EMBEDDING FOUR-DIRECTIONAL PATHS ON CONVEX POINT SETS

TU Graz

Oswin Aichholzer Thomas Hackl Birgit Vogtenhuber

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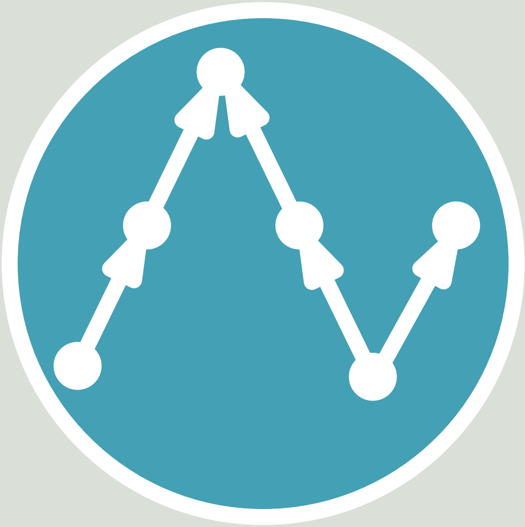
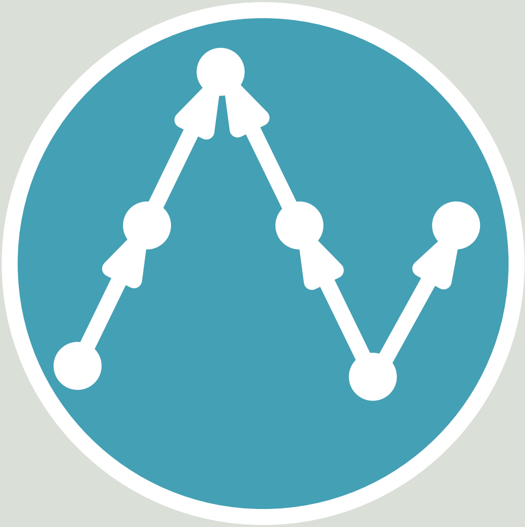


ORIENTED PATH

POINT SET

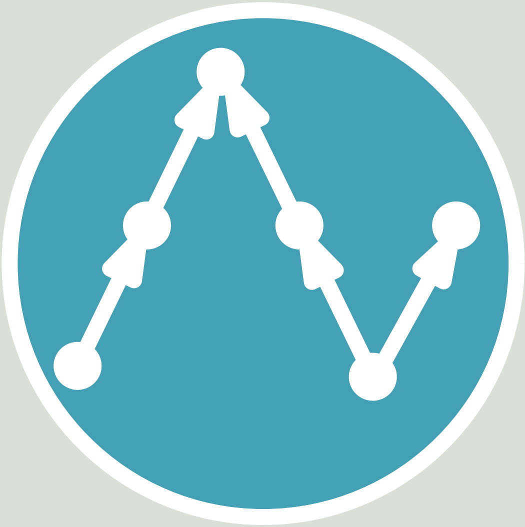


ORIENTED PATH

POINT SET

UPWARD PLANAR EMBEDDING

MOTIVATION & PREVIOUS WORK



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KNOWN RESULTS

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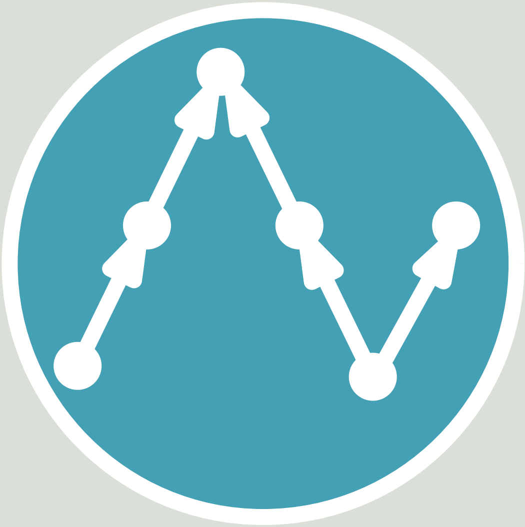

ORIENTED PATH

POINT SET

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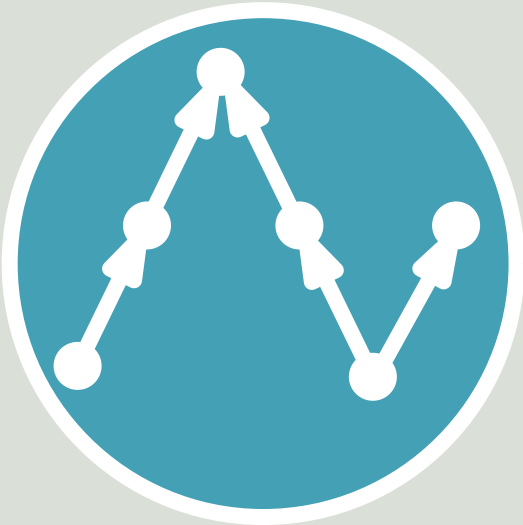
KNOWN RESULTS

Always possible for ≤ 10

Directed order types

MOTIVATION & PREVIOUS WORK



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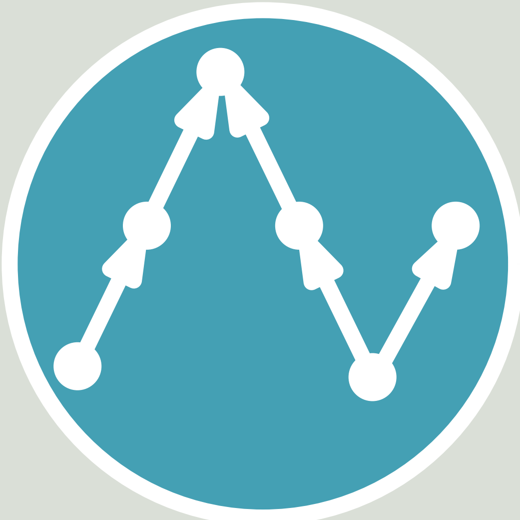
KNOWN RESULTS

Always possible for ≤ 10

Several special cases of paths

Directed order types

Binucci et al. CGTA10
Angelini el al. GD10



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KNOWN RESULTS

Always possible for ≤ 10	Directed order types
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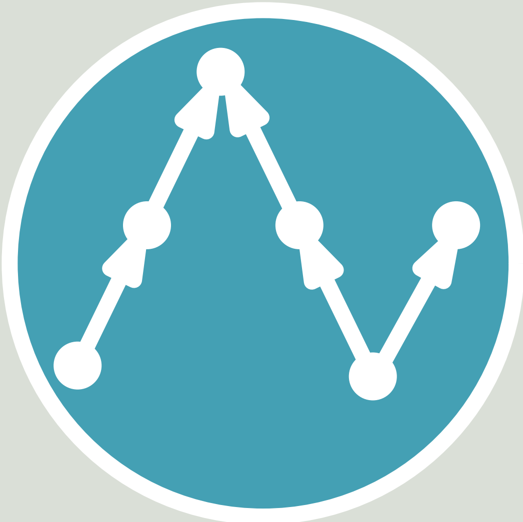
Convex point sets	Binucci et al. CGTA10

MOTIVATION & PREVIOUS WORK



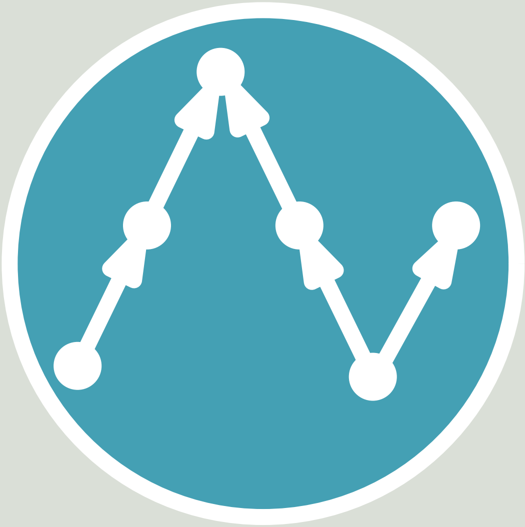
ORIENTED PATH

POINT SET

UPWARD PLANAR EMBEDDING

QUESTION

MOTIVATION & PREVIOUS WORK



ORIENTED PATH

POINT SET

UPWARD PLANAR EMBEDDING

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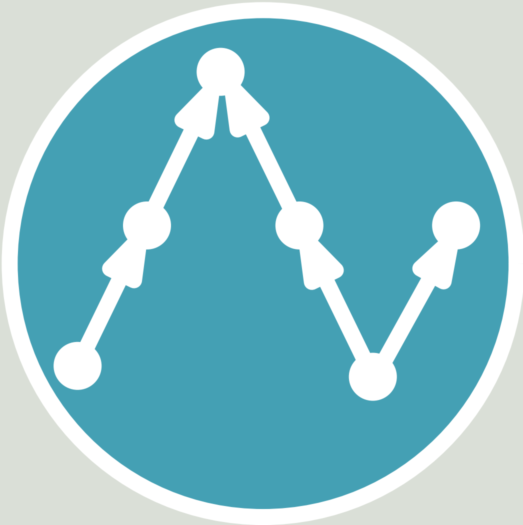
Is it possible for any point set in general position?

MOTIVATION & PREVIOUS WORK



ORIENTED PATH

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We still do not know

LET'S LOOK AT NUMBERS

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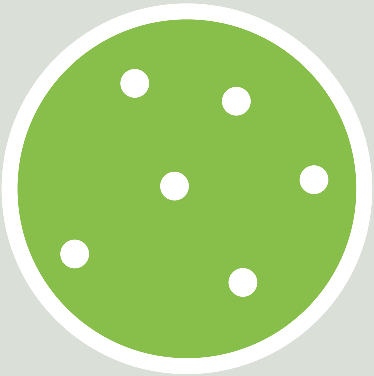

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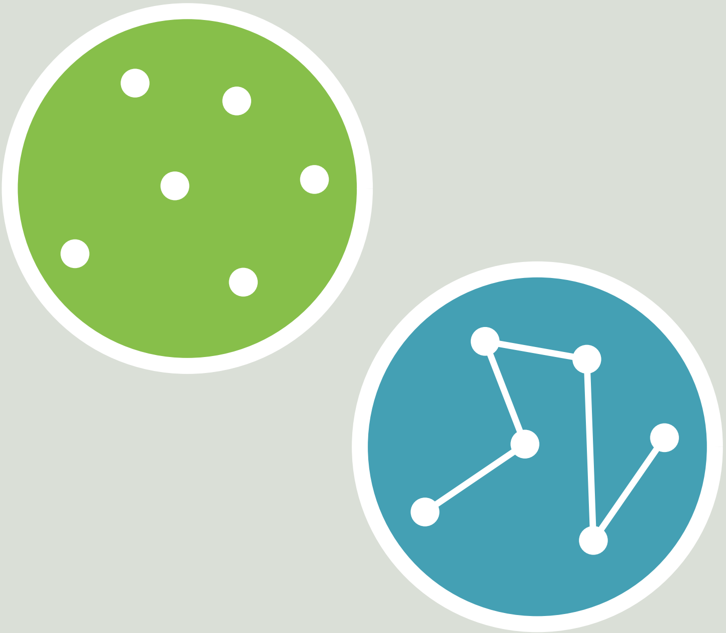

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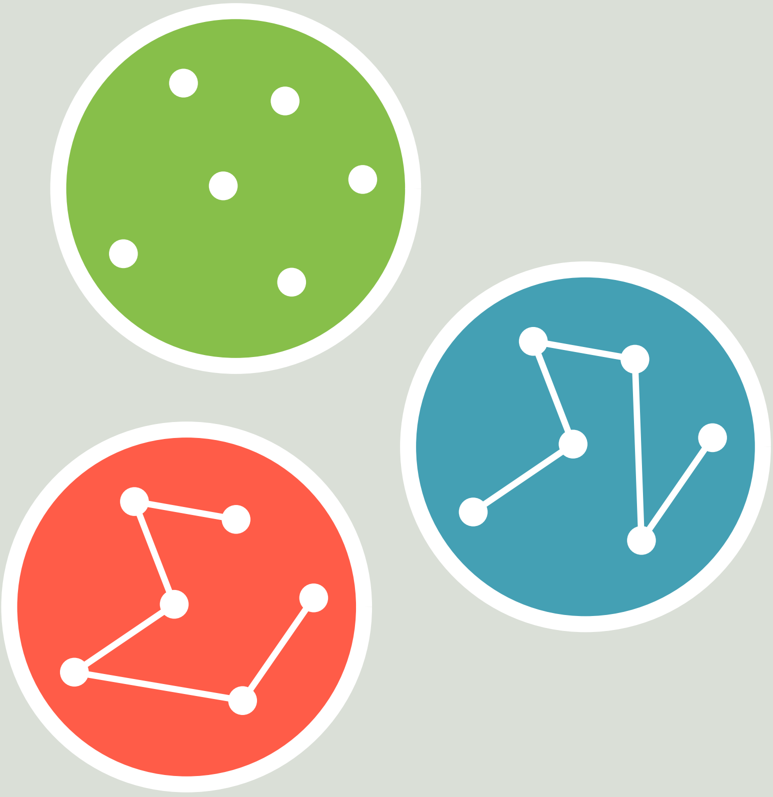

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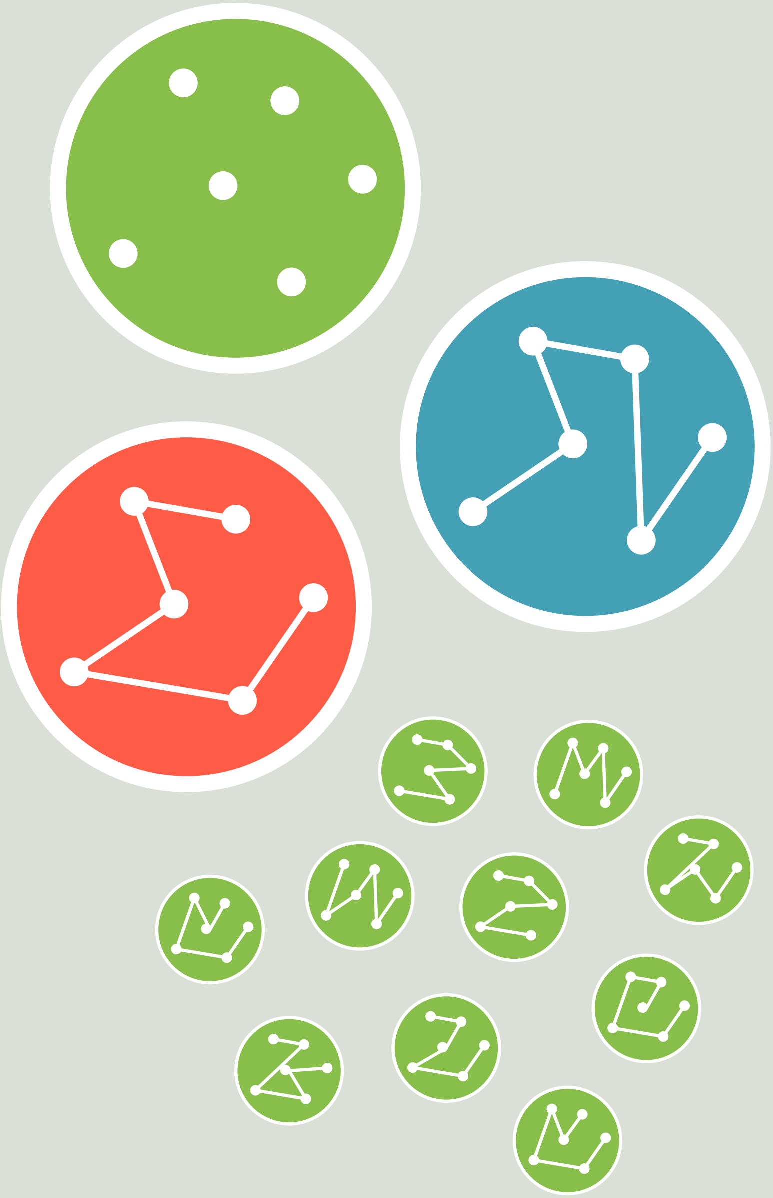

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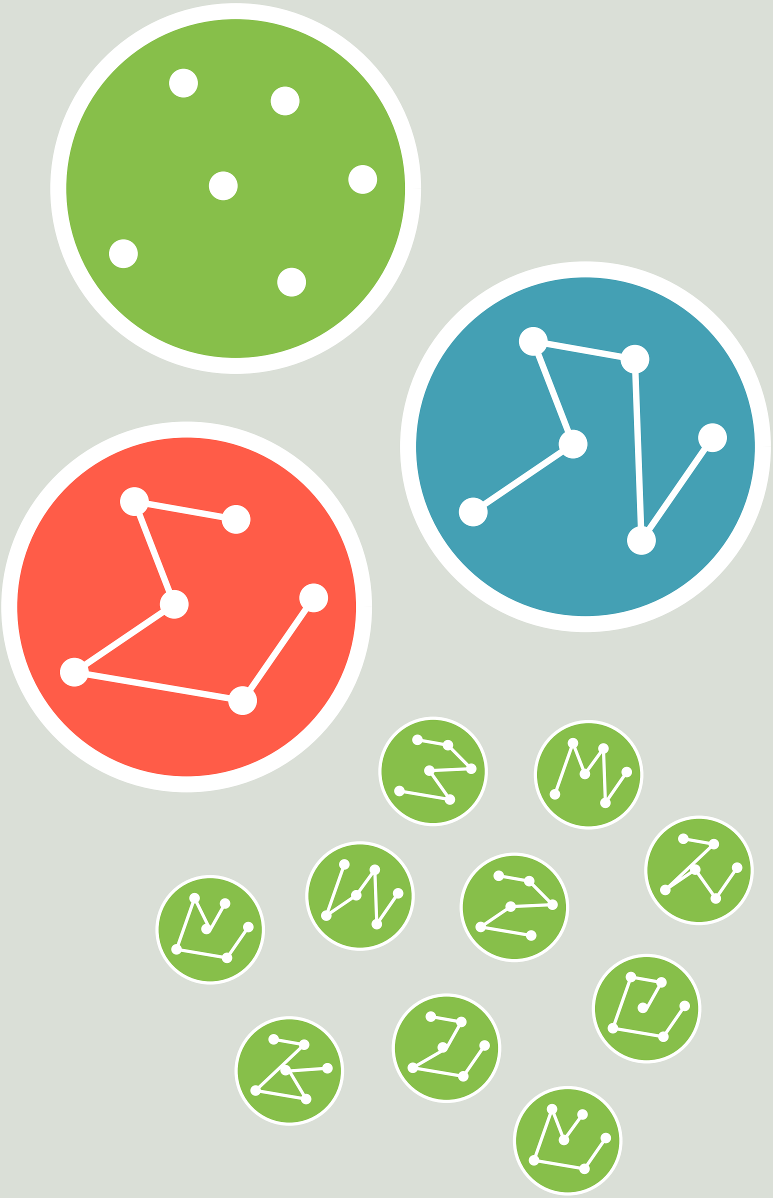
How many distinct plane spanning paths has a point set?

