Bitonic *st*-orderings of biconnected planar graphs

Martin Gronemann

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September 24, 2014

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Introduction

Backstory

- Drawing undirected planar graphs

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- Canonical ordering for incremental drawing algorithms

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- Limited to triconnected (Kant) or maximal planar (FPP)

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Solutions for triconnected $\Rightarrow$ biconnected

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1. Augmentation to triconnected or maximal planar

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   ⚡ maximum degree related problems ⚡

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2. Biconnected canonical & shelling ordering

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   ⚡ not every internal node has a successor ⚡

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   ⚡ not every internal node has a successor ⚡
3. SPQR-tree approach

Biconnected planar graphs and *st*-orderings

What about *st*-orderings?

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- Single source s and single sink t

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Can we use *st*-orderings in the same way as canonical orderings?

Canonical vs. *st*-ordering

Experiment

"Let's use an st-ordering for the FPP-algorithm"

Canonical vs. *st*-ordering

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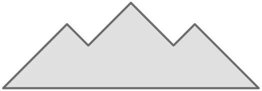

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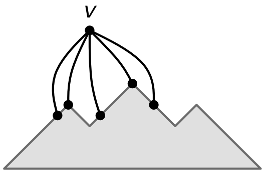

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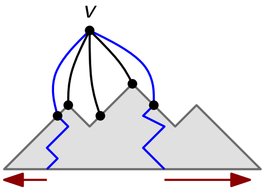

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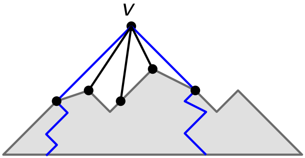

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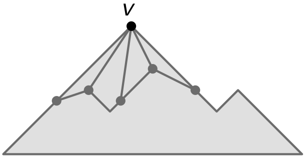

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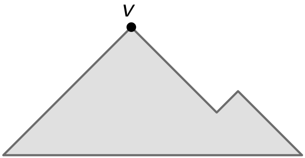

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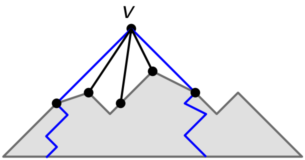

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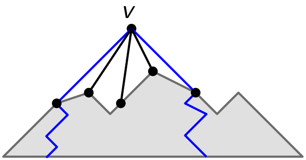

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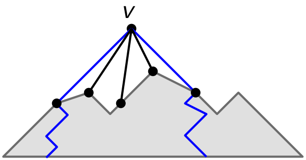

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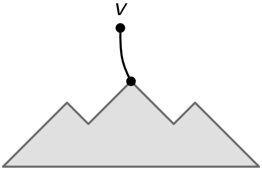

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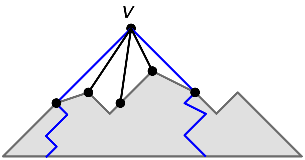

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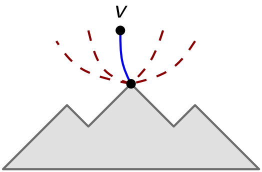

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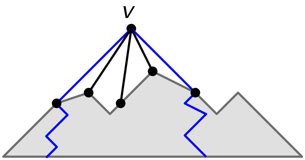

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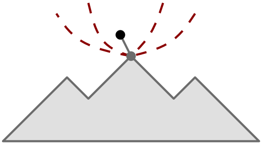

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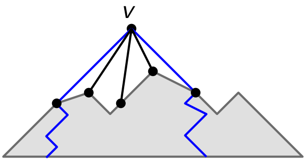

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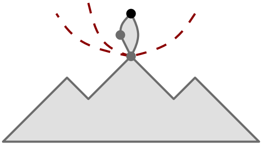

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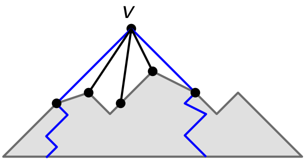

