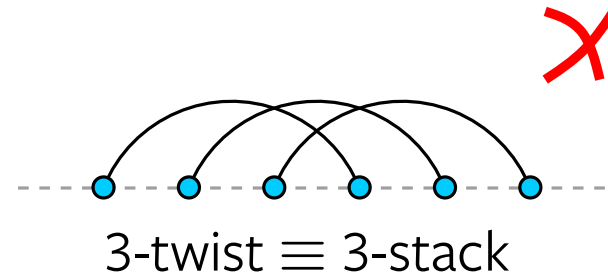
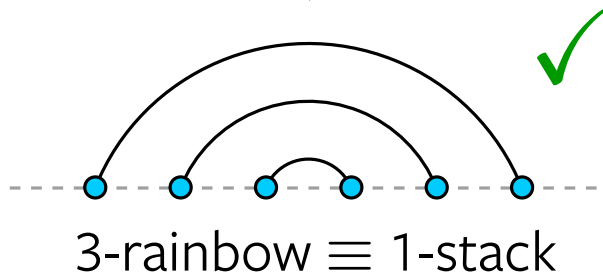


On Families of Planar DAGs with Constant Stack Number

Martin Nöllenburg and **Sergey Pupyrev**

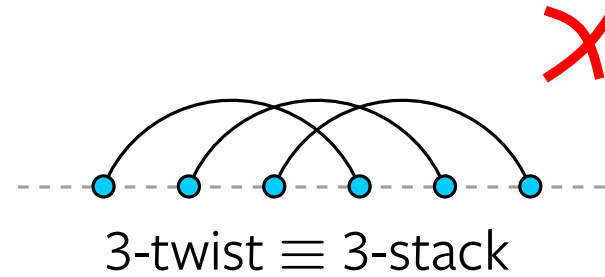
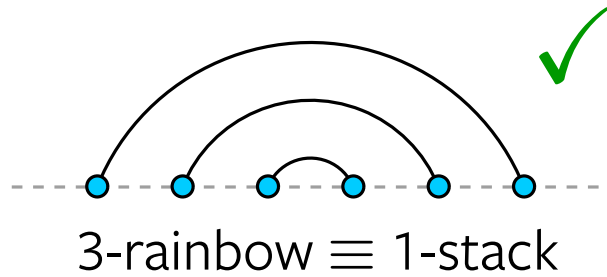
Definitions

Stack Layout (Book Embedding)

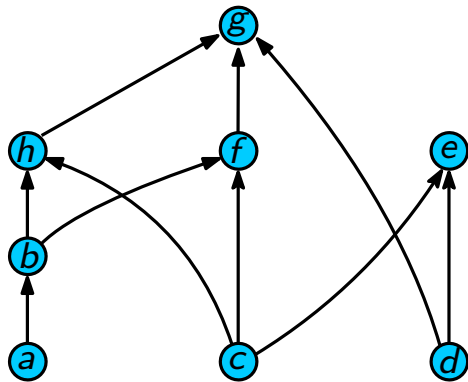


Definitions

Stack Layout (Book Embedding)

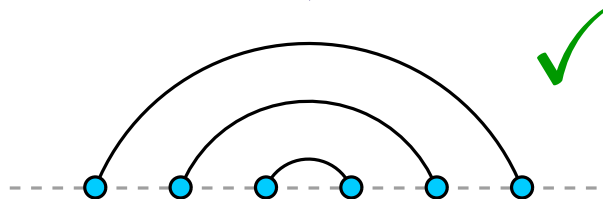


Directed Acyclic Graph

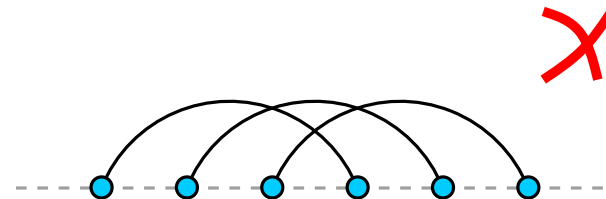


Definitions

Stack Layout (Book Embedding)