At least $n2^{n-3}$, which is achieved by convex point sets



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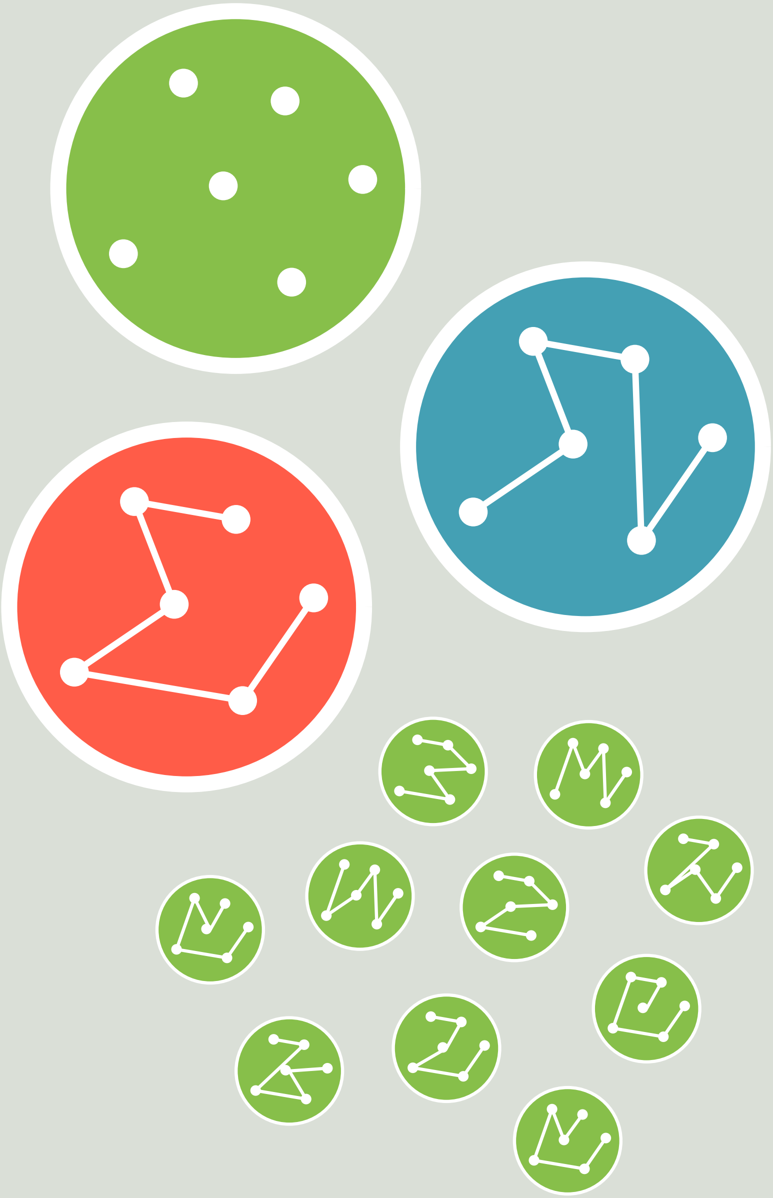

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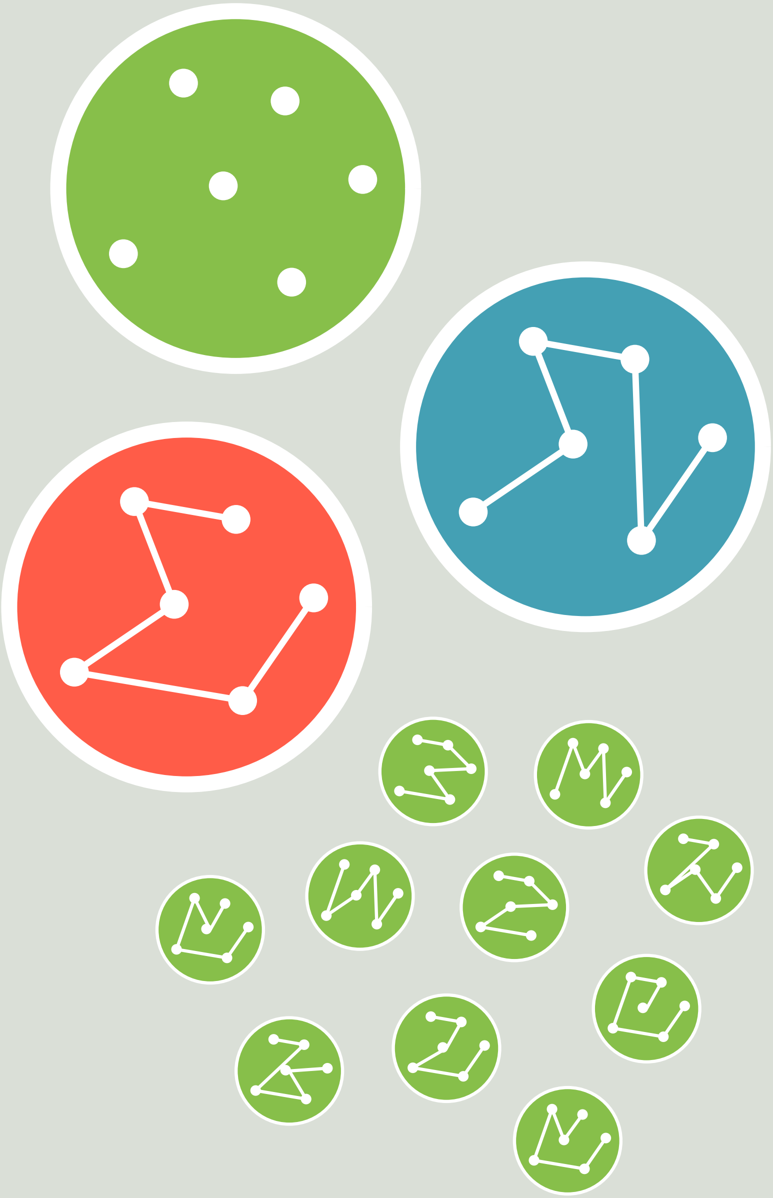
Even for convex point sets it is surprising that embedding always exists



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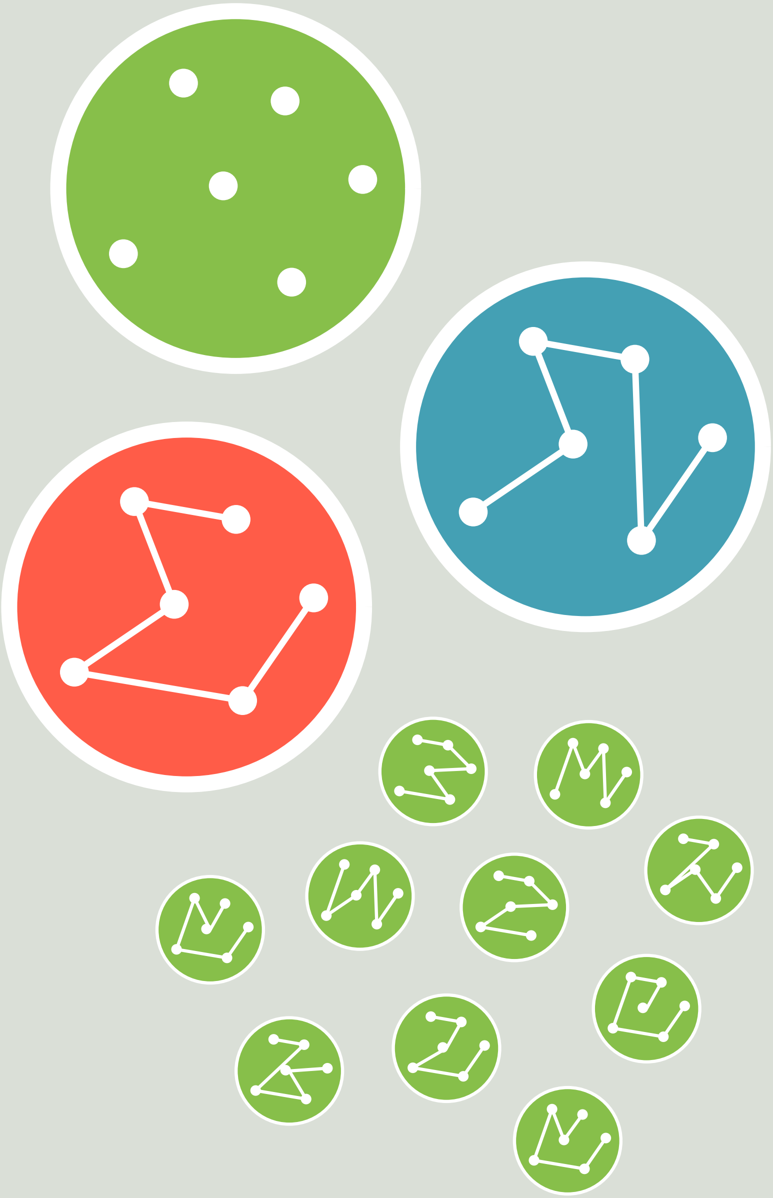

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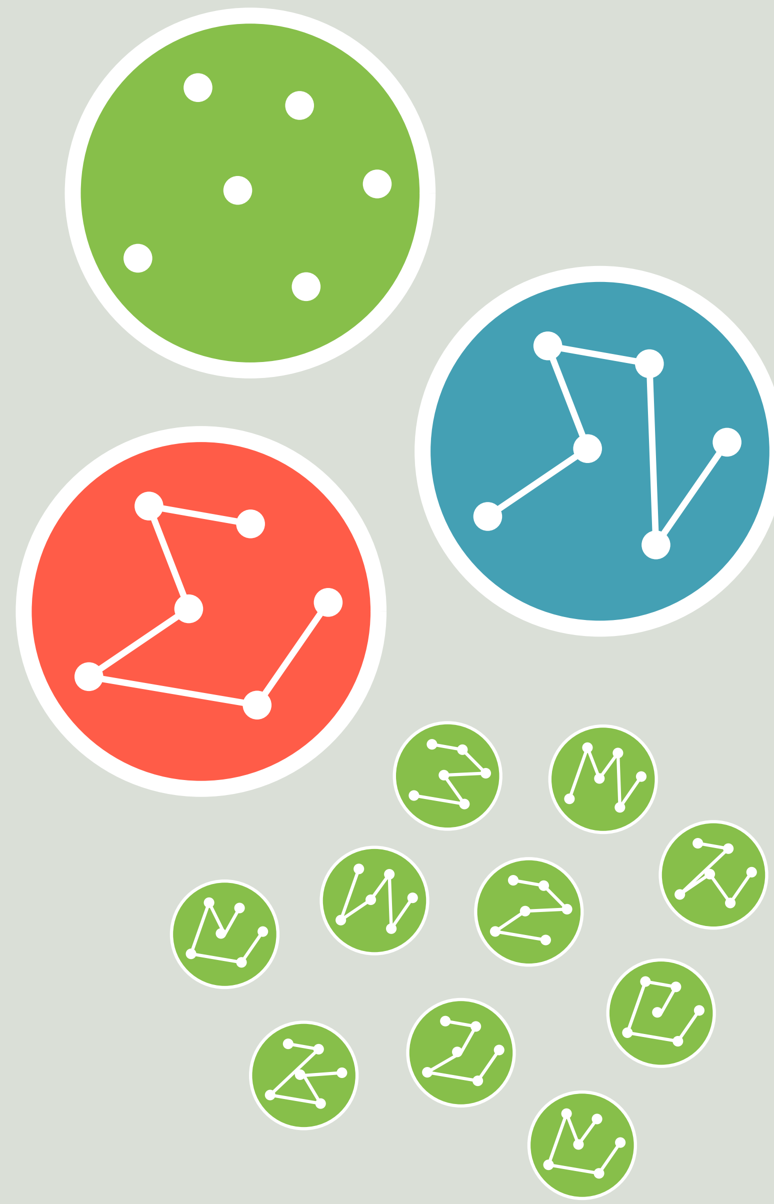

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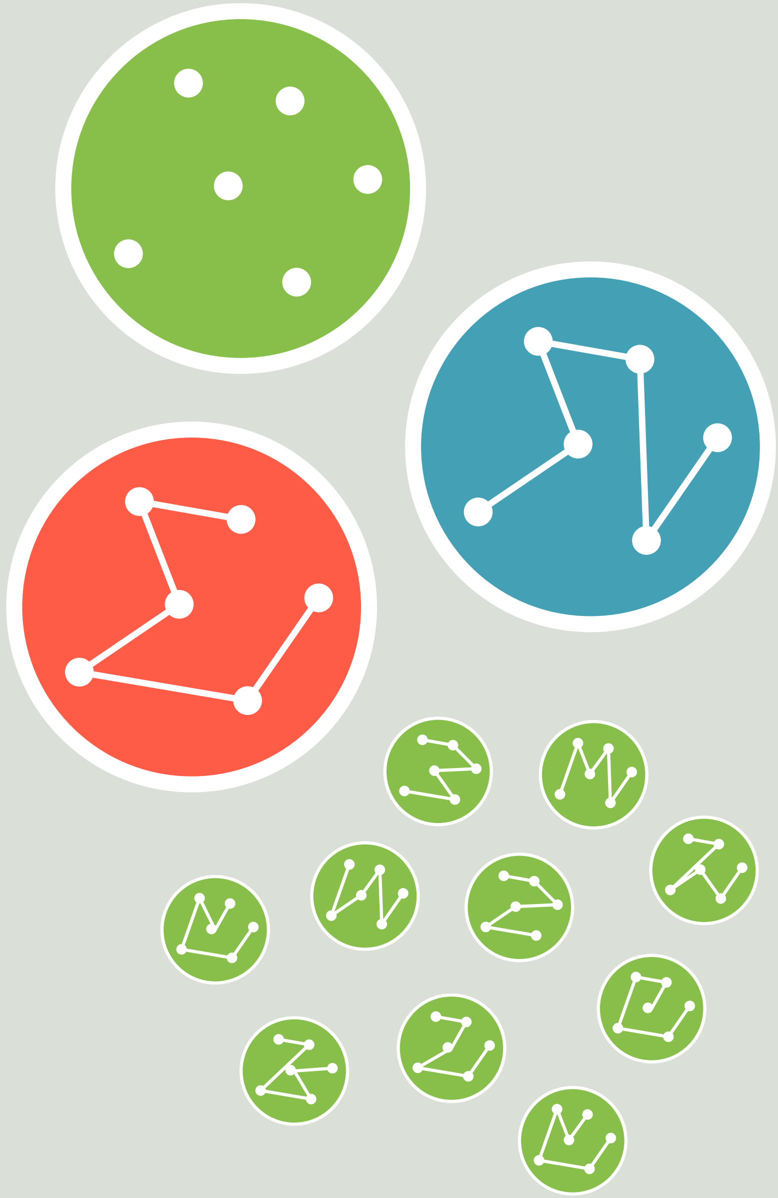

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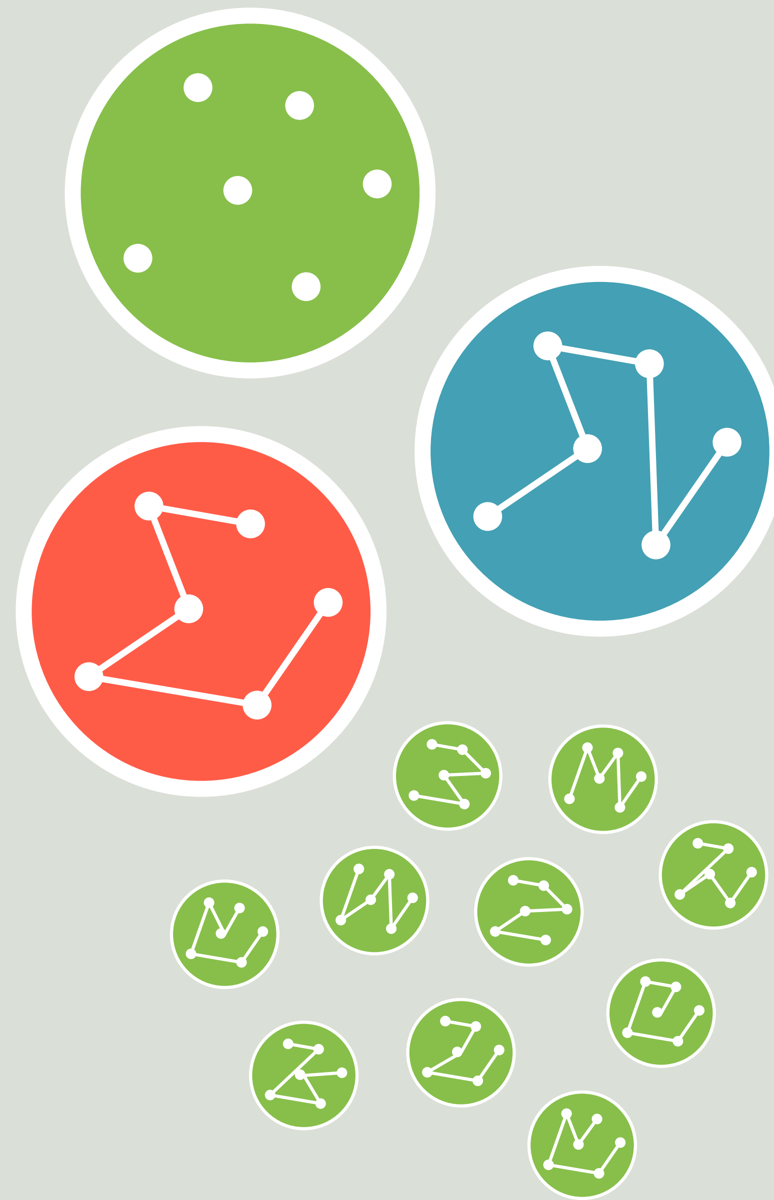

FOR CONVEX POINT SETS THE NUMBERS ARE TIGHT

WHAT HAPPENS WITH FOUR?

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PROBLEM DEFINITION

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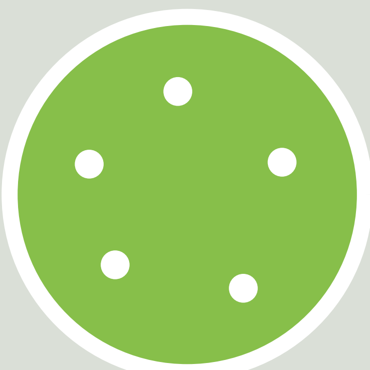

PROBLEM DEFINITION & RESULTS

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DIRECTION-CONSISTENT EMBEDDING

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Not always possible for four directions

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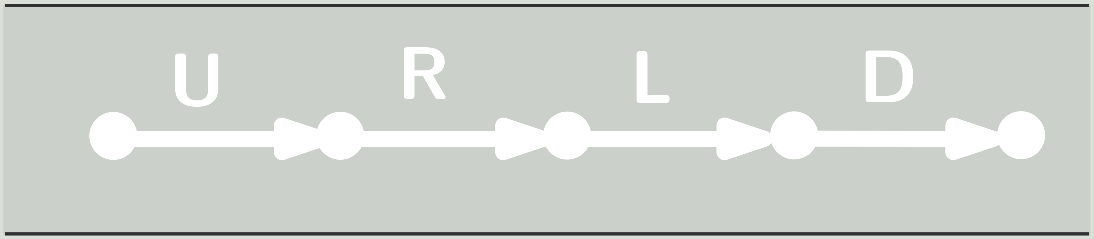

DIRECTION-CONSISTENT EMBEDDING

RESULTS

Not always possible for four directions
Always possible for three directions
Can be decided in $O(n^2)$ time for four directions.

COUNTING?

There are $n2^{n-3}$ oriented paths

Each can be labeled in 2^{n-1} ways
and
read from 2 end-vertices

In total at most $n2^{2n-3}$ plane
4-directional paths
on a convex point set

To compare with 2^{2n-2}
4-directional paths

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COUNTEREXAMPLE

FOUR-DIRECTIONAL

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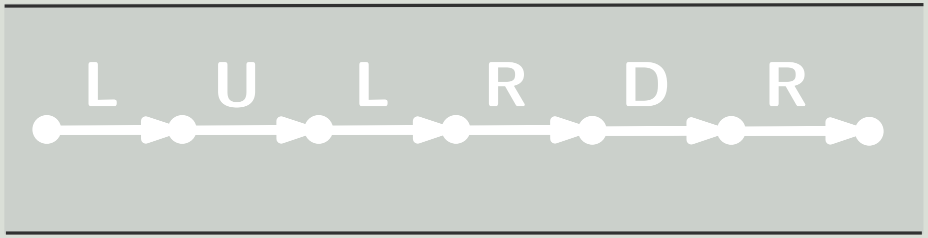

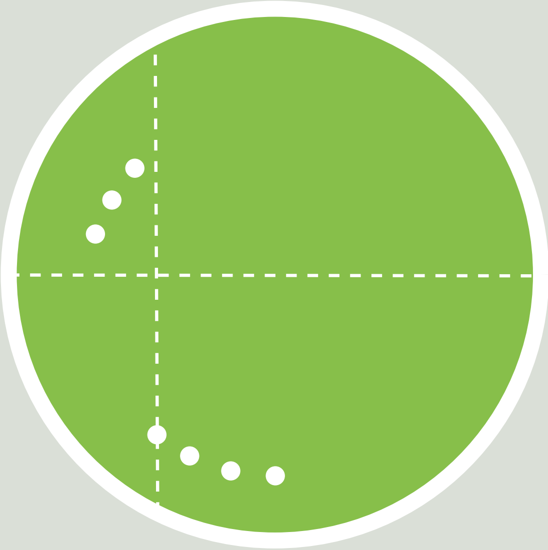

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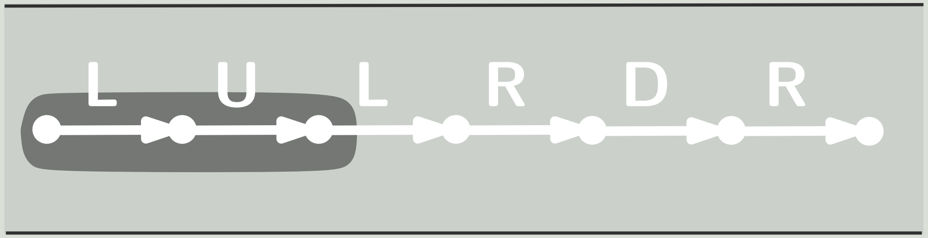

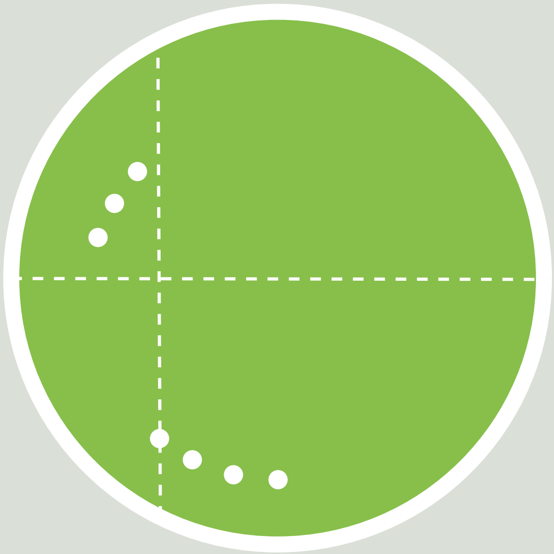

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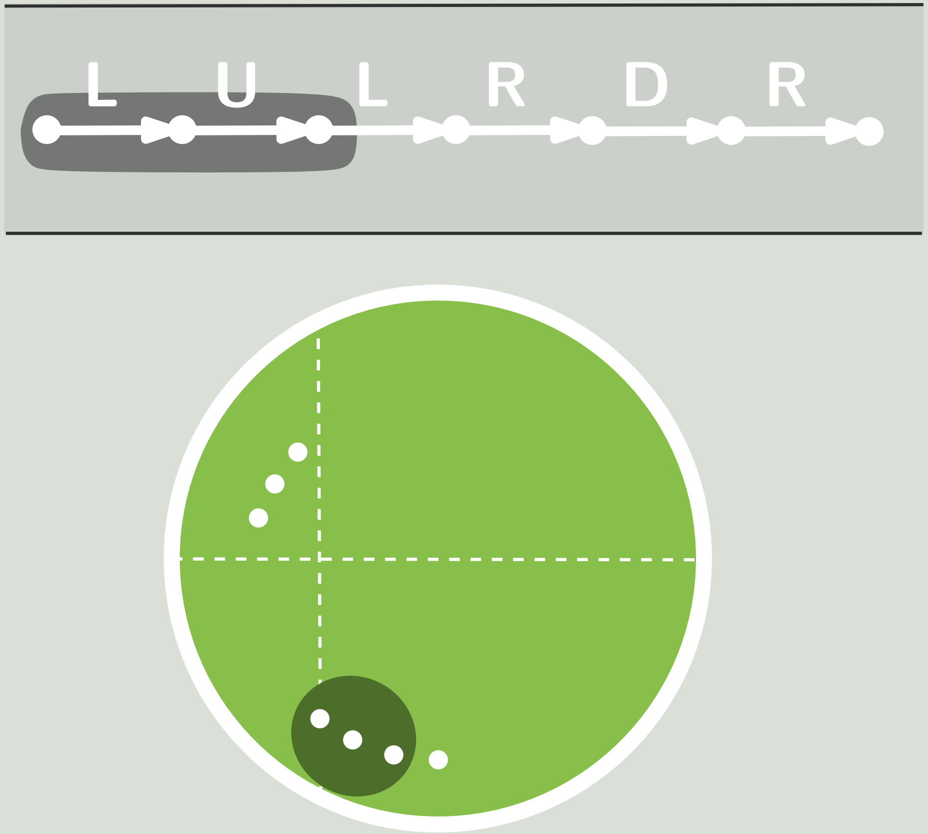

