Planar Octilinear Drawings with One Bend Per Edge

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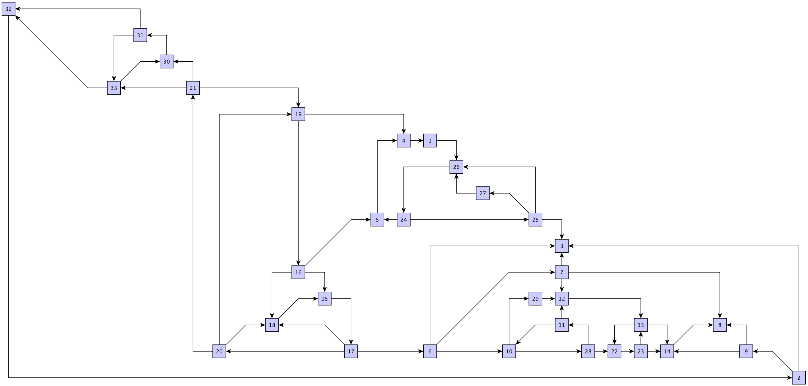
26.09.2014

Motivation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Previous- and Related Work

- M. Nöllenburg: Automated drawings of metro maps [2005]
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- E. Di Giacomo et al.: The planar slope number of subcubic graphs [2014]
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  - All vertices of P_k have at least one neighbor in a P_j with $j > k$
  - $|P_k| = 1$ is called *singleton*, $|P_k| > 1$ is called *chain*

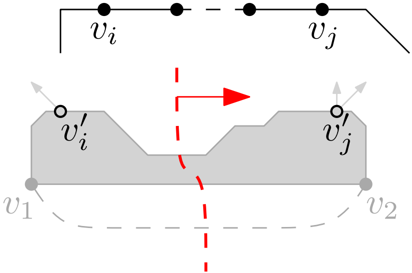
The Triconnected Case

Start of the construction

The Triconnected Case

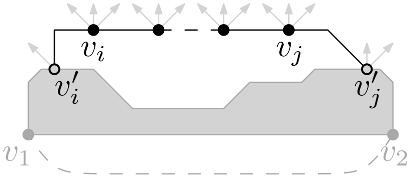
First Partition

The Triconnected Case



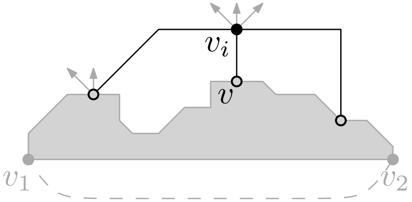
Placing a chain may require stretching

The Triconnected Case



Placing a chain

The Triconnected Case



Placing a singleton

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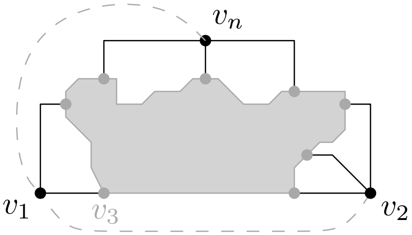
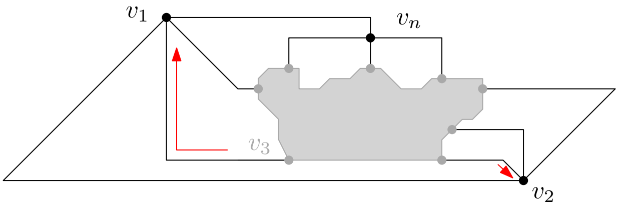


Placing of v_n step 1

The Triconnected Case



Final layout

Results for 4-planar Graphs

Theorem

There exists an infinite class of 4-planar graphs which do not admit bendless octilinear drawings and if they are drawn with at most one bend per edge, then a linear number of bends is required

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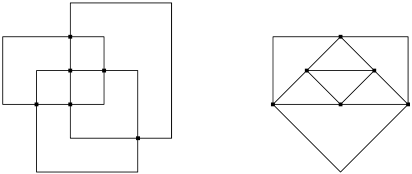
Theorem

Given a triconnected 4-planar graph G, we can compute in $O(n)$ time an octilinear drawing of G with at most one bend per edge on an $O(n^2) \times O(n)$ integer grid.



Non-triconnected Graphs

- Extend to biconnected by using *SPQR*-trees and the triconnected algorithm for the *R*-nodes

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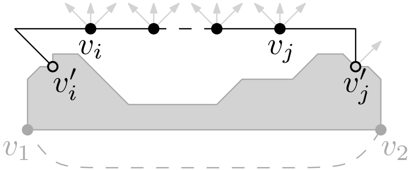

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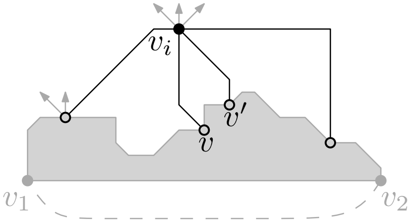