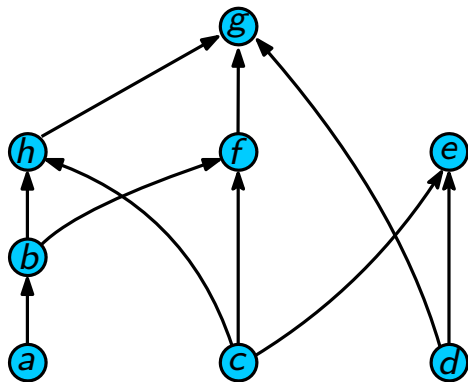


3-rainbow \equiv 1-stack

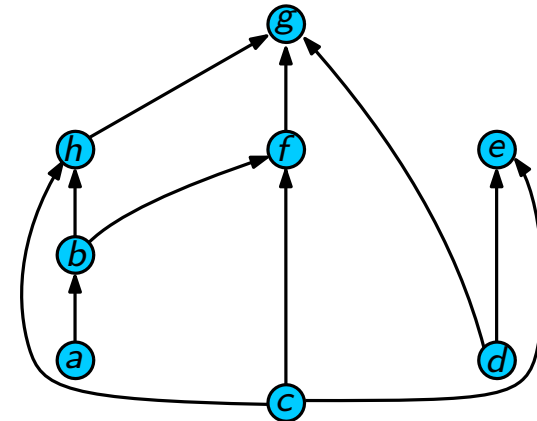


3-twist \equiv 3-stack

Directed Acyclic Graph

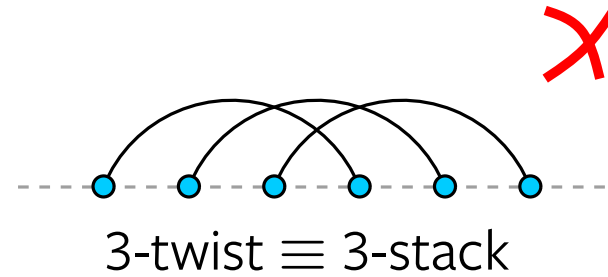
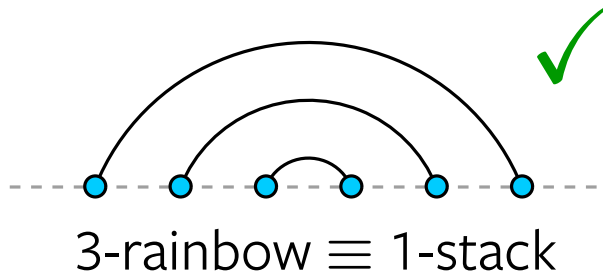


Upward (Planar) Graph

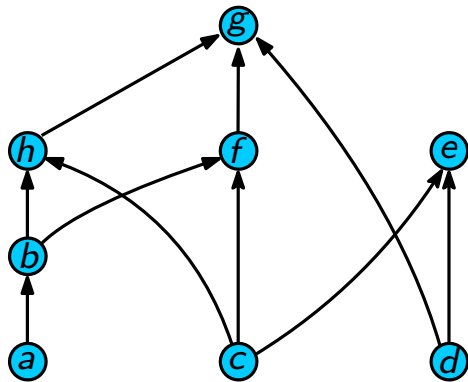


Definitions

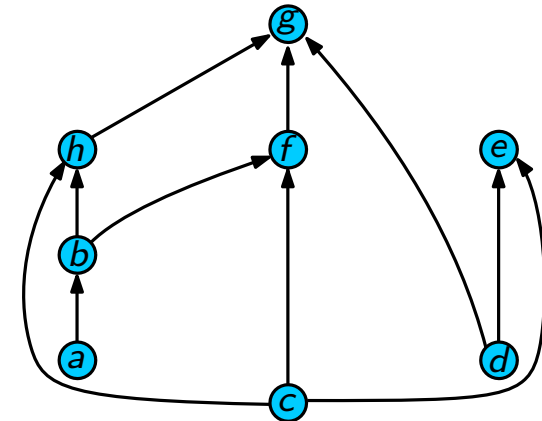
Stack Layout (Book Embedding)



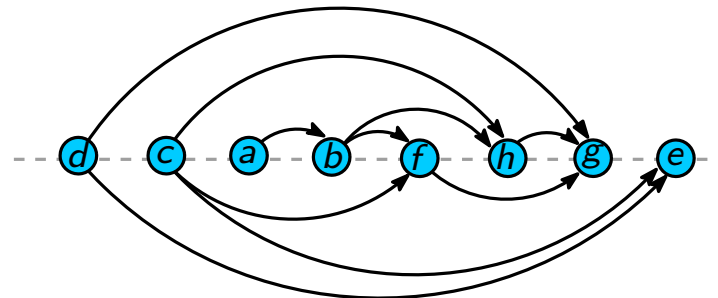
Directed Acyclic Graph



Upward (Planar) Graph



Stack Layout of DAGs



FAQ

Question 1

What is the (undirected) stack number of graphs in class X?

- outerplanar
- 2-tree (series-parallel)
- planar 3-tree
- planar

FAQ

Question 1

What is the (undirected) stack number of graphs in class X?

SP

What is the stack number of planar graphs?