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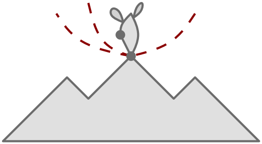

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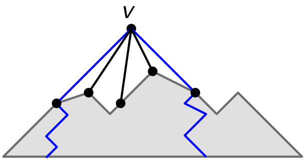

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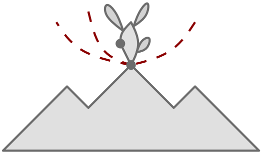

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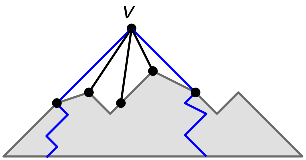

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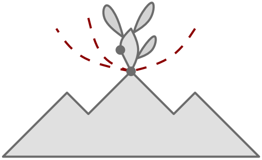

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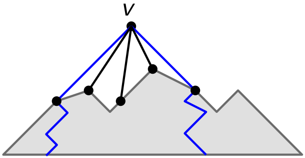

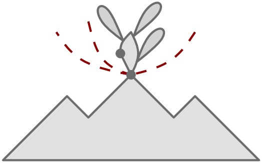

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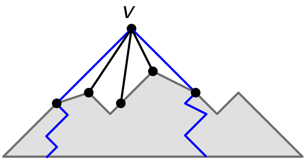

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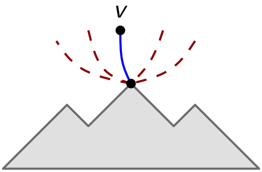

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(Harel & Sardas)

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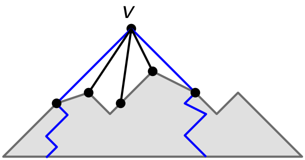

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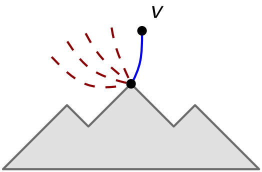

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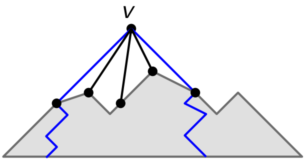

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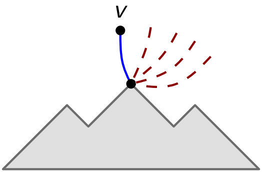

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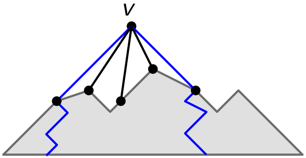

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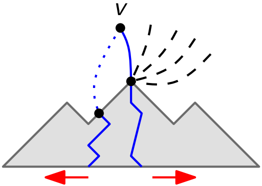

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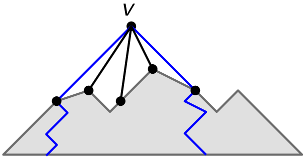

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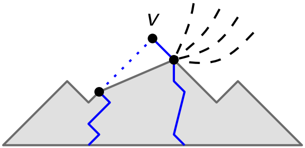

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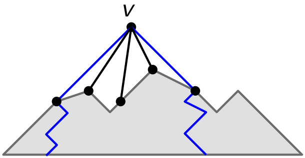

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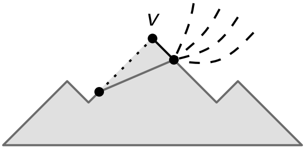

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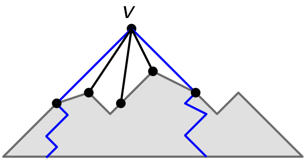

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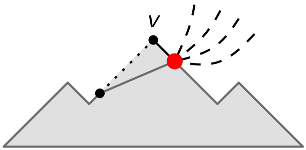

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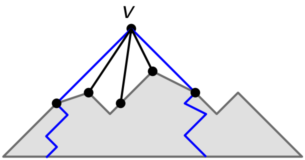

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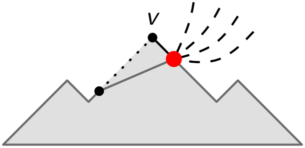

Single predecessor
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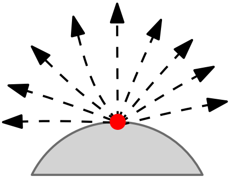
Observation

The unattached edges are the problem, not the single predecessor property.