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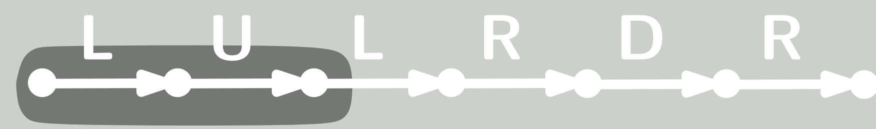

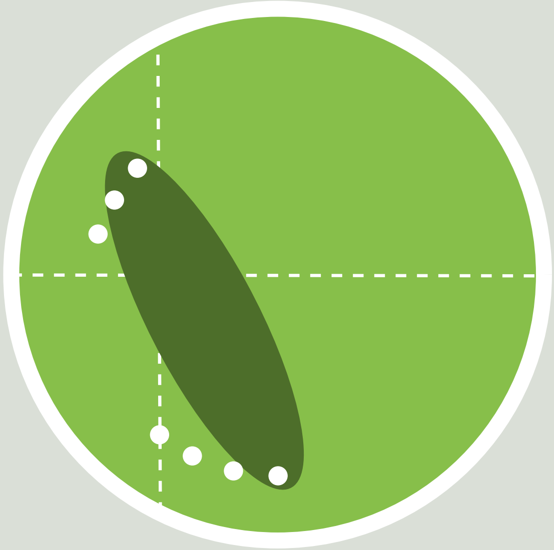

THEOREM

Any three-directional path admits a direction-consistent embedding on any convex point set

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"ONE-SIDED" LEMMA

A $\{U,D,R\}$-path admits a direction-consistent embedding on a one-sided convex point set

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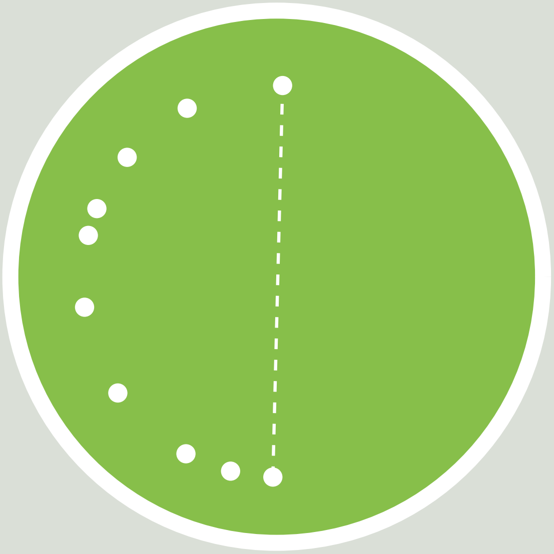

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THREE-DIRECTIONAL

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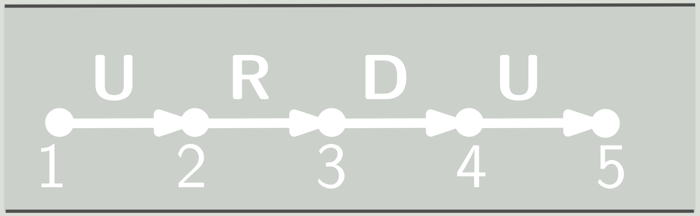

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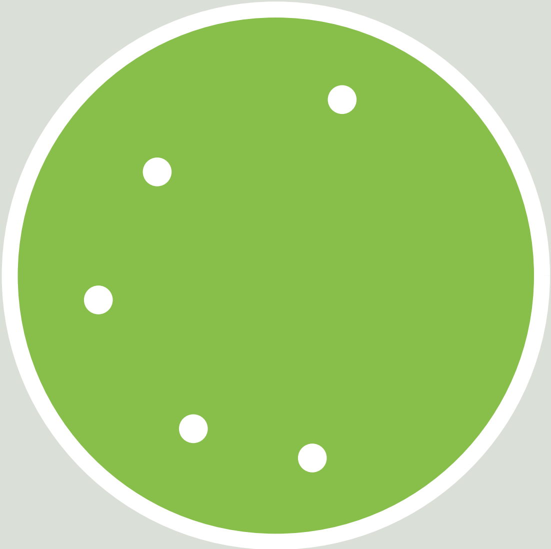

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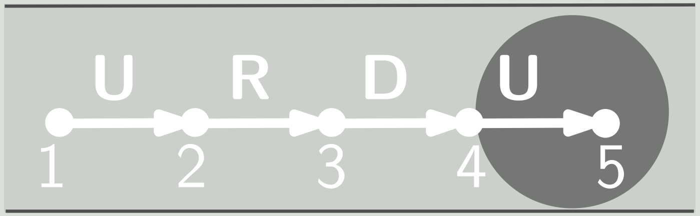

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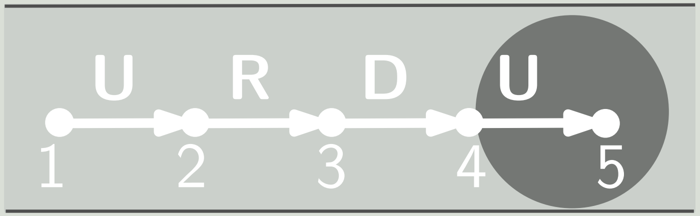

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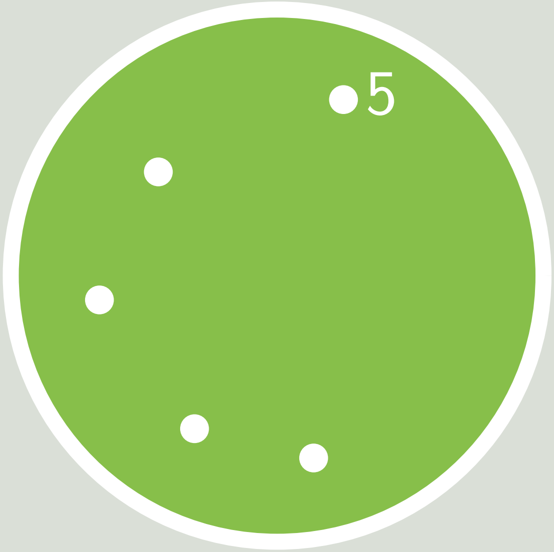

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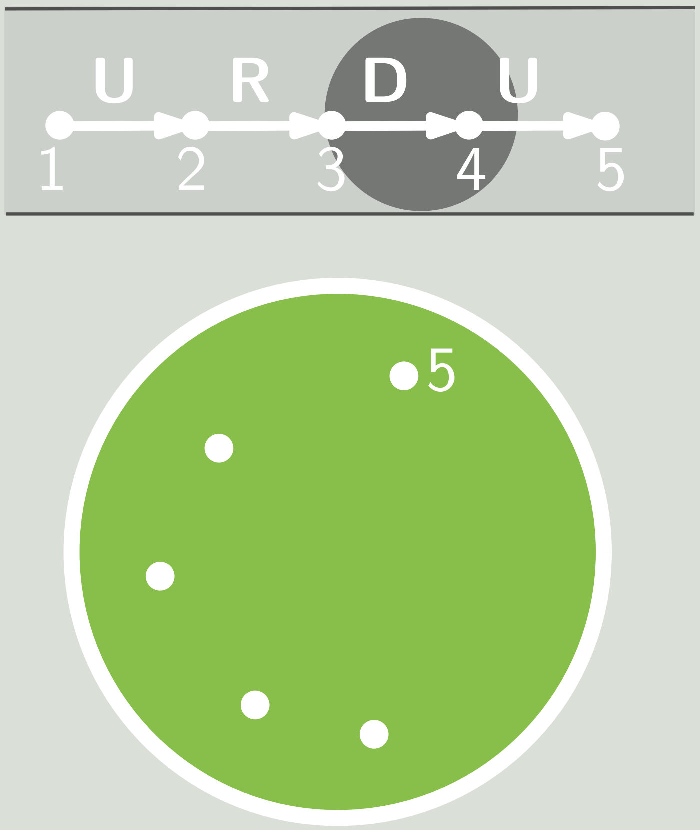

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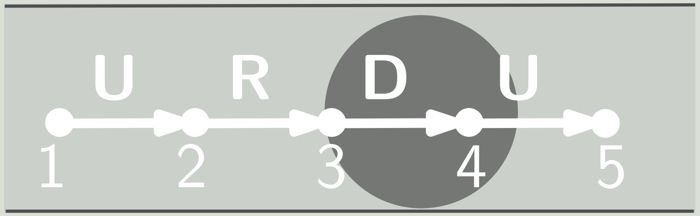

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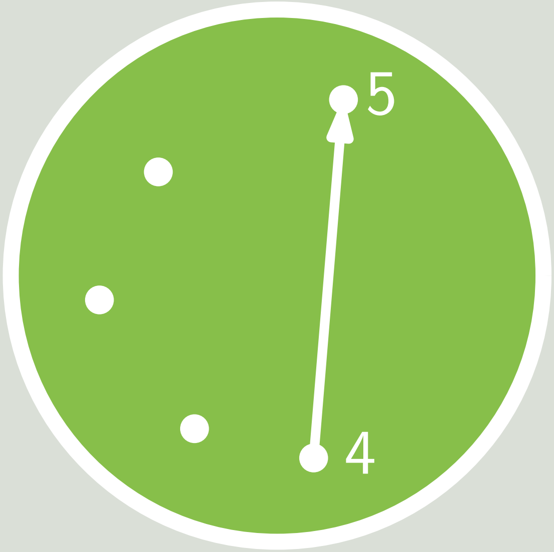

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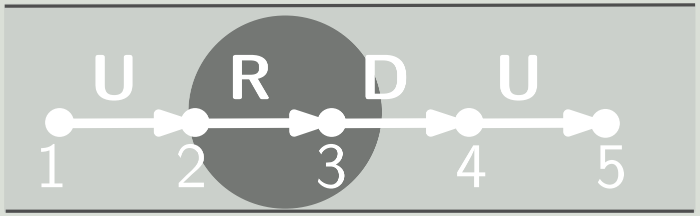

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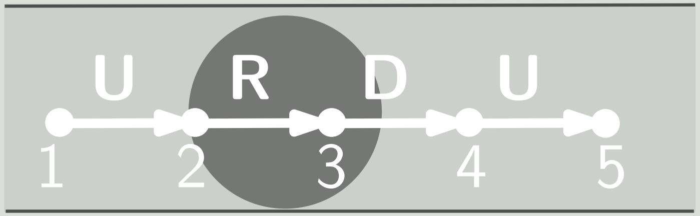

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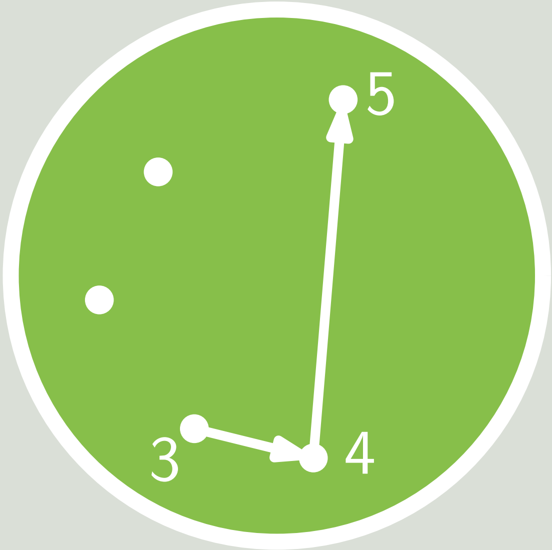

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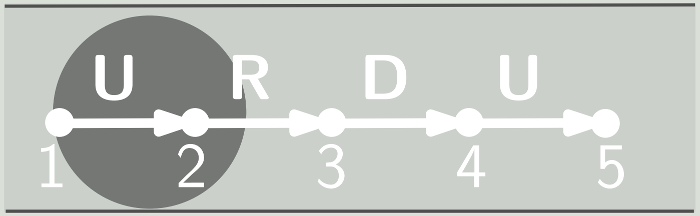

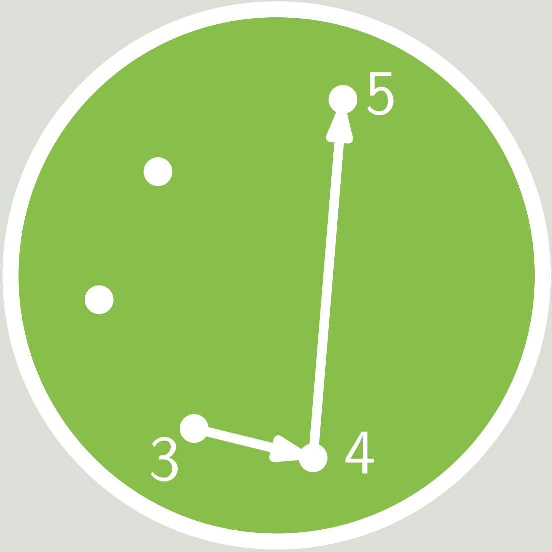

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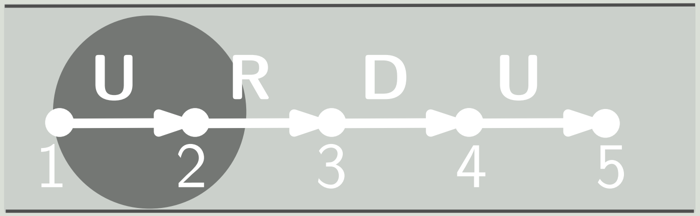

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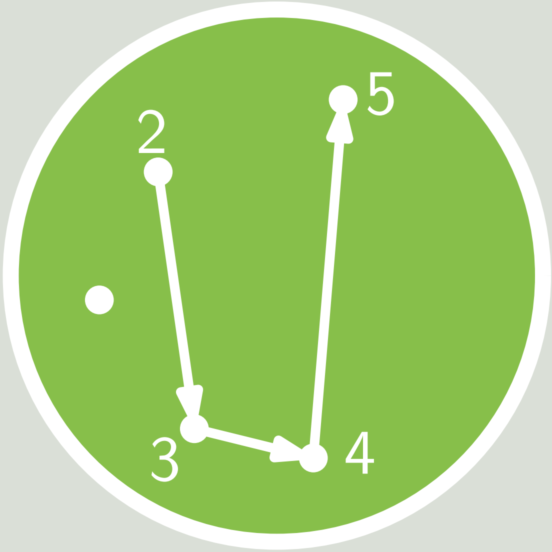

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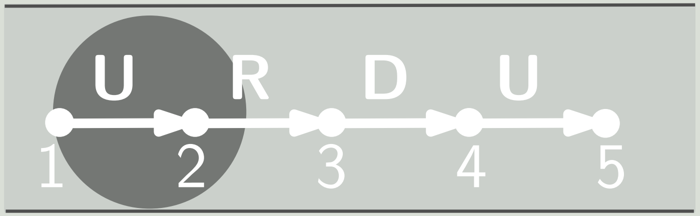
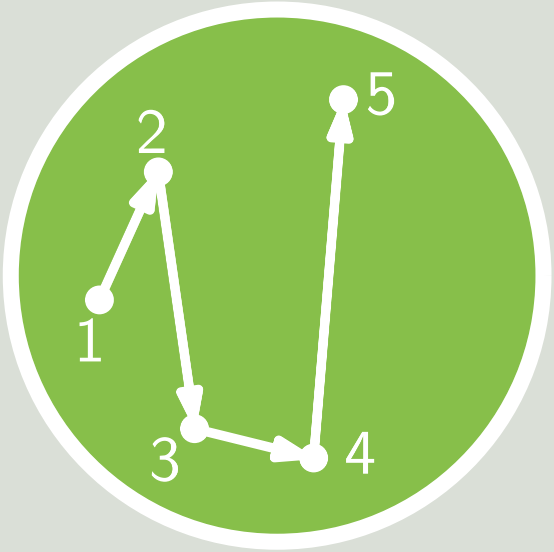


“ONE-SIDED” LEMMA

A {U,D,R}-path admits a direction-consistent embedding on a one-sided convex point set



“PROOF”

Proceed the path backward. Choose the topmost (bottommost, rightmost) free point, if the previous edge has label U (D,R).

THEOREM

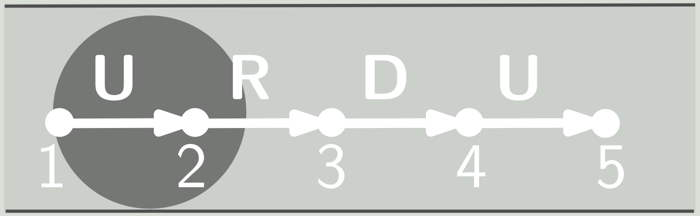
Any three-directional path admits a direction-consistent embedding on any convex point set

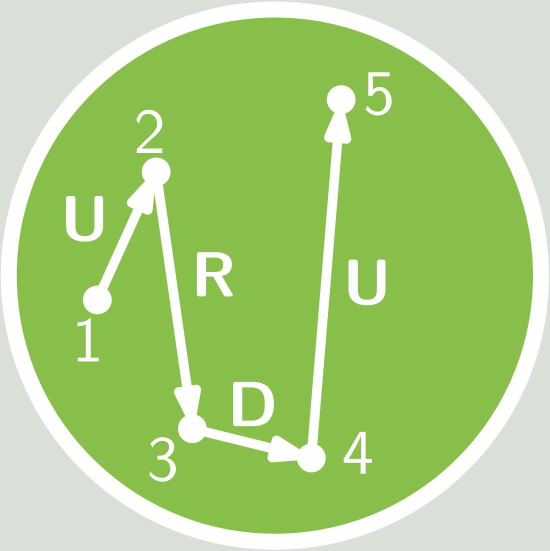
"ONE-SIDED" LEMMA

A {U,D,R}-path admits a direction-consistent embedding on a one-sided convex point set

"PROOF"

Proceed the path backward. Choose the topmost (bottomost, rightmost) free point, if the previous edge has label U (D,R).





