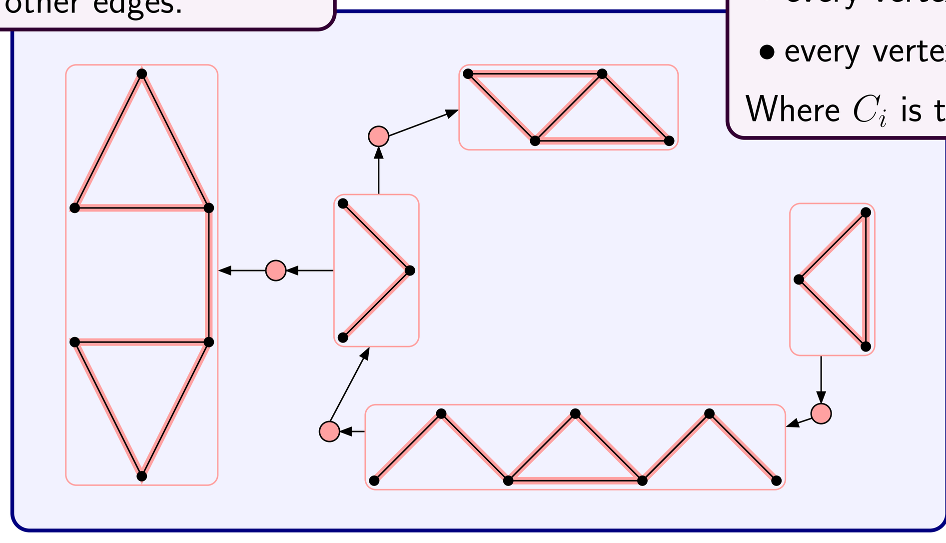


Venation Graph

The vertices of VENATION(G) are:

- Components (c) of the graph of interior edges of G
- And the faces (f) connecting the components.

There is an edge (c, f) iff the chord c is on the boundary of the face f. There are no other edges.



Valid Orientation

An orientation of the edges of VENATION(G) is called *valid* if all edges are oriented and:

- every vertex in C_2 has only incoming arcs,
- every vertex in C_1 has at most one outgoing arc,
- every vertex in C_0 has at most two outgoing arcs,
- every vertex in F has at precisely one outgoing arc.

Where C_i is the set of components with i interior faces.

Theorem

Let G be a biconnected outerplanar graph and v_2 the number of degree two vertices in G. Let k be an integer such that $2 < k \leq v_2$ if $v_2 \geq 3$ and let $k = 3$ otherwise. A biconnected outerplanar graph has a kTTR if and only if

1. Each component of interior edges of G has at most two interior faces.
2. The graph VENATION(G) admits a valid orientation.
3. There is a way to select k vertices of degree 2 in G such that, for every component $c \in C_2$, there are two representatives in this set, v_i, v_j, such that, between the representatives there is a dividing path for c.

Idea of Proof

For each component of the interior edges of G, an assignment of flat angles is made for the accompanying part of the auxiliary graph H. The assignment comes from a chord-to-endpoint assignment, and we can assure that:

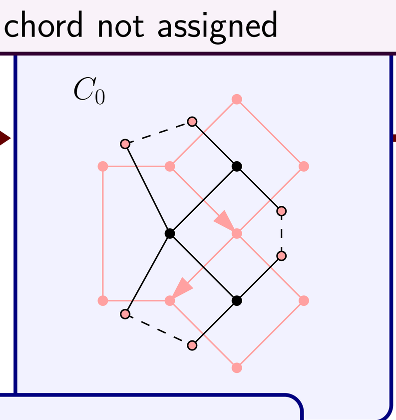
- A component with at most one interior face, does not require a segment connecting two boundary vertices in the representation.
- A component with two interior faces needs precisely one such segment, and we can construct the flat angle assignment such that the segment crosses the dividing path garuanteed by 3.

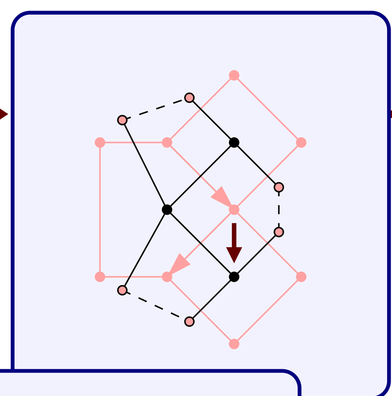
The wrap-up is to find k representatives for the vertices chosen in 3. The assignment is then extended with the assignments of the other boundary vertices to the outer face.