Canonical vs. *st*-ordering

Intuition

At any time, all incident edges that are **not yet present** in the current drawing, appear **consecutively** in the embedding.



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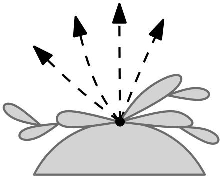

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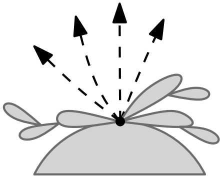

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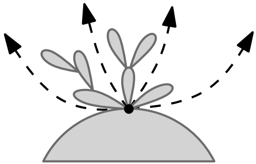

st-ordering

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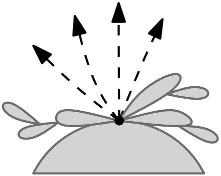

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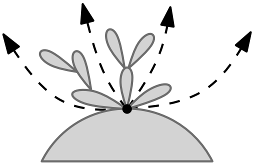

st-ordering

Question

Is there an *st*-ordering with this property?

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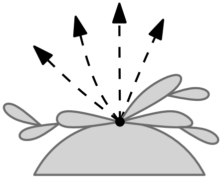

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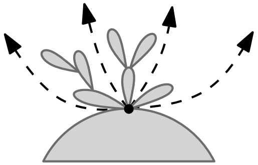

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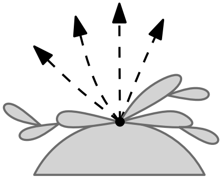

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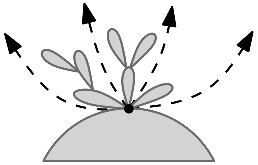

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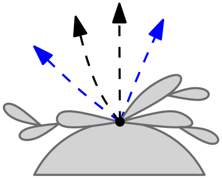
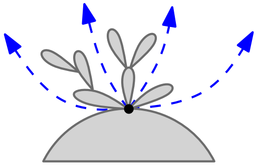


consecutive



st-ordering

Question

Is there an *st*-ordering with this property? $\rightarrow$ **bitonic** ***st*-ordering**

Canonical vs. *st*-ordering

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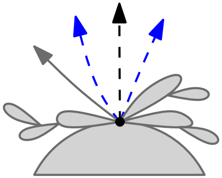

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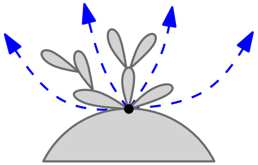

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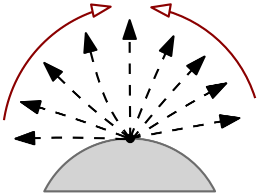

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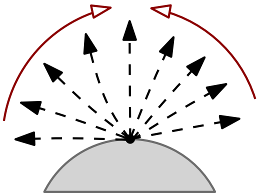

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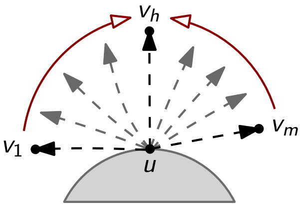

$$S(u) = \{v_1, \dots, v_m\}$$

Successors of u ordered as in the embedding

Canonical vs. *st*-ordering

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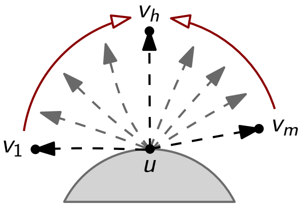
Given an *st*-ordering $\pi : V \mapsto \{1, \ldots, |V|\}$ with $\pi(v)$ being the rank of $v \in V$ in the ordering.