THEOREM

Any three-directional path admits a direction-consistent embedding on any convex point set

"ONE-SIDED" LEMMA

A {U,D,R}-path admits a direction-consistent embedding on a one-sided convex point set

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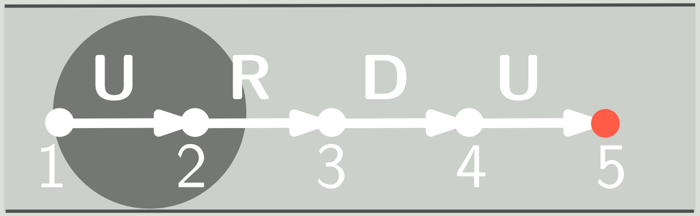

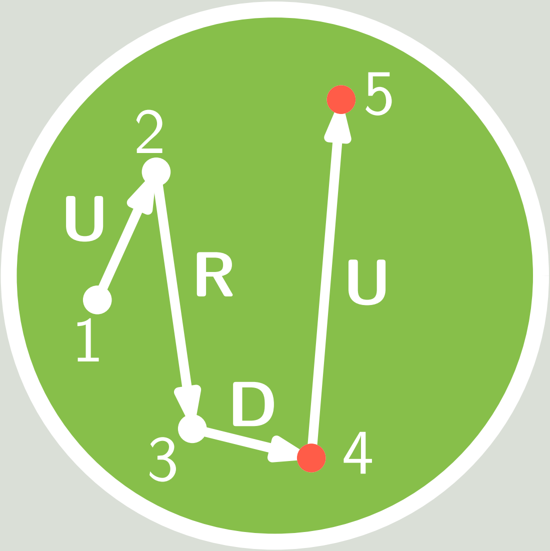

THEOREM

Any three-directional path admits a direction-consistent embedding on any convex point set

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Any three-directional path admits a direction-consistent embedding on any convex point set

"STRIP-CONVEX" LEMMA

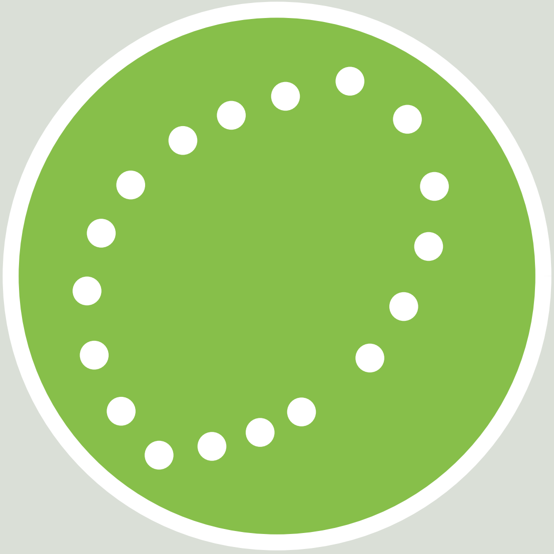
A $\{U,R\}$-path admits a direction-consistent embedding on a strip-convex point set

THEOREM

Any three-directional path admits a direction-consistent embedding on any convex point set

“STRIP-CONVEX” LEMMA

A {U,R}-path admits a direction-consistent embedding on a strip-convex point set



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"STRIP-CONVEX" LEMMA

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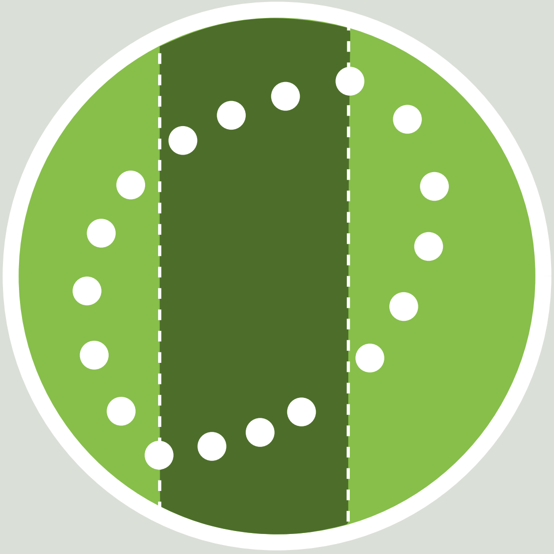

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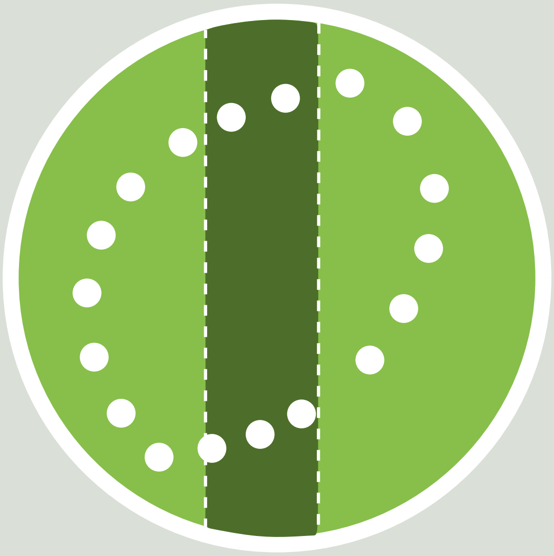
A {U,R}-path admits a direction-consistent embedding on a strip-convex point set



THEOREM

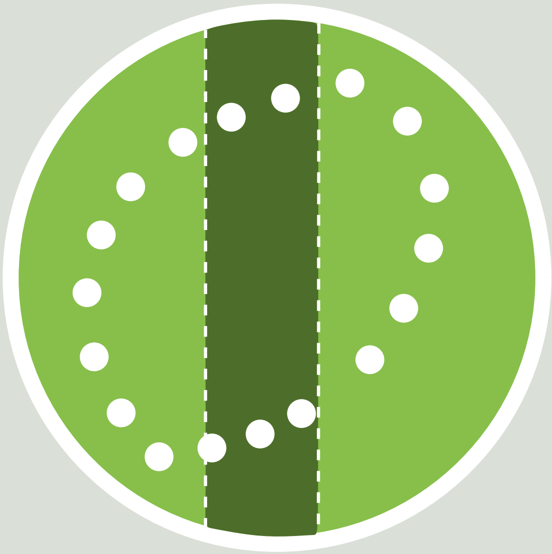
Any three-directional path admits a direction-consistent embedding on any convex point set

"STRIP-CONVEX" LEMMA

A {U,R}-path admits a direction-consistent embedding on a strip-convex point set

"PROOF"

Apply the same algorithm. Observe that the identified points are consecutive.



THEOREM

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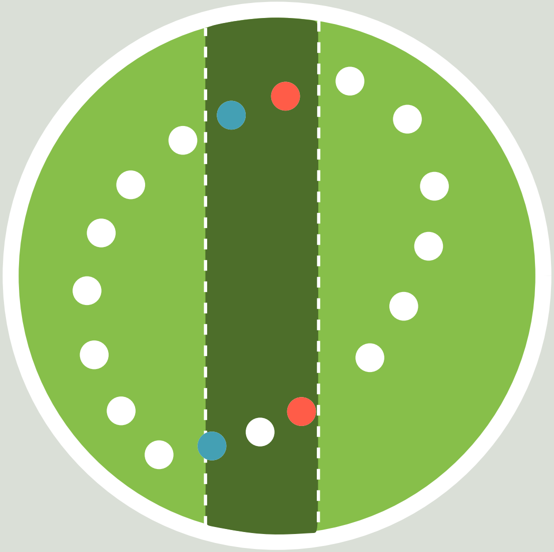

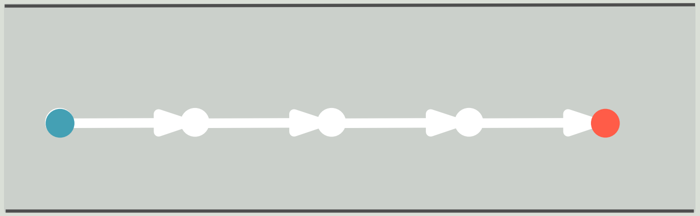

THEOREM

Any three-directional path admits a direction-consistent embedding on any convex point set

{U,D,R}-LEMMA

A {U,D,R}-path admits a direction-consistent embedding on a convex point set*

{U,D,R}-LEMMA

A {U,D,R}-path admits a direction-consistent embedding on a convex point set*

“PROOF”

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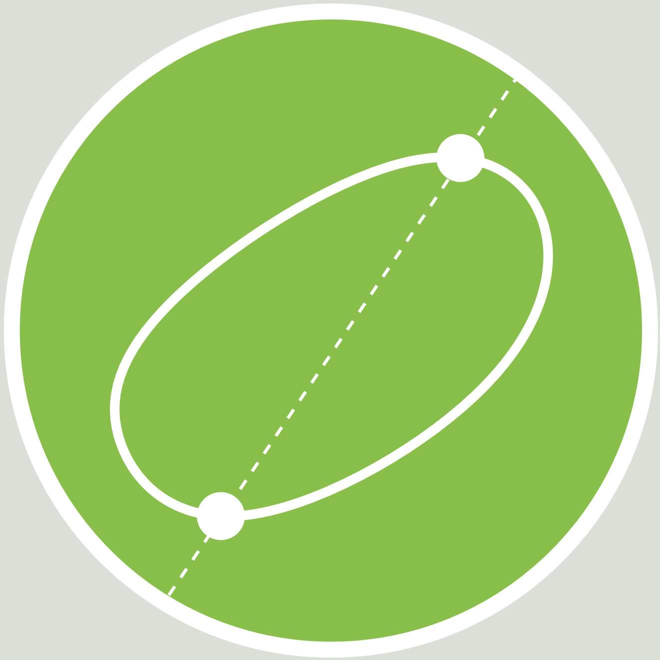
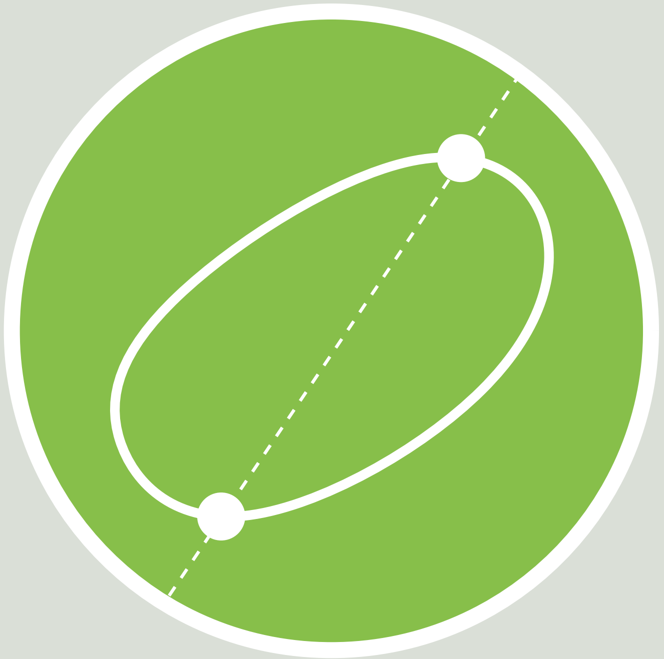


{U,D,R}-LEMMA

A {U,D,R}-path admits a direction-consistent embedding on a convex point set*

"PROOF"



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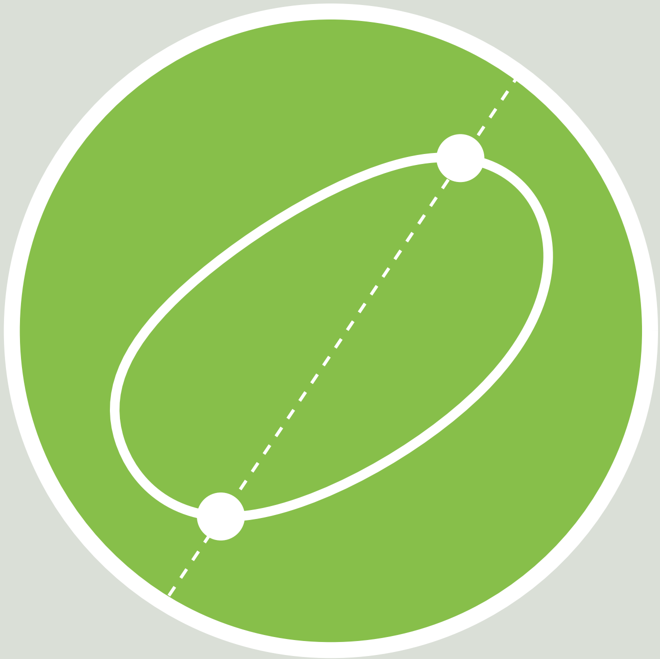
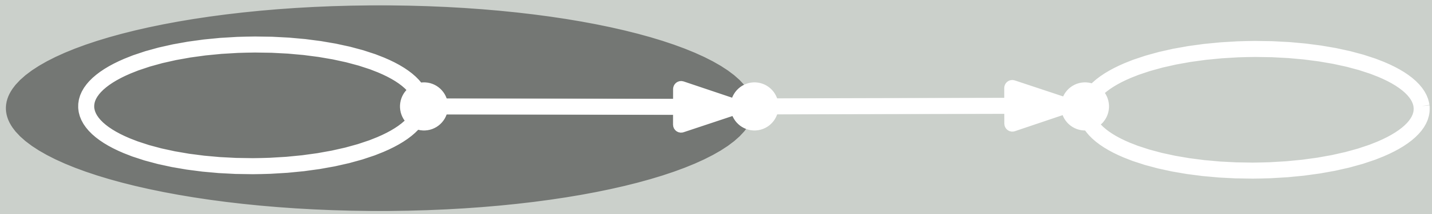
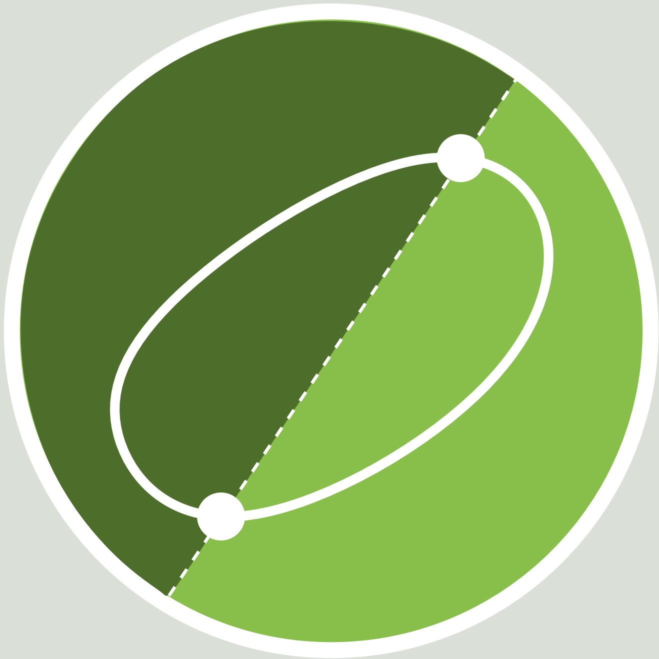


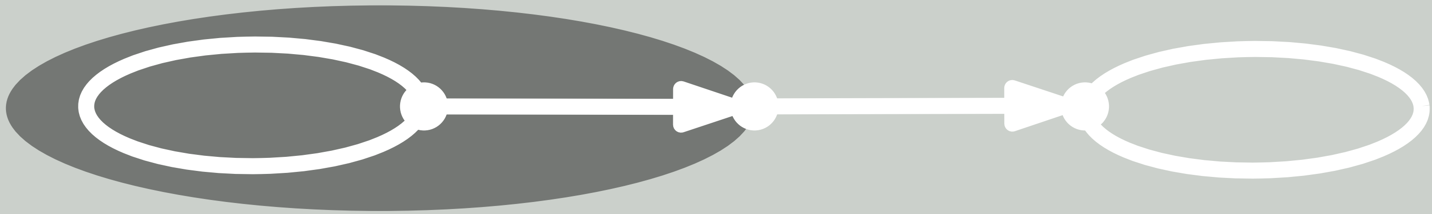


{U,D,R}-LEMMA

A {U,D,R}-path admits a direction-consistent embedding on a convex point set*

"PROOF"



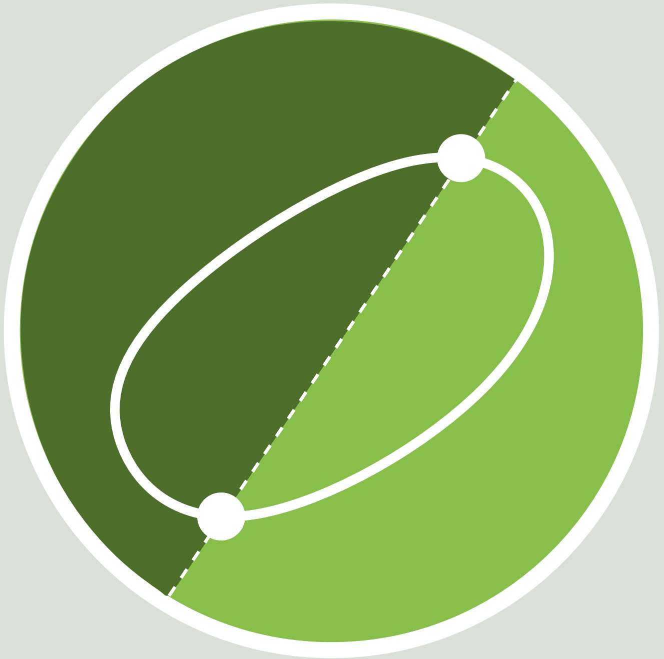


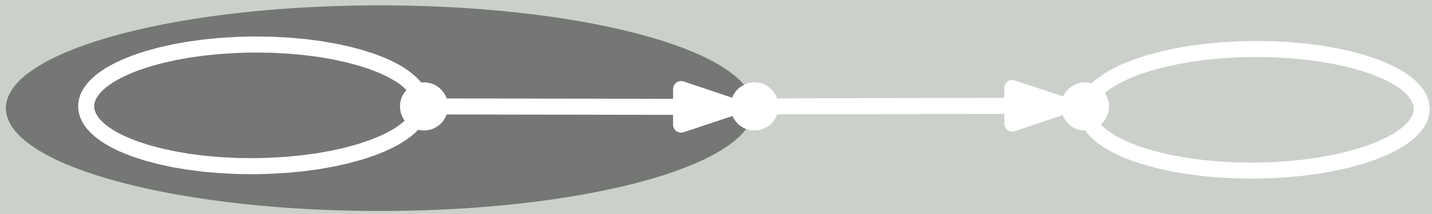
{U,D,R}-LEMMA

A {U,D,R}-path admits a direction-consistent embedding on a convex point set*

"PROOF"

One of the boundary edges is **D**





{U,D,R}-LEMMA

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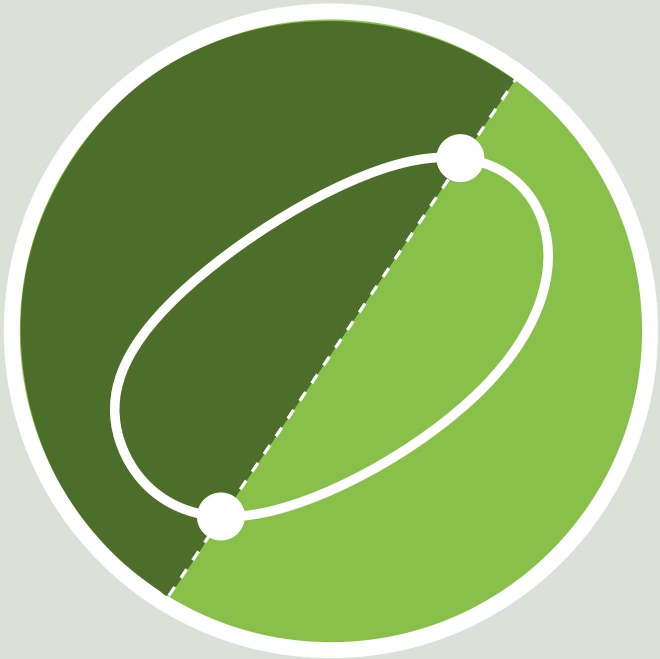
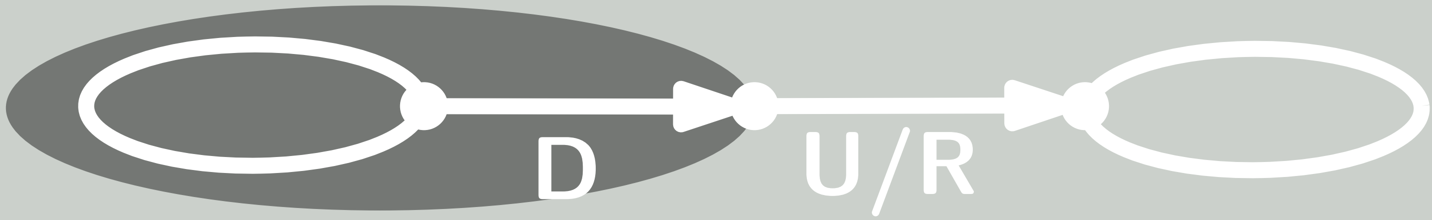
One of the boundary edges is **D**





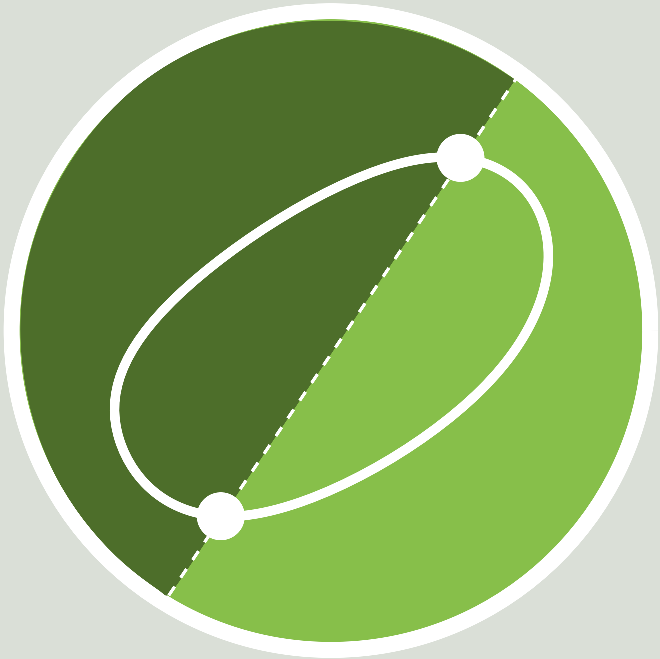
{U,D,R}-LEMMA

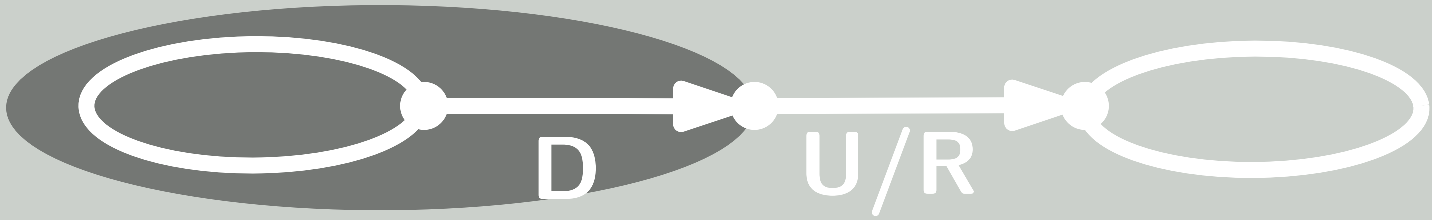
A {U,D,R}-path admits a direction-consistent embedding on a convex point set*

"PROOF"