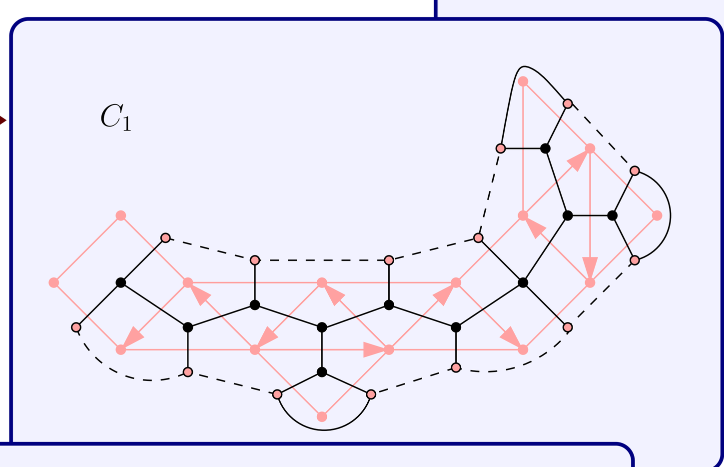
Chord-to-Endpoint Assignment[2] of G

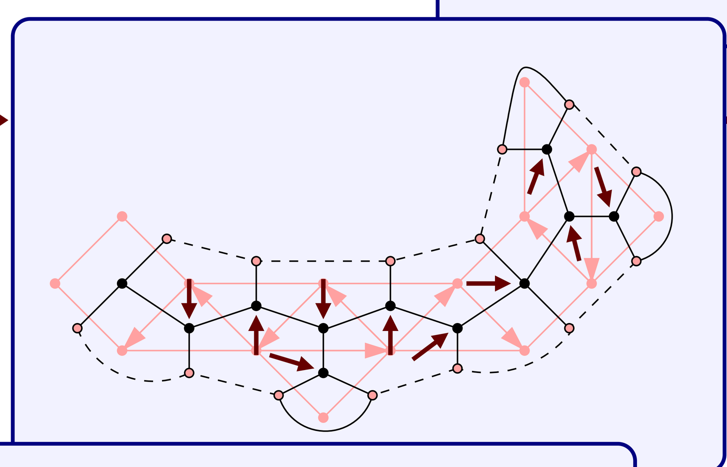
- Each endpoint has at most one chord assigned to it
- Each component of interior edges has at most one chord not assigned

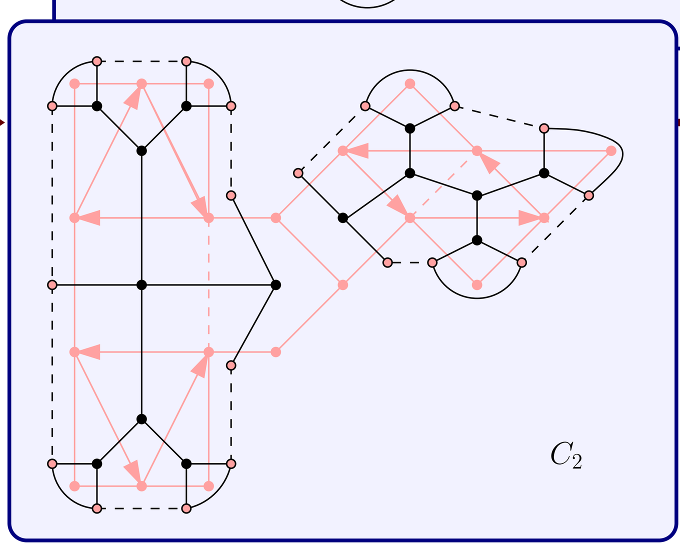
- All chords of $c \in C_0, c \in C_1$ are assigned
- All but one of the chords of $c \in C_2$ are assigned