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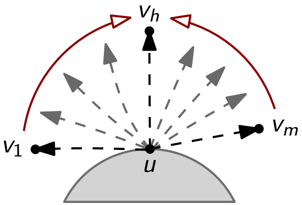

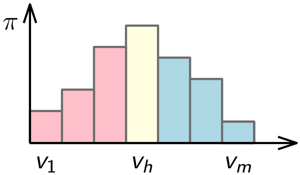

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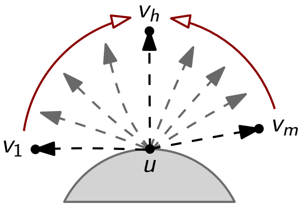

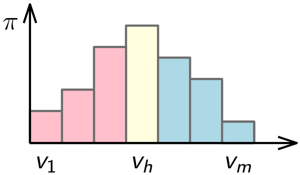

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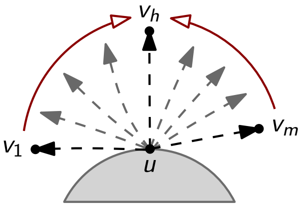

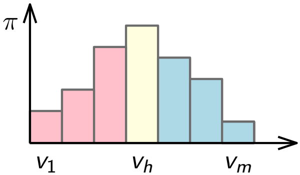

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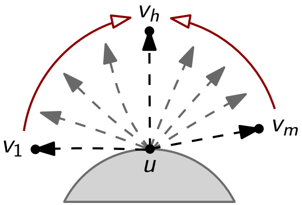

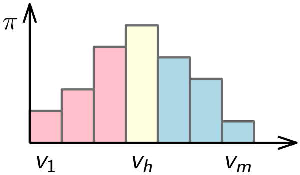

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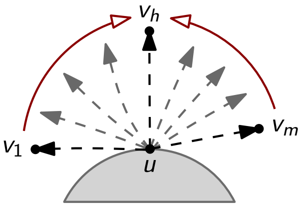

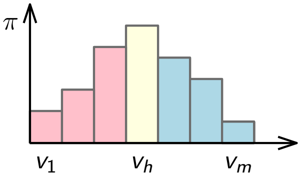

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Bitonic *st*-ordering

Theorem

Biconnected planar graph $G = (V, E)$

$+$

st-edge $(s, t) \in E$

Bitonic *st*-ordering

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counterexample

~~bitonic *st*-ordering π~~

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$\Downarrow$ linear-time algorithm

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Sketch of the algorithm

- SPQR-tree approach to derive π
- Canonical ordering for the R-nodes
- P-nodes may require a change in the embedding

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Experiment (revisited)

"Let's use a **bitonic** *st-ordering for the FPP-algorithm"*

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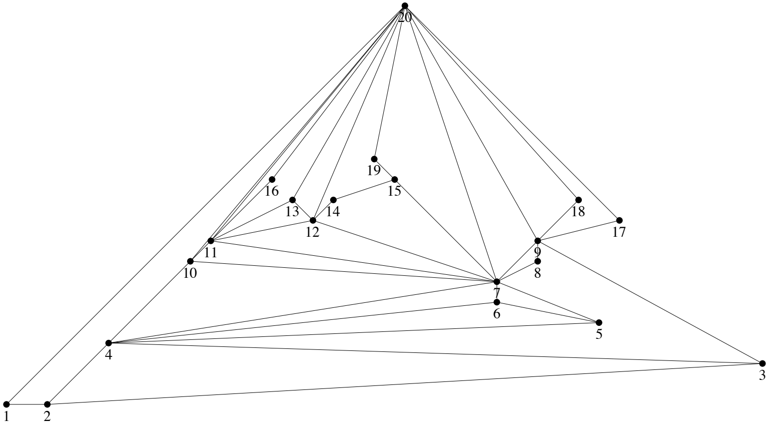

Rectilinear T-shaped contact representation

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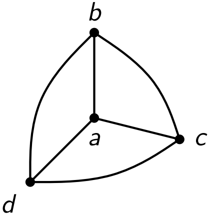