One of the boundary edges is **D**

Apply "one-sided" Lemma





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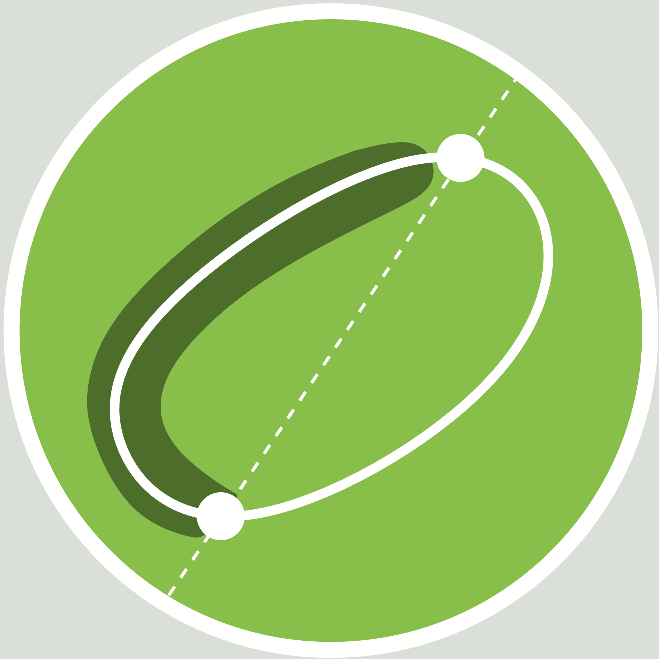

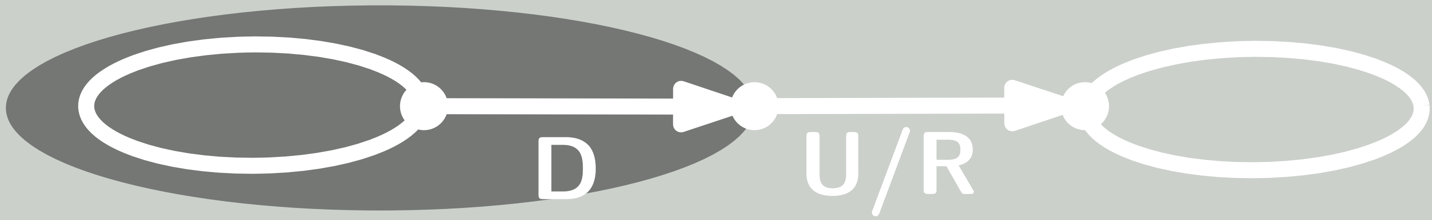

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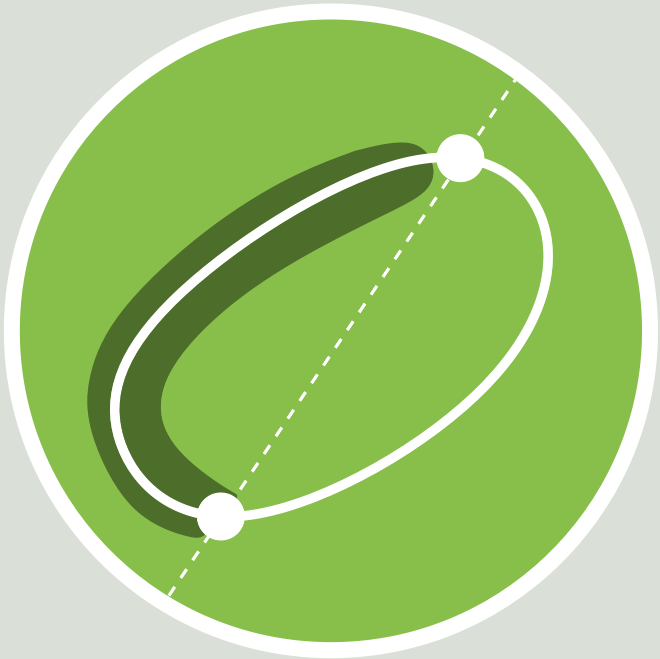

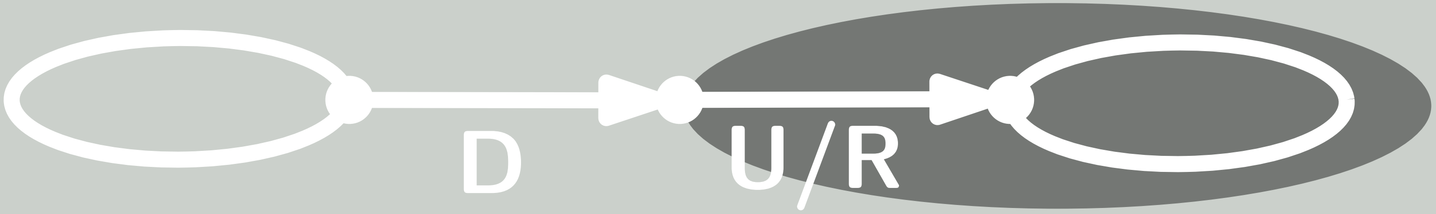

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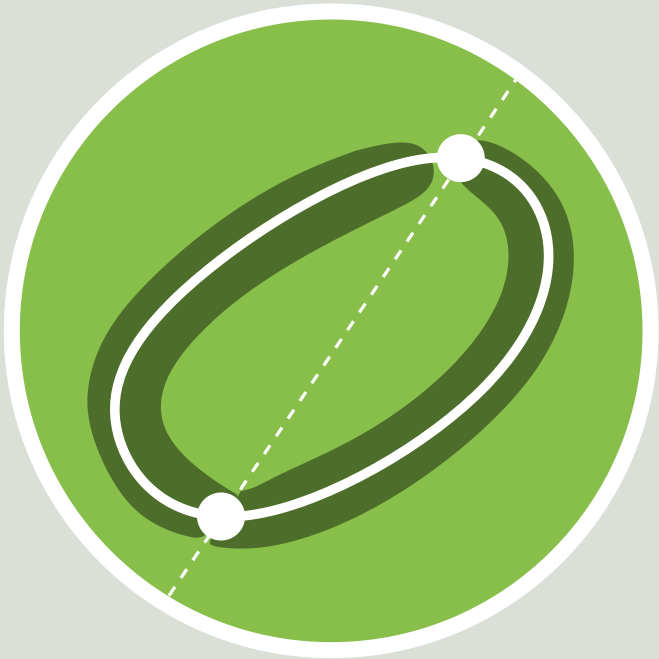

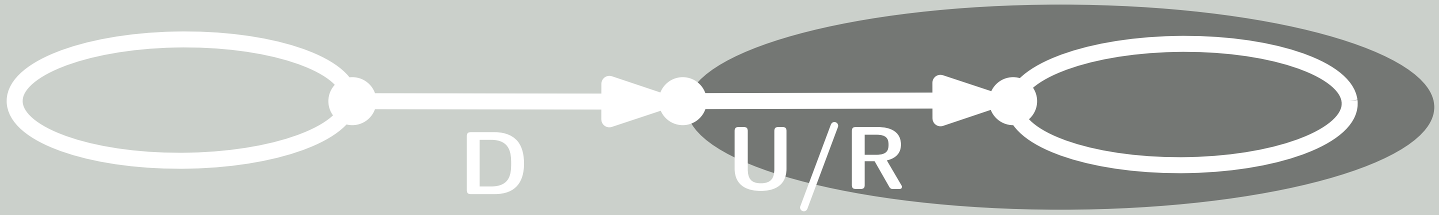

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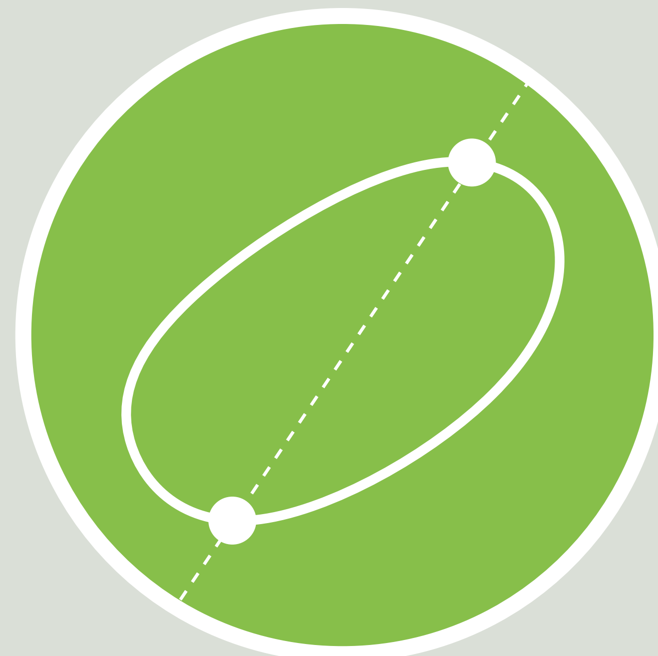

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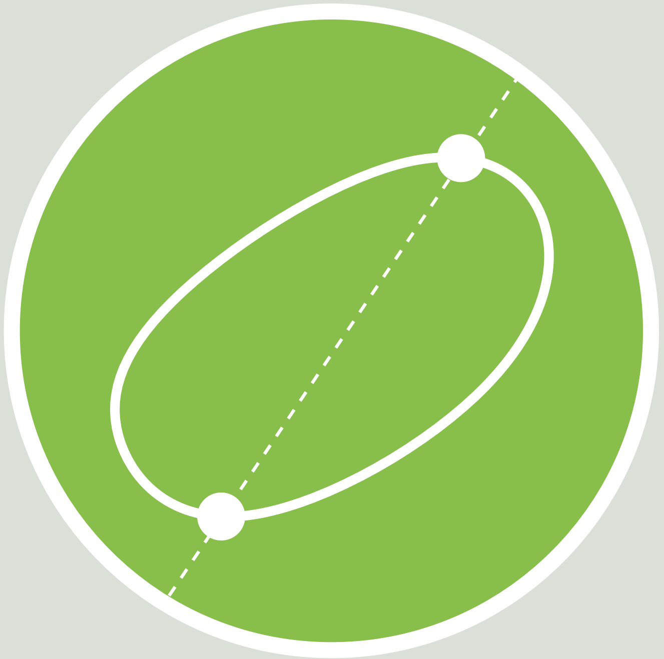

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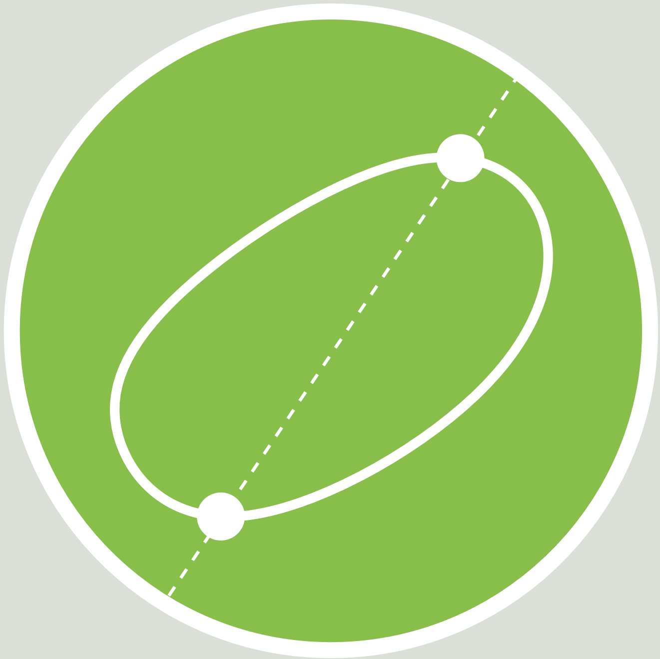

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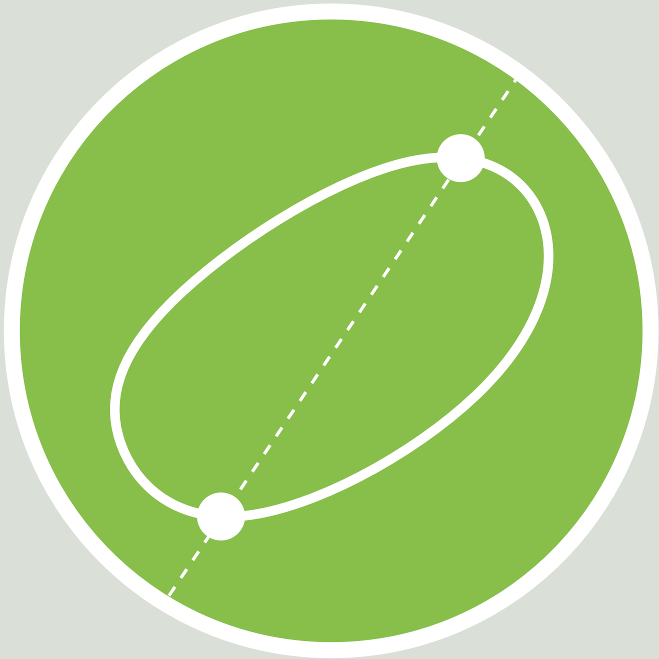


{U,D,R}-LEMMA

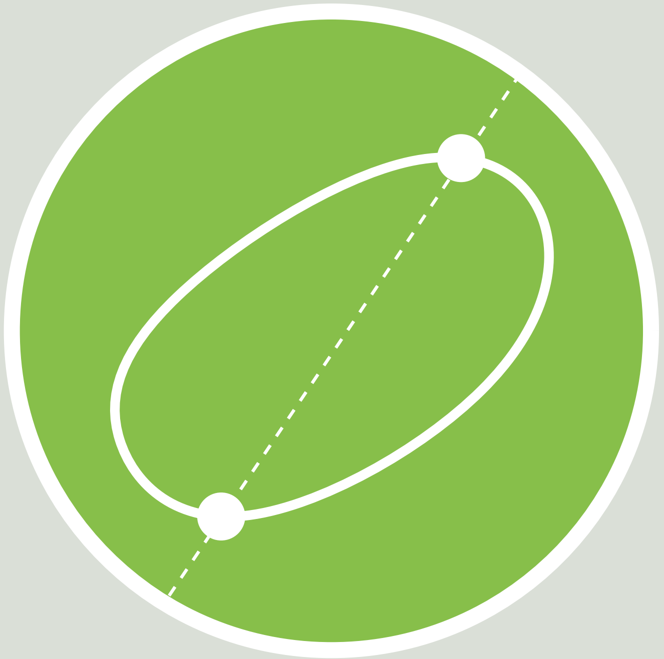
A {U,D,R}-path admits a direction-consistent embedding on a convex point set*

"PROOF"

One of the boundary edges is **D**

Apply "one-sided" Lemma

Both boundary edges are **D**



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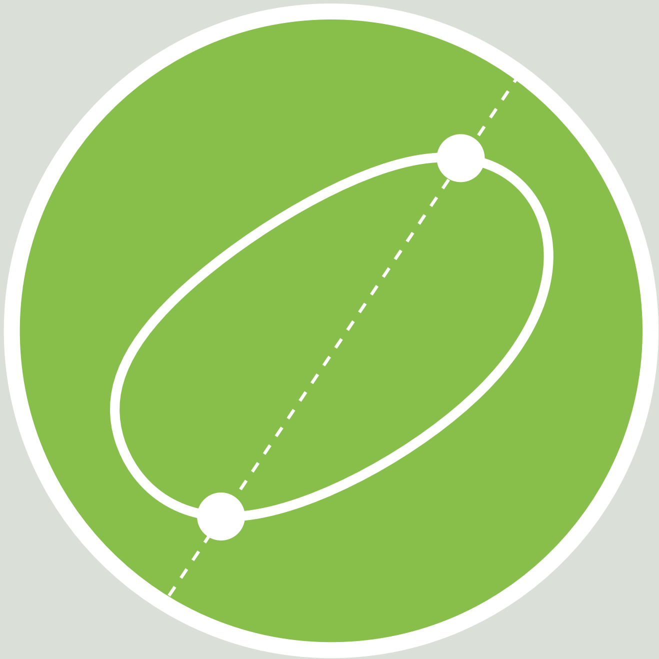

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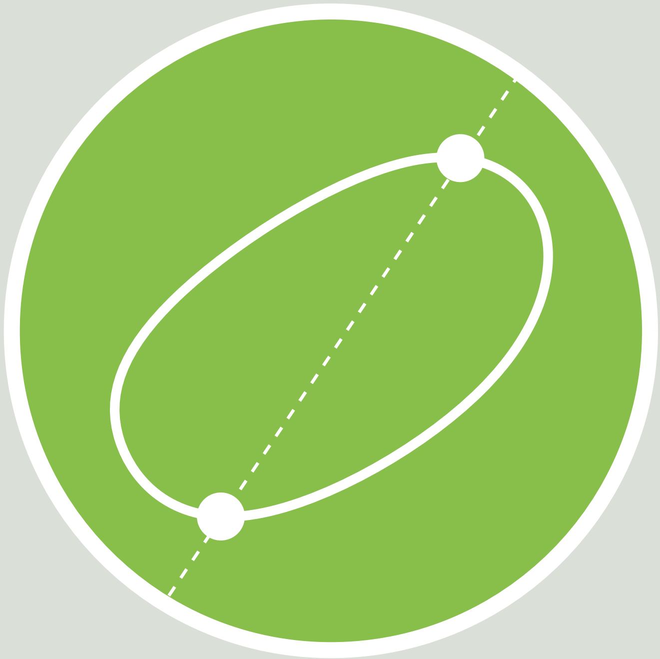

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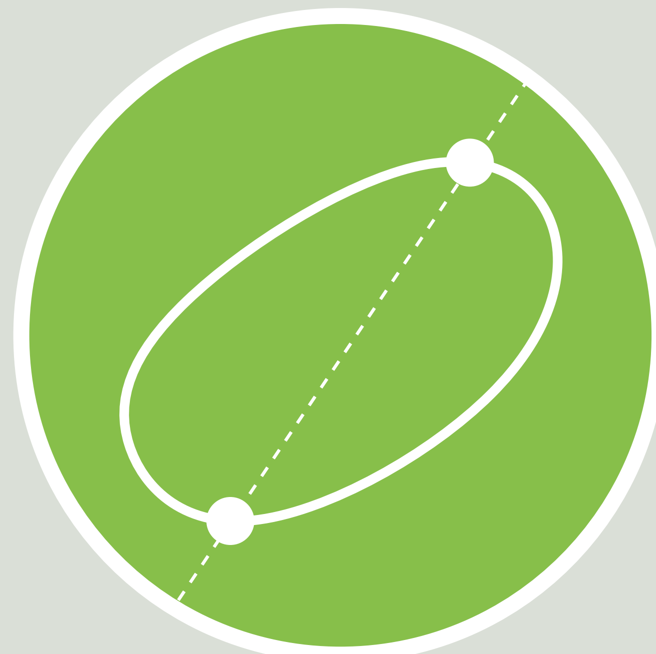
Apply "one-sided" Lemma



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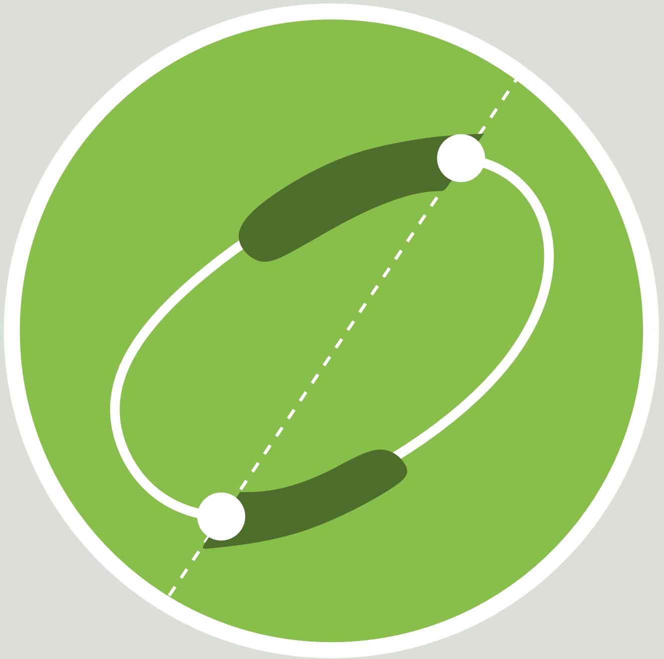

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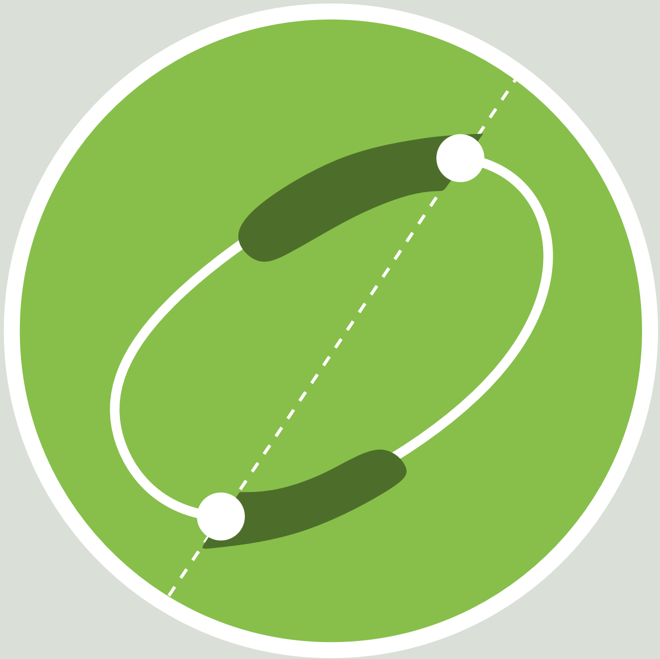
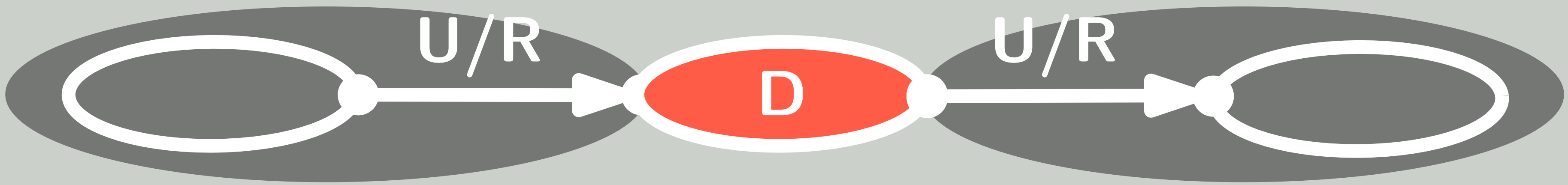
Apply "one-sided" Lemma