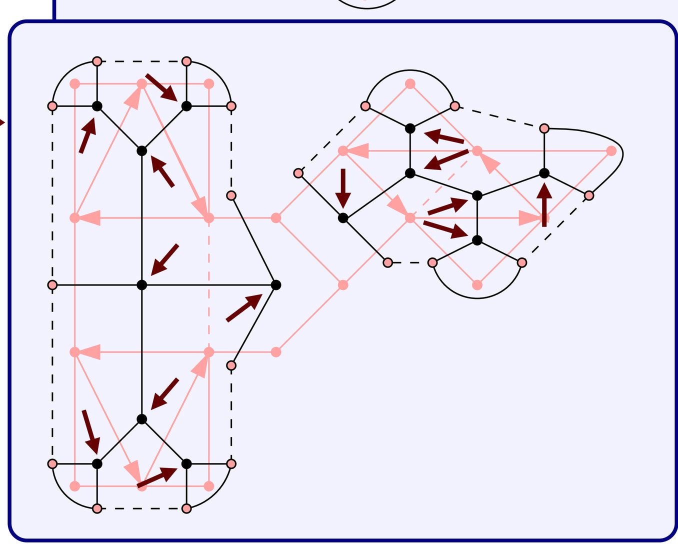






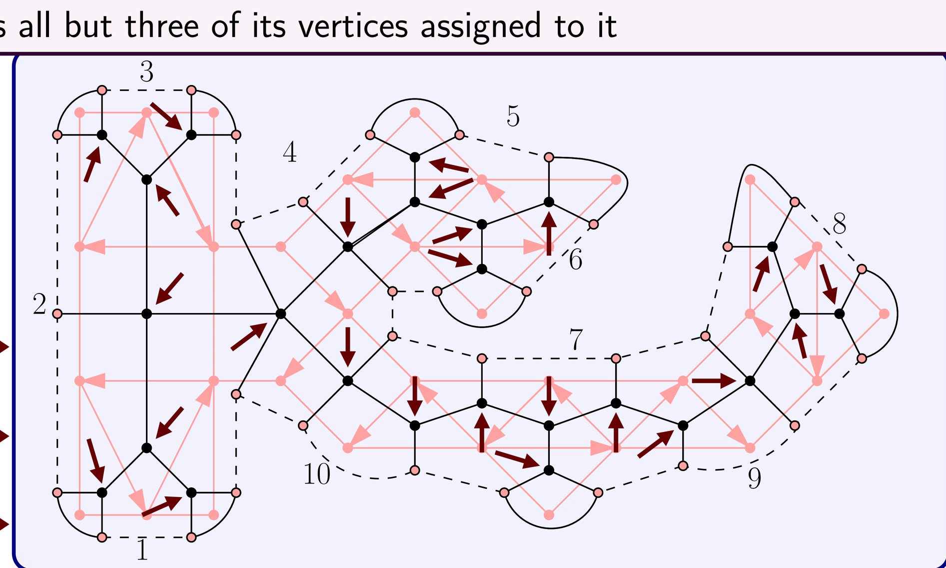




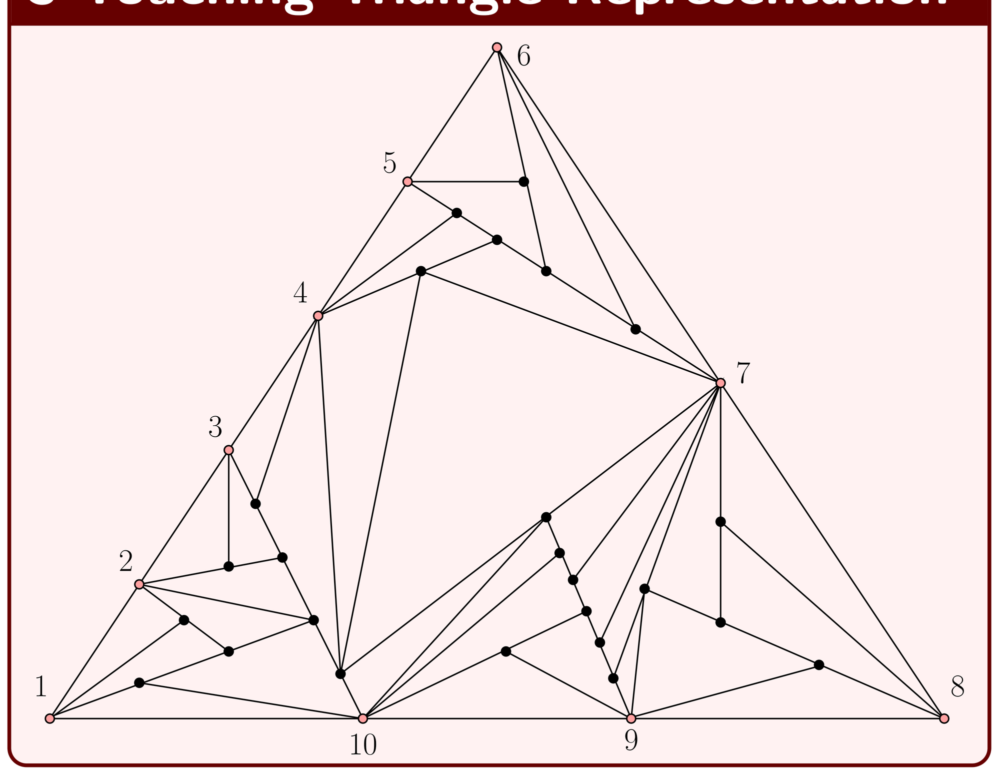


Flat Angle Assignment[1] of H

- Each interior vertex is assigned at most once
- Boundary vertices are not assigned
- Each face has all but three of its vertices assigned to it



3-Touching Triangle Representation



References

[1] N. Aerts and S. Felsner, *Straight Line Triangle Representations*, in Proc. Graph Drawing, S. K. Wismath and A. Wolff, eds., vol. 8242 of *LNCS*, Springer, 2013, pp. 119–130.

[2] J. J. Fowler, *Strongly-Connected Outerplanar Graphs with Proper Touching Triangle Representations*, in Proc. Graph Drawing, S. K. Wismath and A. Wolff, eds., vol. 8242 of *LNCS*, Springer, 2013, pp. 156–161.

[3] E. R. Gansner, Y. Hu, and S. G. Kobourov, *On Touching Triangle Graphs*, in Proc. Graph Drawing, U. Brandes and S. Cornelsen, eds., vol. 6502 of *LNCS*, Springer, 2010, pp. 250–261.