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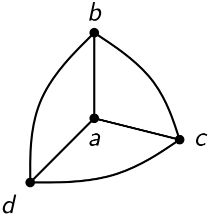

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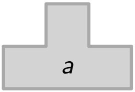
- Given a (biconnected) planar graph
- Vertices drawn as rectilinear T-shaped polygons



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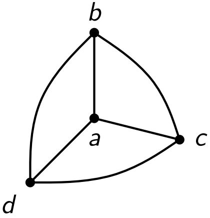

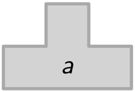

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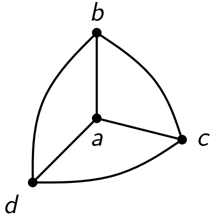

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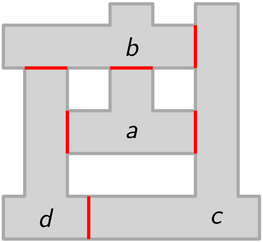

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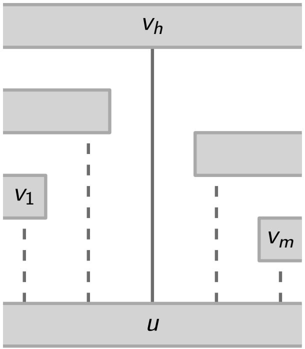

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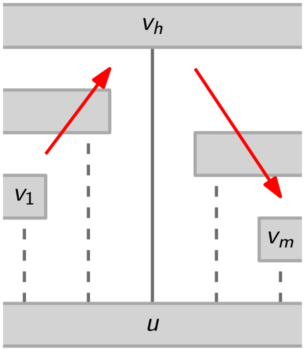

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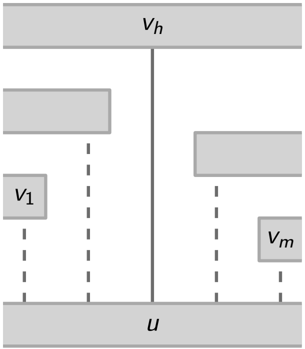

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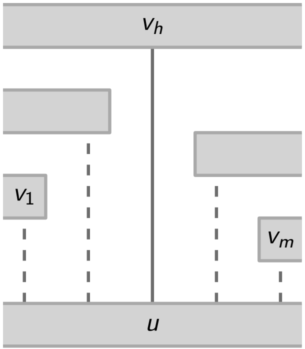
1. Grow a pole touching the highest successor from below



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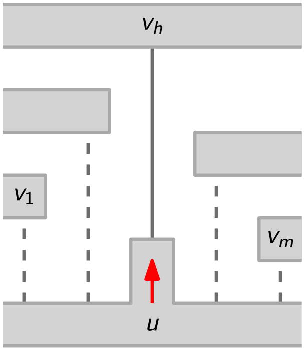

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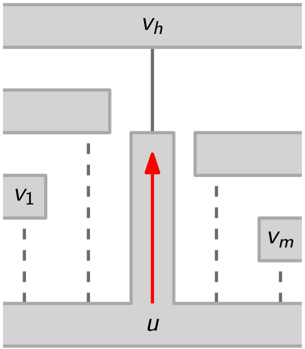

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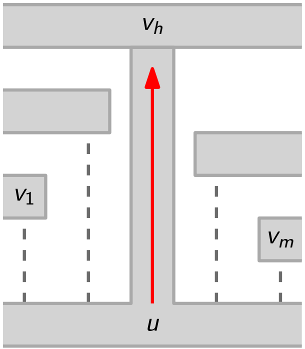

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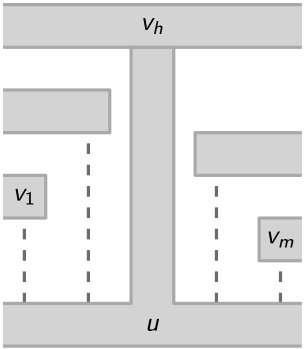