{U,D,R}-LEMMA

A {U,D,R}-path admits a direction-consistent embedding on a convex point set*

"PROOF"

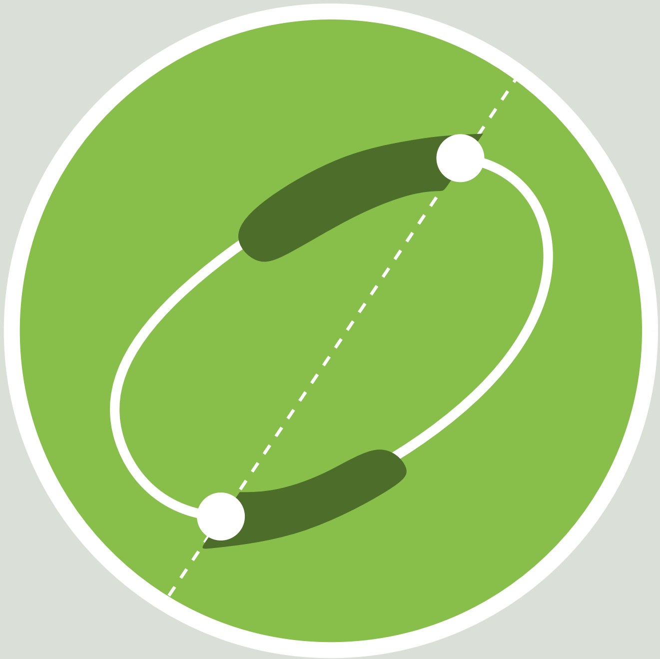
One of the boundary edges is **D**

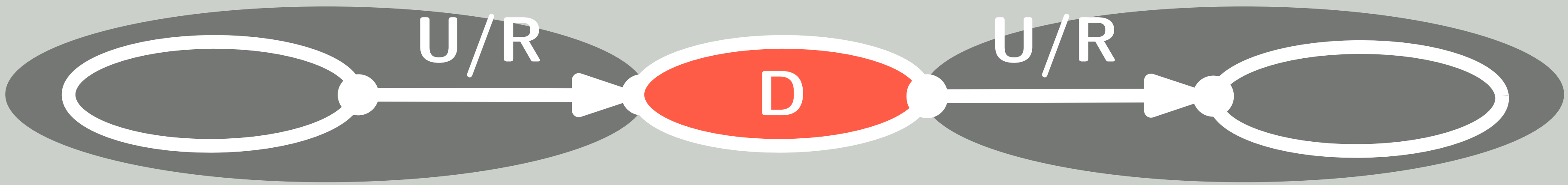
Apply "one-sided" Lemma

Both boundary edges are **D**

Apply "one-sided" Lemma

Sort by y-coordinate





{U,D,R}-LEMMA

A {U,D,R}-path admits a direction-consistent embedding on a convex point set*

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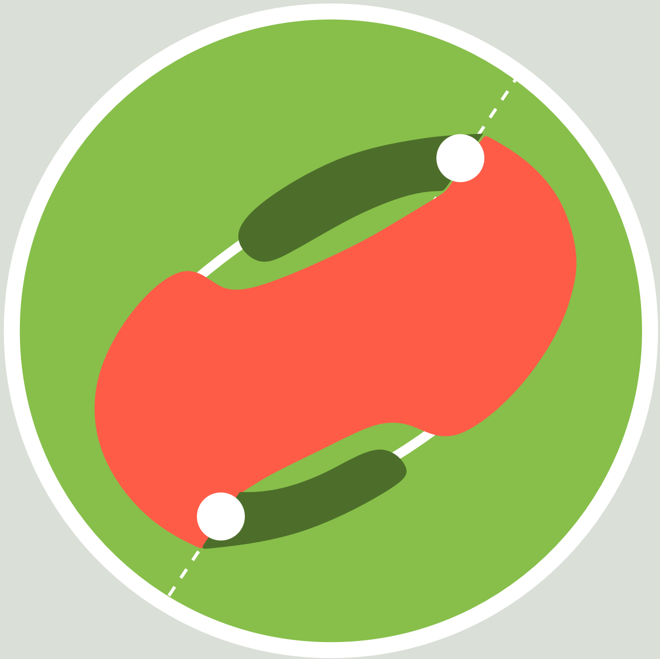
One of the boundary edges is **D**

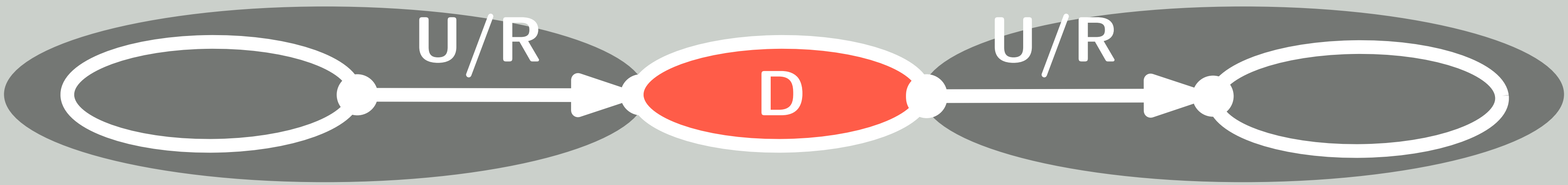
Apply "one-sided" Lemma

Both boundary edges are **D**

Apply "one-sided" Lemma

Sort by y-coordinate





{U,D,R}-LEMMA

A {U,D,R}-path admits a direction-consistent embedding on a convex point set*

“PROOF”

{U,D,R}-LEMMA

A {U,D,R}-path admits a direction-consistent embedding on a convex point set*

"PROOF"

Both boundary edges are **U/R**

{U,D,R}-LEMMA

A {U,D,R}-path admits a direction-consistent embedding on a convex point set*

"PROOF"

Both boundary edges are **U/R**

{U,D,R}-LEMMA

A {U,D,R}-path admits a direction-consistent embedding on a convex point set*

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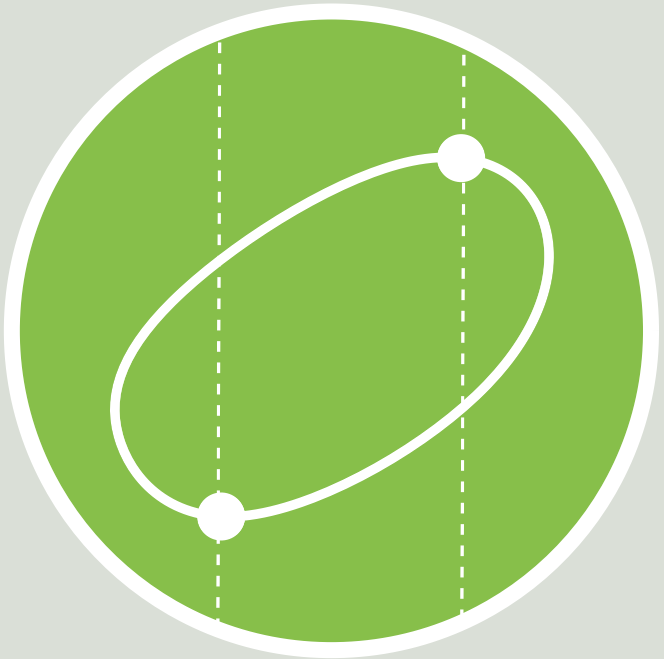

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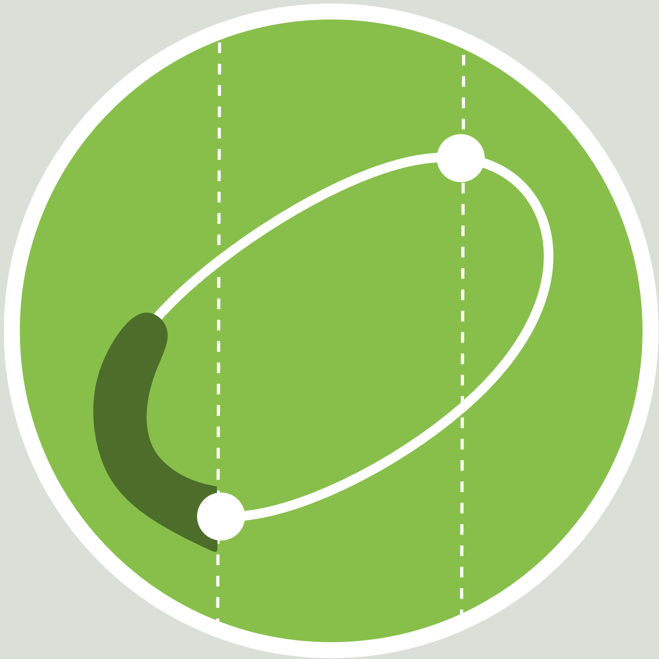

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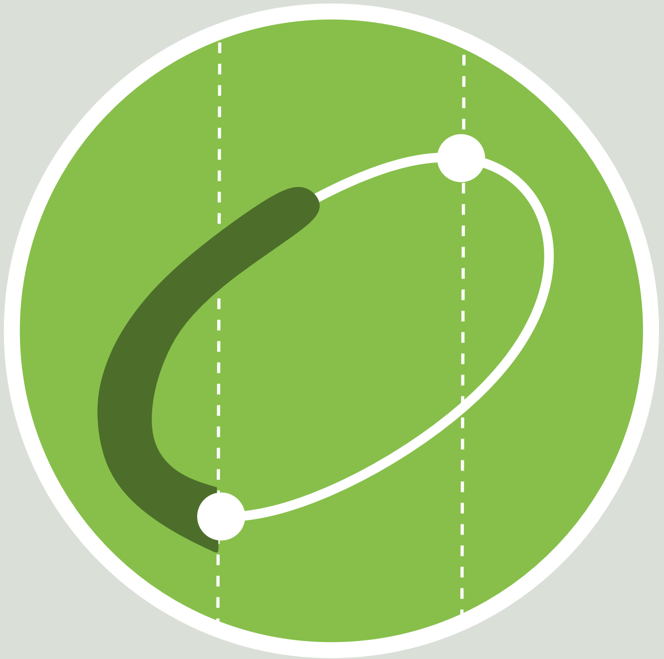

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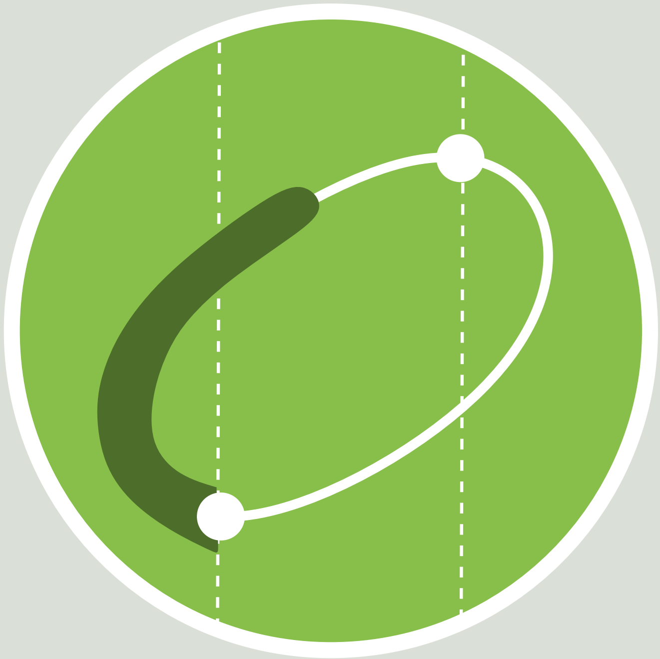
A {U,D,R}-path admits a direction-consistent embedding on a convex point set*

"PROOF"

Both boundary edges are **U/R**

None fit - One fit - Both fit



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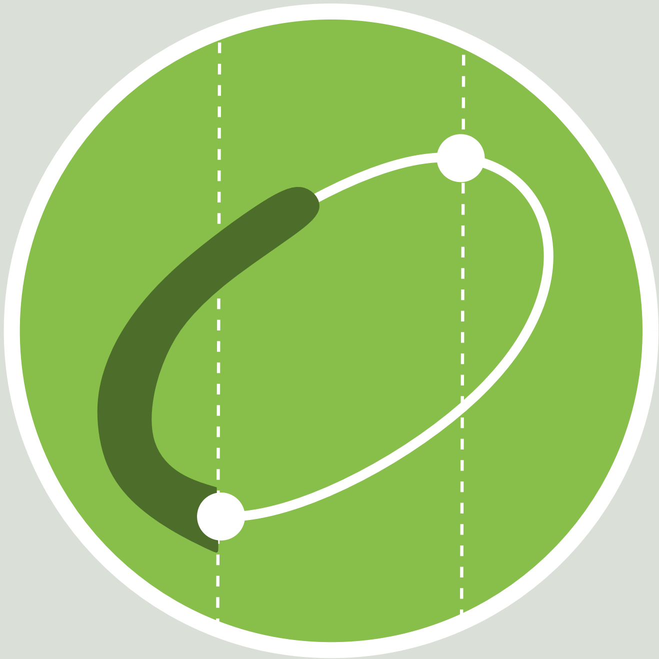

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A {U,D,R}-path admits a direction-consistent embedding on a convex point set*

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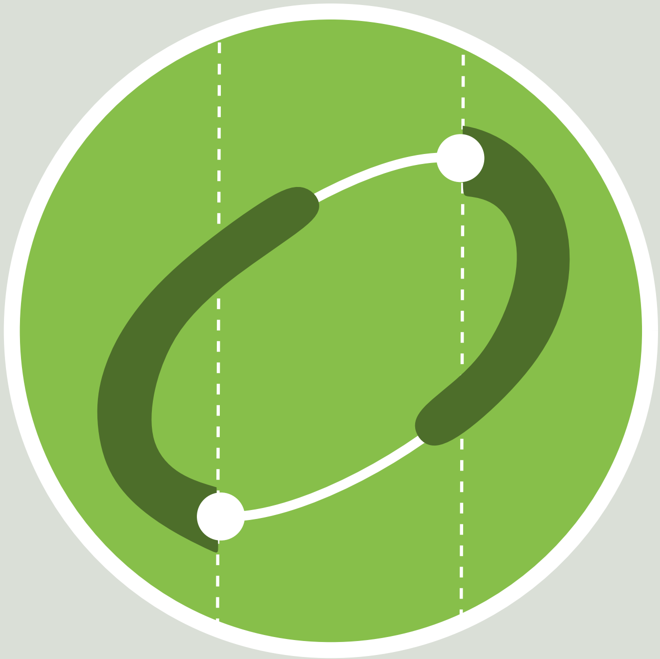
None fit - One fit - Both fit

Apply "one-sided" Lemma



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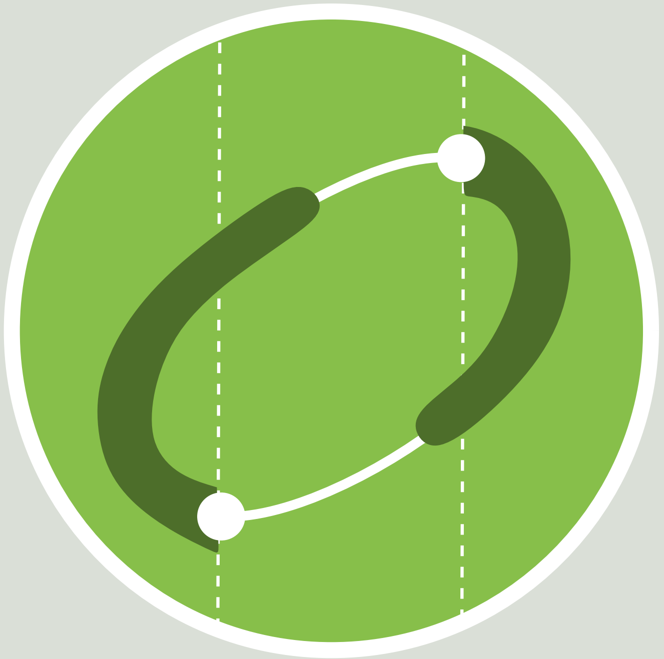

{U,D,R}-LEMMA

A {U,D,R}-path admits a direction-consistent embedding on a convex point set*

"PROOF"

Both boundary edges are **U/R**

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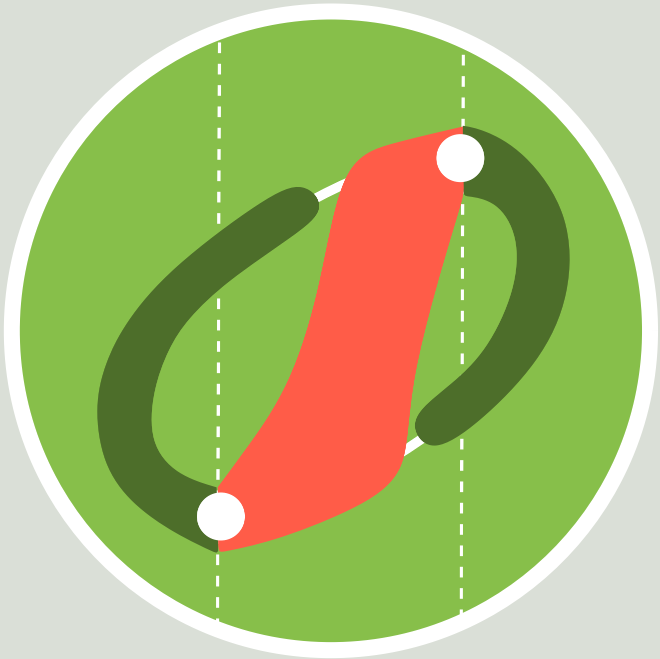

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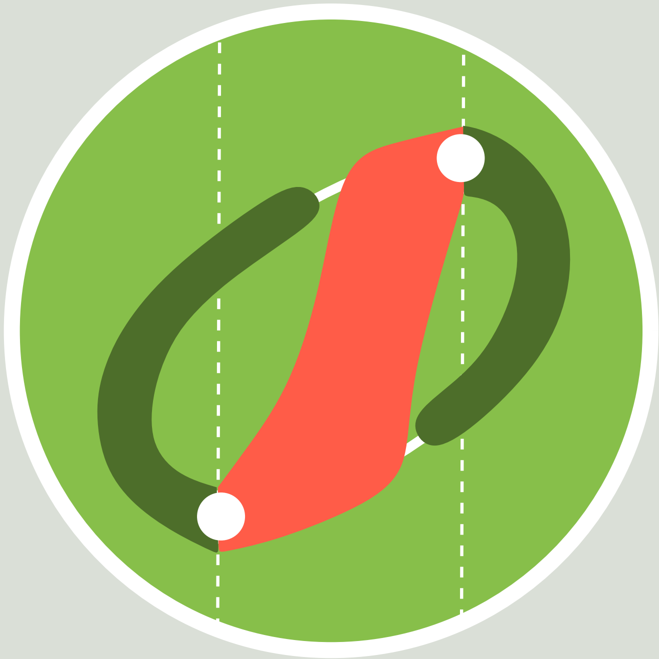

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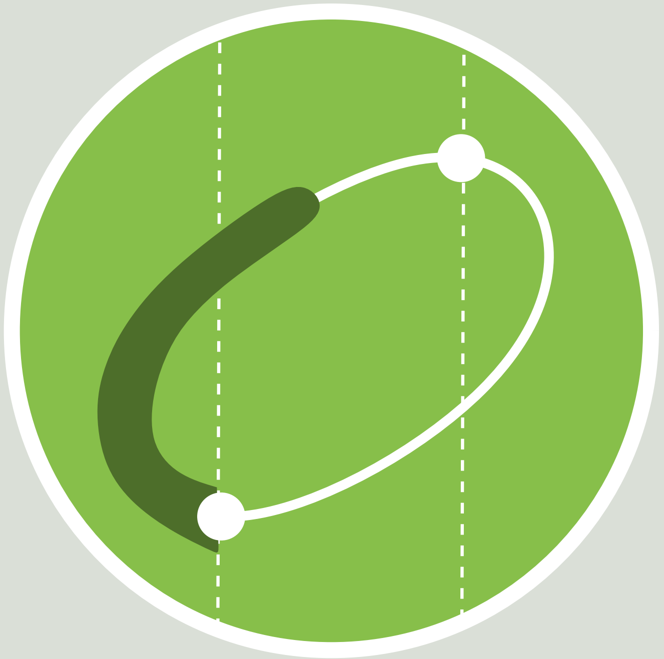

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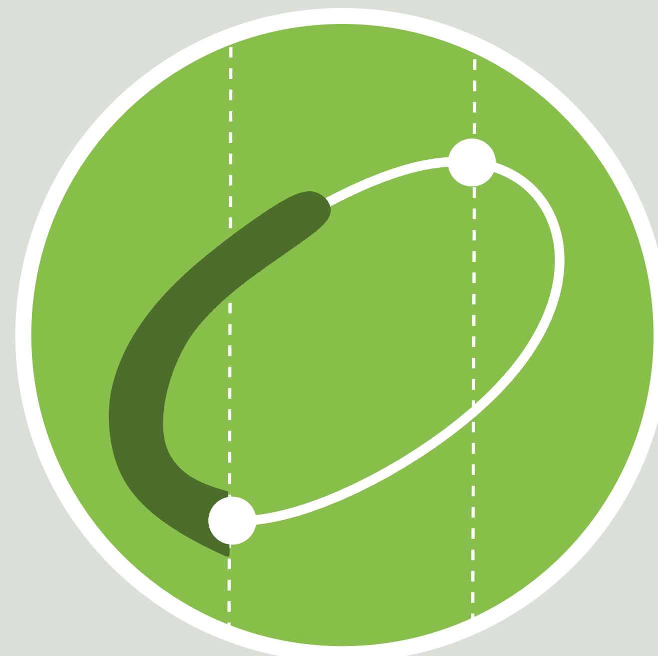