Placing a singleton

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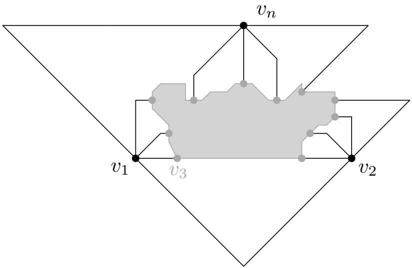
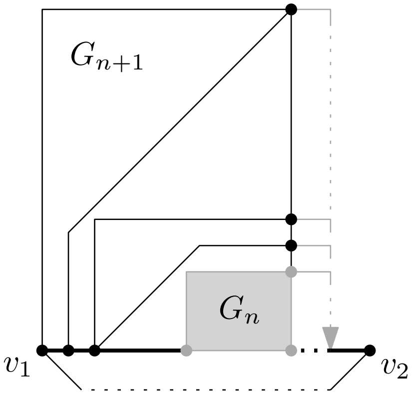


Final layout

Bad news



Super-polynomial area requirement

$$h(G_n) > w(G_n)$$
$$w(G_{n+1}) \geq 2w(G_n)$$
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Properties of the 5-planar Algorithm

Theorem

Given a triconnected 5-planar graph G, we can compute in $O(n^2)$ time an octilinear drawing of G with at most one bend per edge.

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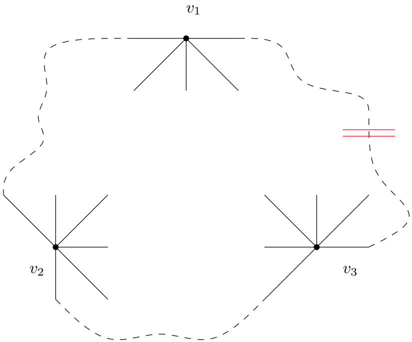
One Bend Per Edge Is Not Always Enough



Outer Face that does not admit a one-bend drawing

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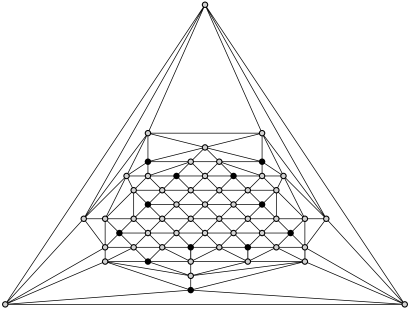

6-planar triangulation in which each is adjacent to only degree 6 (grey) vertices and at most one degree 5 (black) vertex

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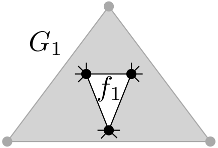

Construction of an infinite family of graphs

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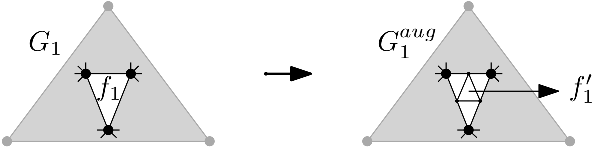

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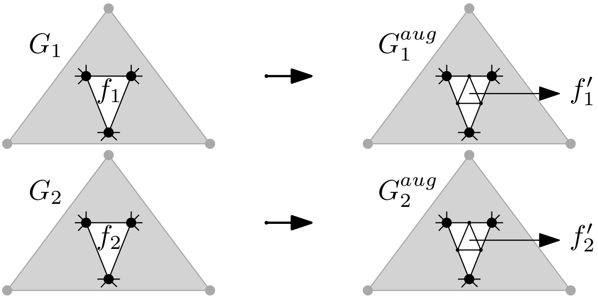

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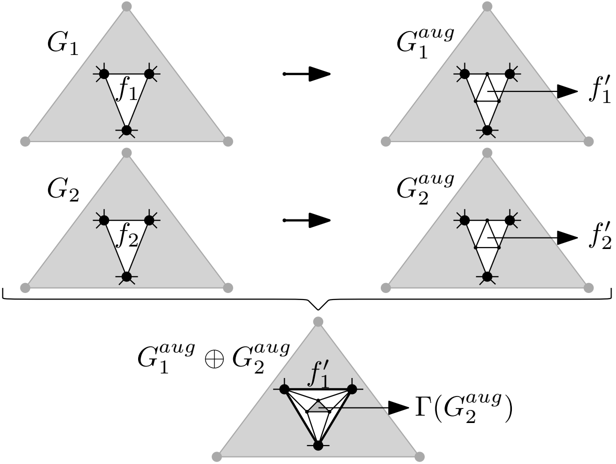

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- 4-planar graphs are octilinear drawable with at most one bend per edge in cubic area in linear time

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- 4-planar graphs are octilinear drawable with at most one bend per edge in cubic area in linear time
- 5-planar graphs are octilinear drawable with at most one bend per edge in super-polynomial area in quadratic time
- There exist 6-planar graphs that do not admit planar octilinear drawings with at most one bend per edge

Open Problems

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- What is the area requirement of 5-planar (triconnected) graphs?
- Do triangle-free 6-planar graph admit one-bend octilinear drawings?
- What is the complexity to determine whether a 6-planar graph admits a one-bend octilinear drawing?