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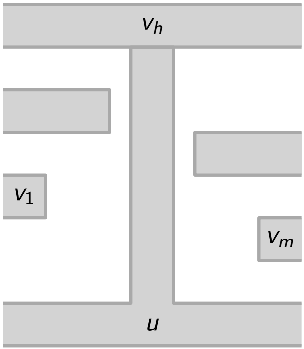

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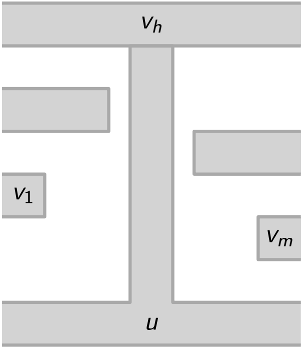

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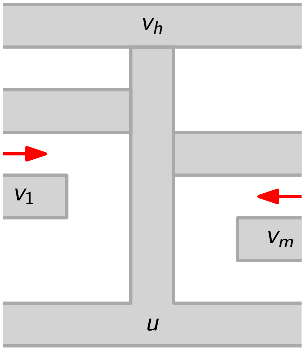

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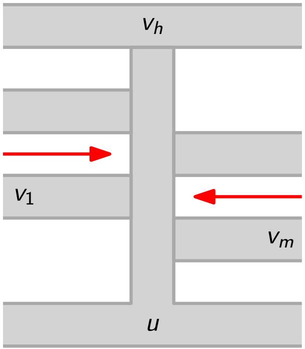

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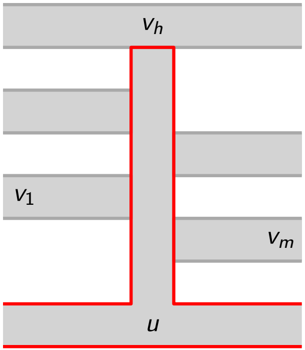

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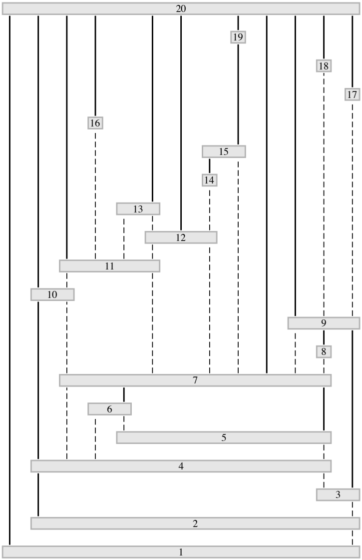

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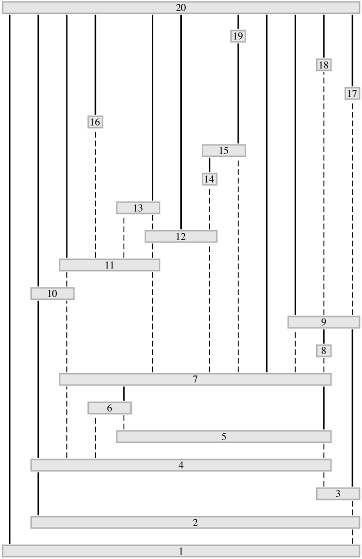

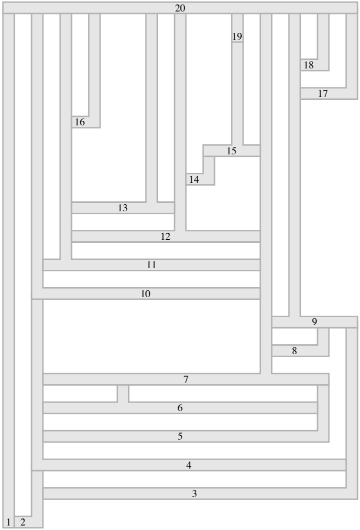

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Bitonic *st*-ordering

- Special *st*-ordering obtainable in linear time
- Can be used similar to canonical orderings
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- Relation to upward straight-line drawings
- More applications, undirected and directed

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