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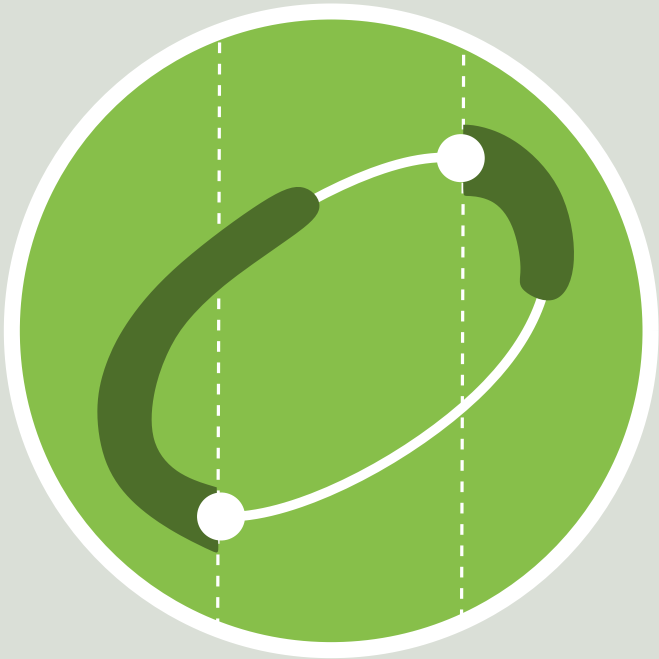

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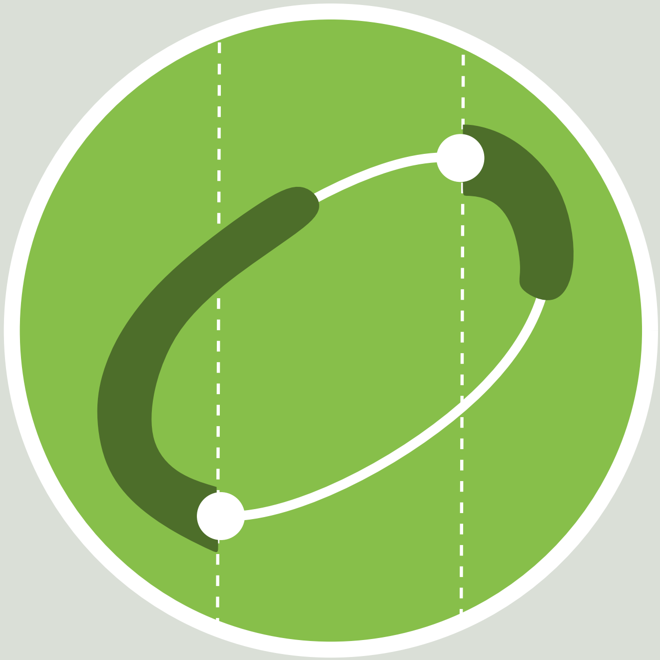

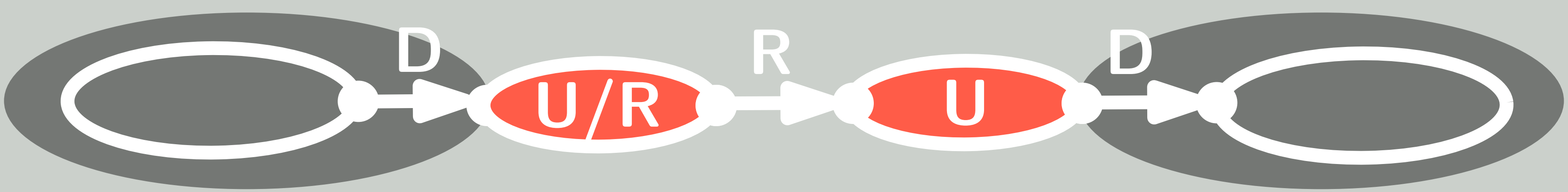

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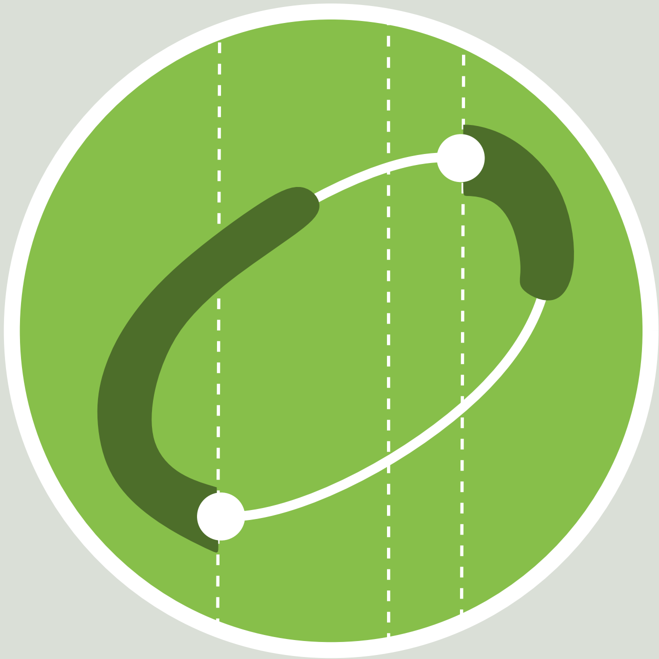

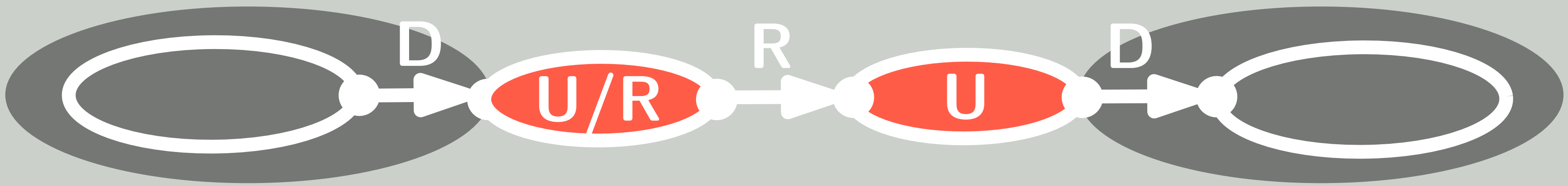

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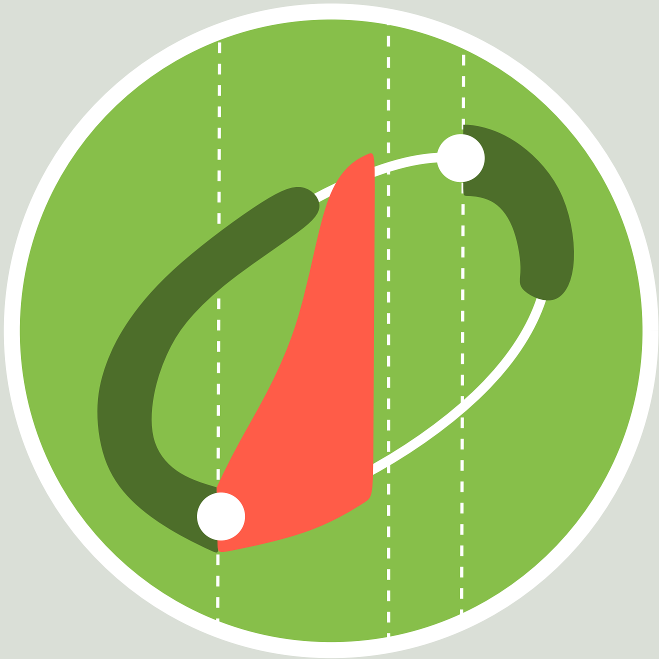

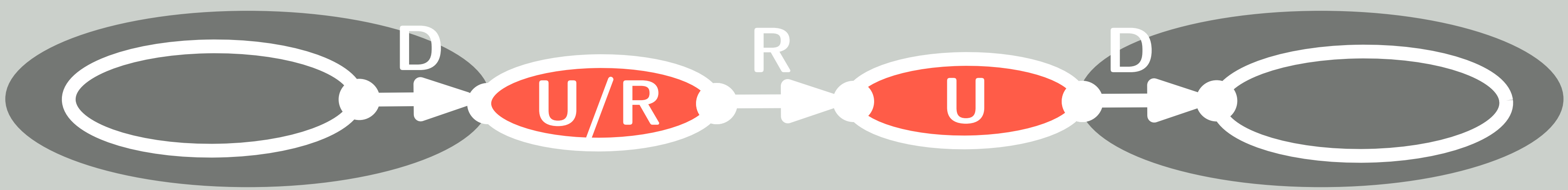

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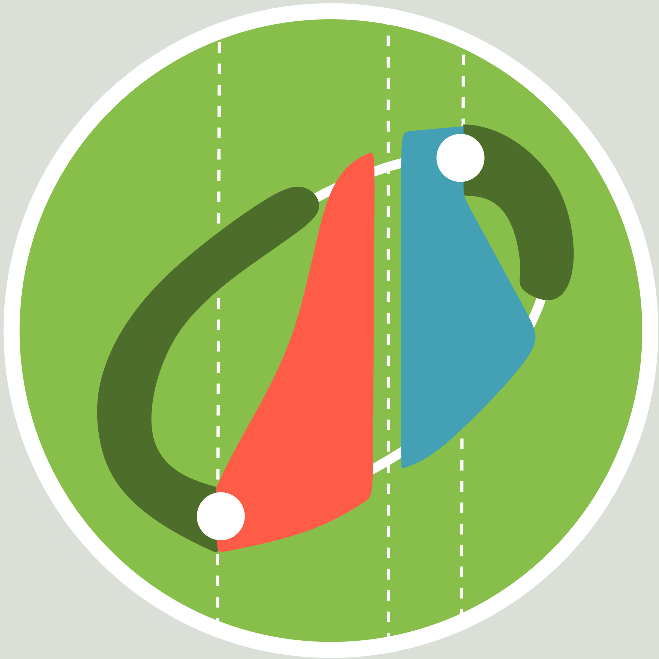

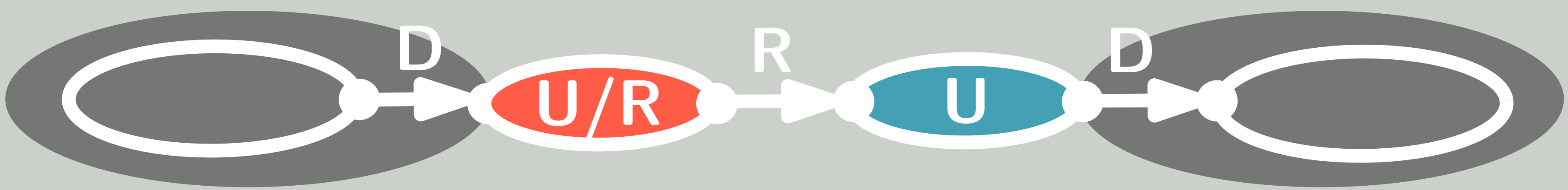

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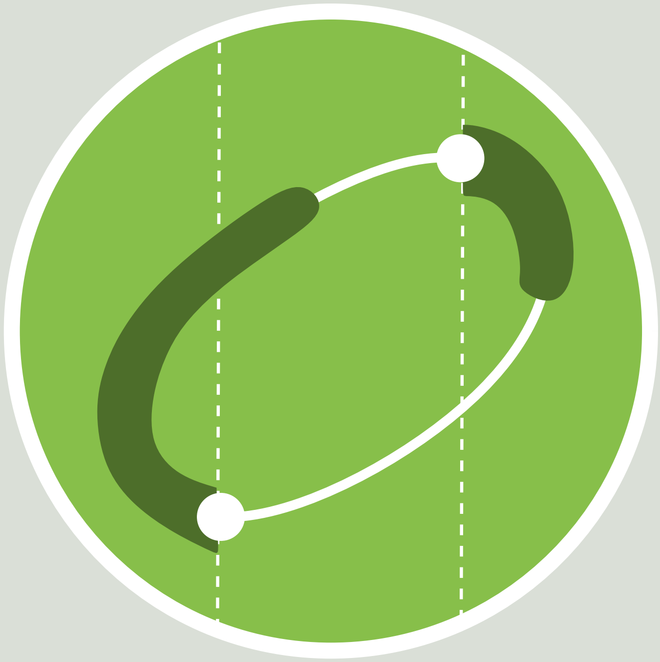

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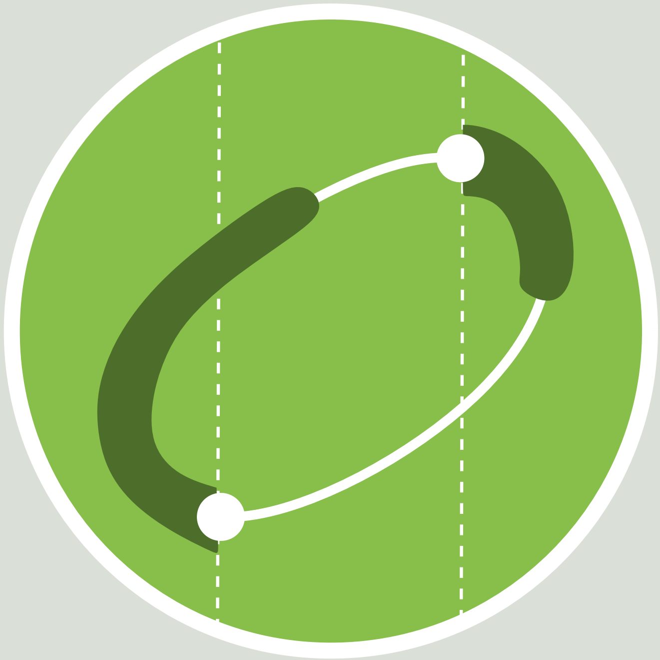

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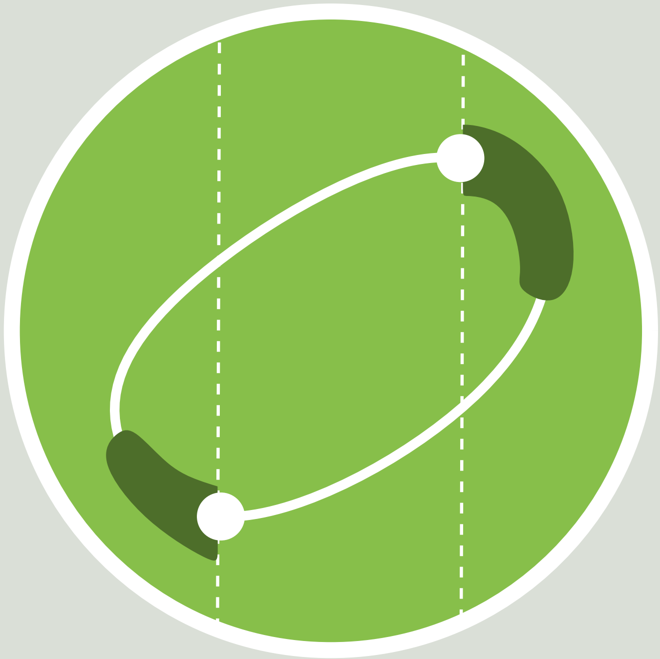

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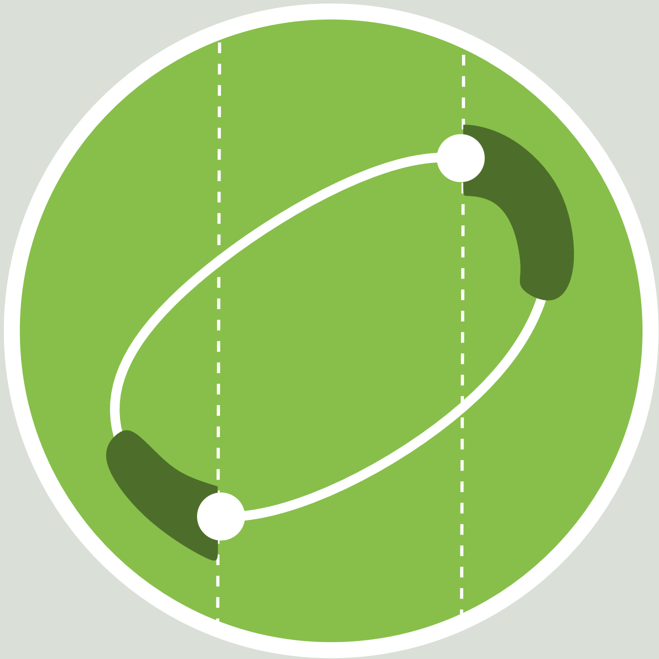

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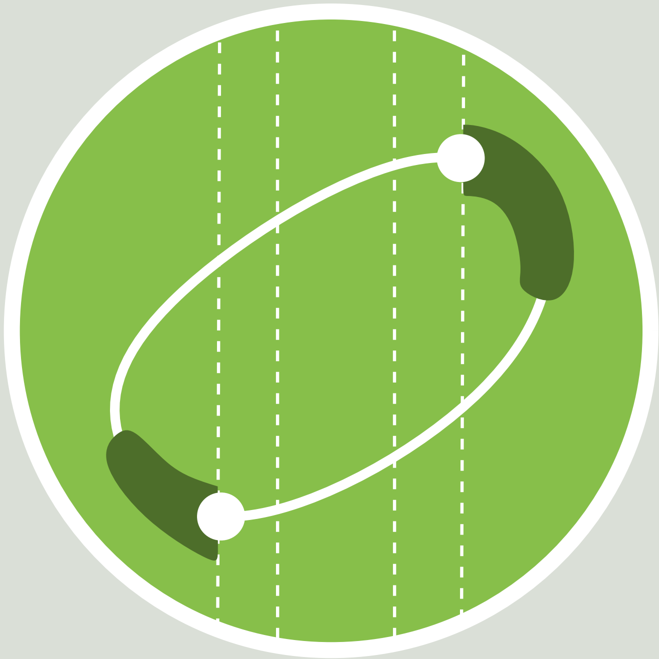

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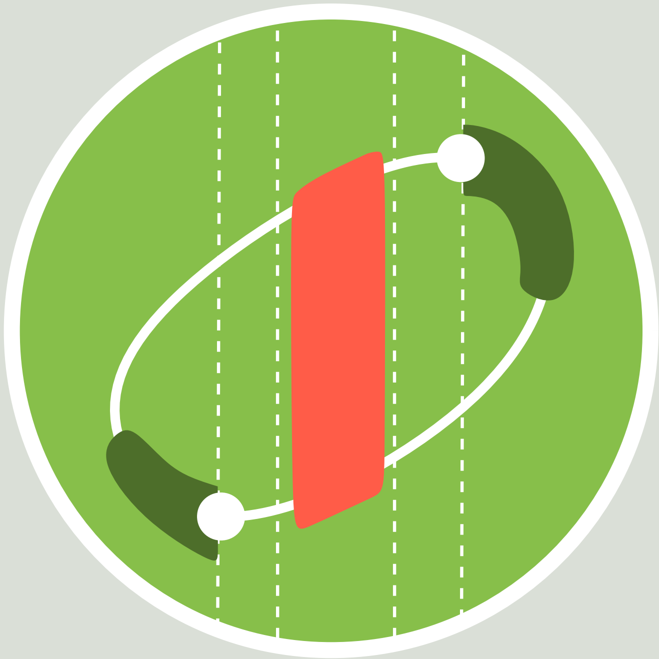

THREE-DIRECTIONAL

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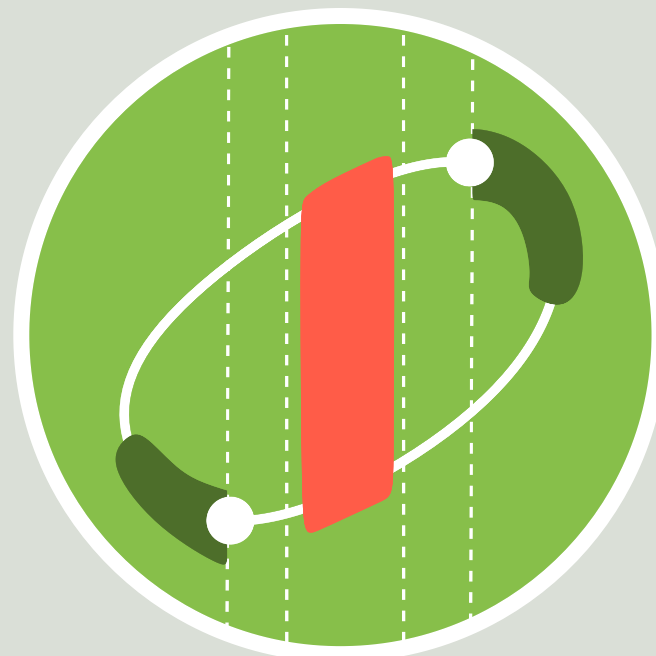

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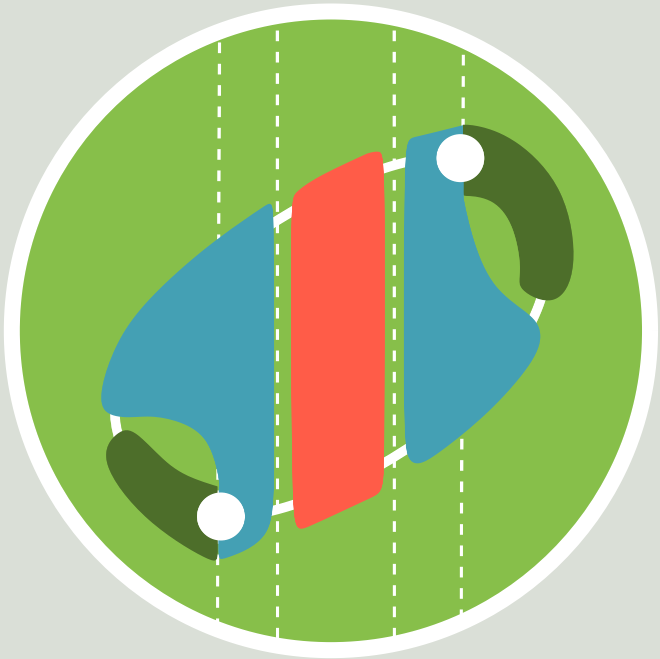

THEOREM

Any three-directional path admits a direction-consistent embedding on any convex point set

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General convex point set and $\{U,D,R\}$-path

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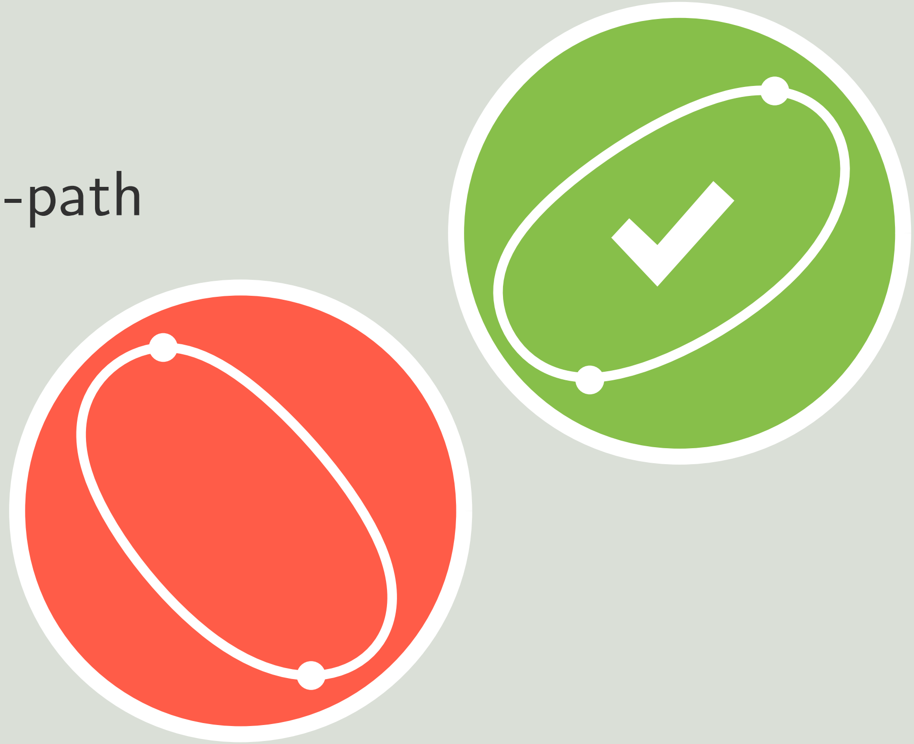

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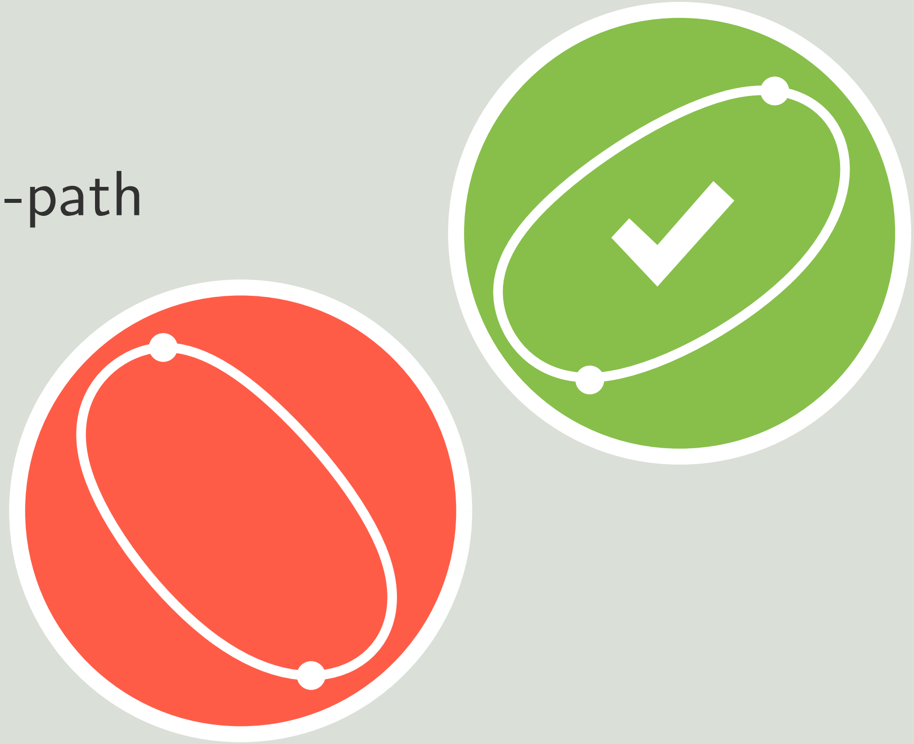
General convex point set and {U,D,R}-path

Mirror the point set and the path.



THEOREM

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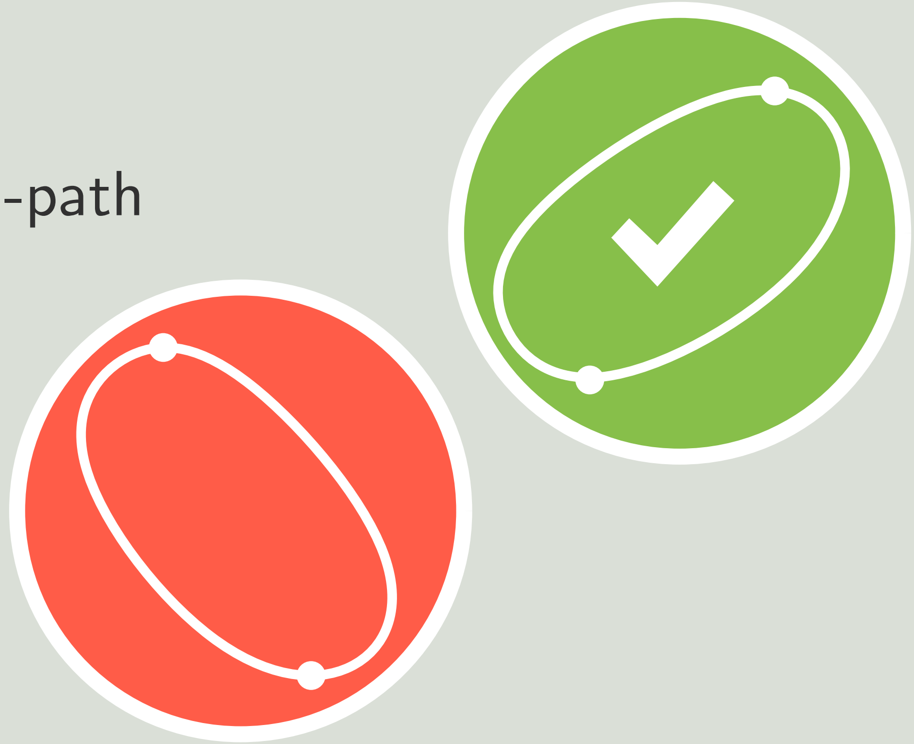
General convex point set and $\{U,D,R\}$-path

Mirror the point set and the path.

Get a $\{U,D,L\}$-path



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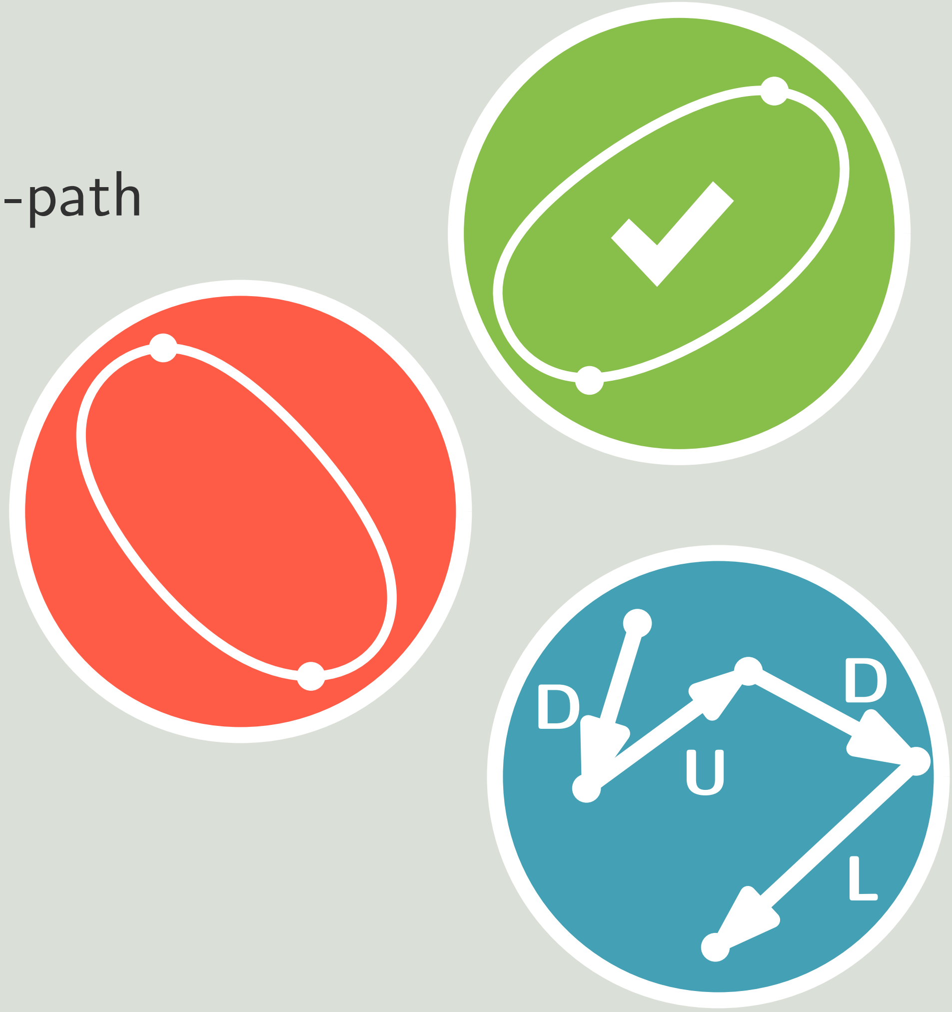

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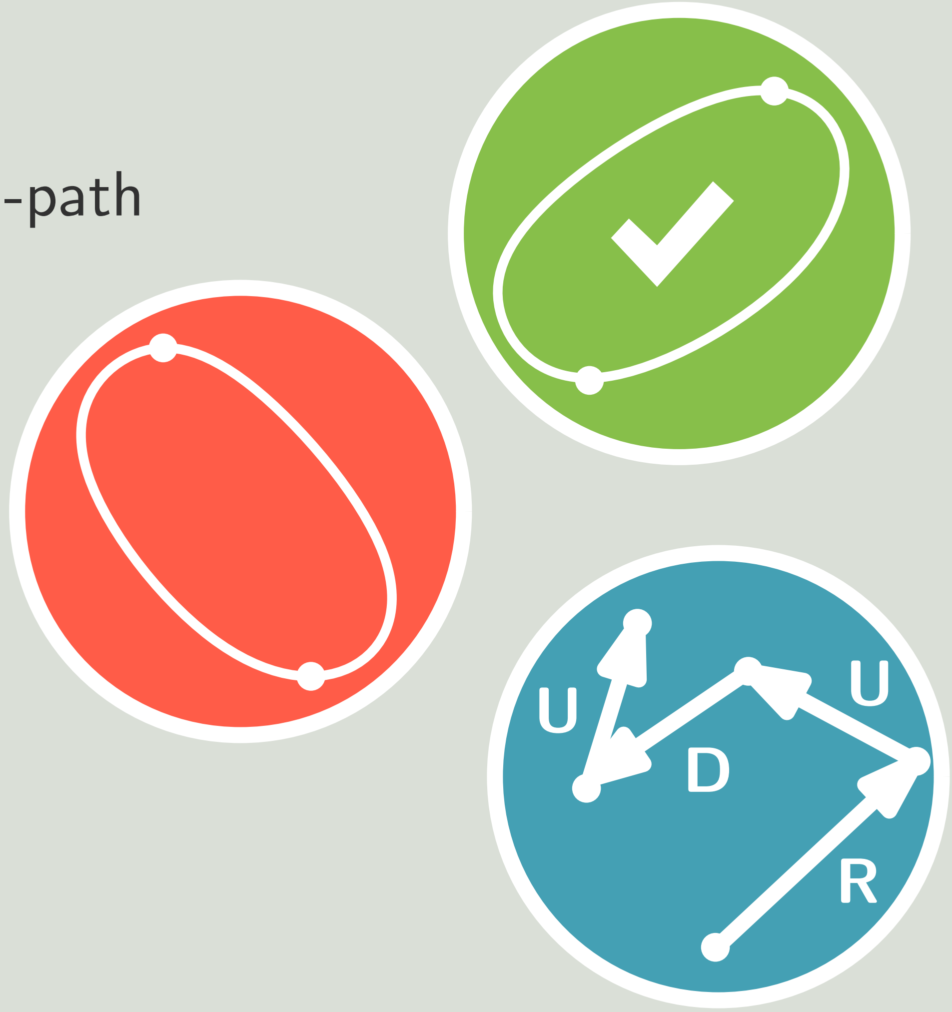

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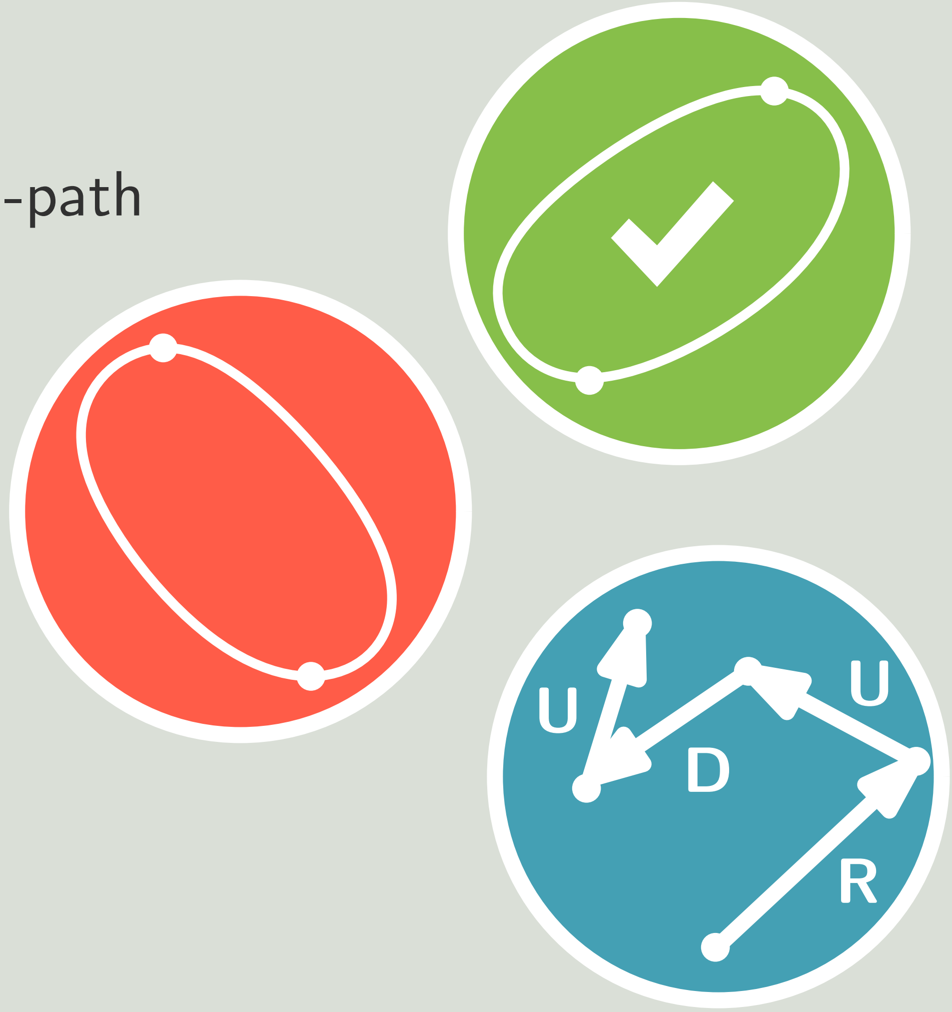
Mirror the point set and the path.

Get a {U,D,L}-path

Reverse the path and the labels, get a {U,D,R}-path



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Reverce the path and the labels,
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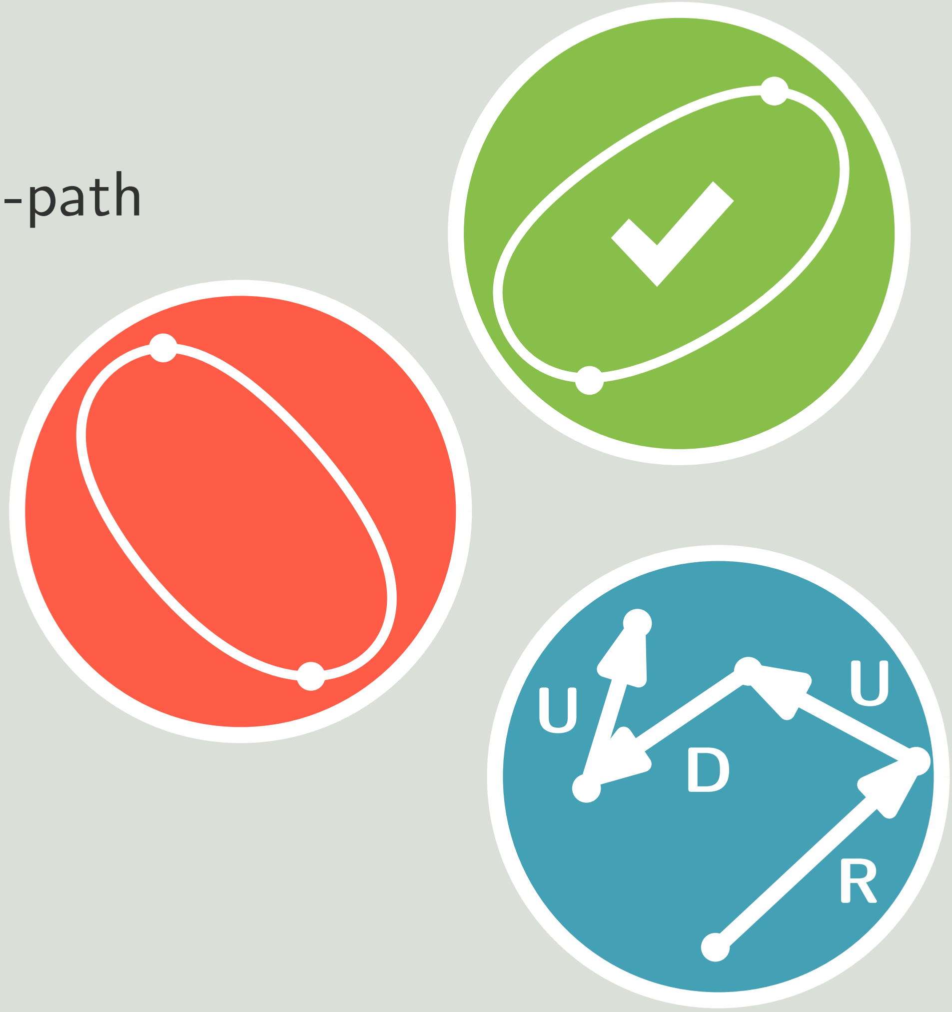
Apply the {U,D,R}-Lemma



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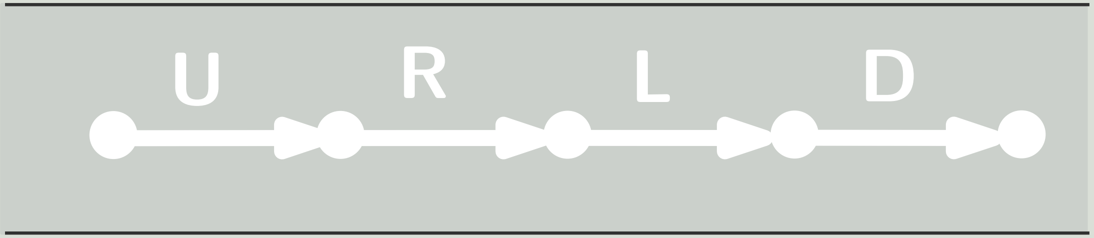
Apply the {U,D,R}-Lemma

Treat {U,D,L}, {R,L,U} and {L,R,D}-paths similarly

EMBEDDING 4-DIRECTIONAL PATHS ON CONVEX POINT SETS



RESULTS

Not always possible for four directions

Always possible for three directions

Can be decided in $O(n^2)$ time for four directions.

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OPEN PROBLEMS

Does every oriented path admit an upward planar embedding on every point set?

CONCLUSION

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Are the four-directional planar drawings interesting by themselves? (no point set given)

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THANK YOU!