GPT-3.5



The stack number of a graph is a combinatorial parameter that measures how "stackable" the graph is. It is also known as the page number or book thickness of a graph. The stack number of a planar graph is well-studied and has been shown to be at most 4.

In other words, any planar graph can be arranged in such a way that it can be represented as a collection of at most four pages in a book, where each page represents a set of non-crossing edges in the graph. This arrangement ensures that no two edges intersect within the same page.

The result that the stack number of planar graphs is at most 4 is a consequence of the fact that planar graphs have a treewidth of at most 3. Treewidth is another graph parameter related to how "tree-like" a graph is, and it is closely related to the stack number. The treewidth of a planar graph is at most 3, and it has been shown that the stack number is at most one more than the treewidth.

FAQ

Question 1

What is the (undirected) stack number of graphs in class X?

- outerplanar $= 1$
- 2-tree (series-parallel) $= 2$
- planar 3-tree $= 3$
- planar $= 4$

FAQ

Question 1

What is the (undirected) stack number of graphs in class X?

- outerplanar = 1
- 2-tree (series-parallel) = 2
- planar 3-tree = 3
- planar = 4

Question 2

What is the (directed) stack number of DAGs in class X?

non-upward DAG

upward planar

FAQ

Question 1

What is the (undirected) stack number of graphs in class X?

- outerplanar = 1
- 2-tree (series-parallel) = 2
- planar 3-tree = 3
- planar = 4

Question 2

What is the (directed) stack number of DAGs in class X?

non-upward DAG		upward planar	
• tree	= 1	• tree	= 1
• outerplanar	?	• outerpath	≤ 16
• 2-tree	?	• outerplanar	?
• planar 3-tree	unbounded	• (maximal) 3-tree	$O(1)$
• planar	unbounded	• planar	open

FAQ

Question 1

What is the (undirected) stack number of graphs in class X?

- outerplanar = 1
- 2-tree (series-parallel) = 2
- planar 3-tree = 3
- planar = 4

Question 2

What is the (directed) stack number of DAGs in class X?

non-upward DAG

- tree = 1
- outerplanar ?
- 2-tree ?
- planar 3-tree unbounded
- planar unbounded

upward planar

- tree = 1
- outerpath ≤ 16
- outerplanar ?
- (maximal) 3-tree $O(1)$
- planar open

Outerplanar DAGs

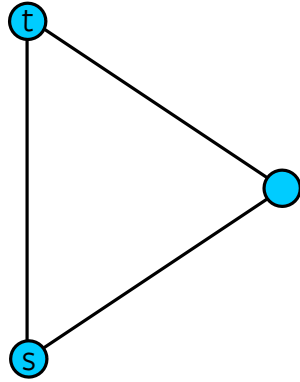
undirected

directed acyclic



Outerplanar DAGs

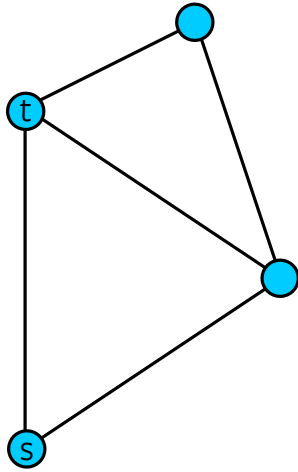
undirected



directed acyclic

Outerplanar DAGs

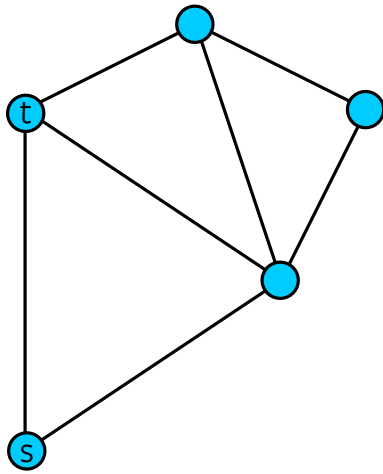
undirected



directed acyclic

Outerplanar DAGs

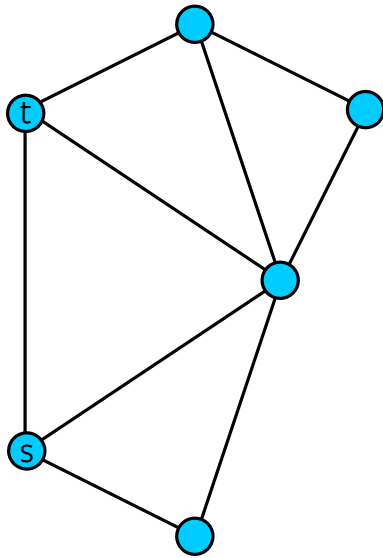
undirected



directed acyclic

Outerplanar DAGs

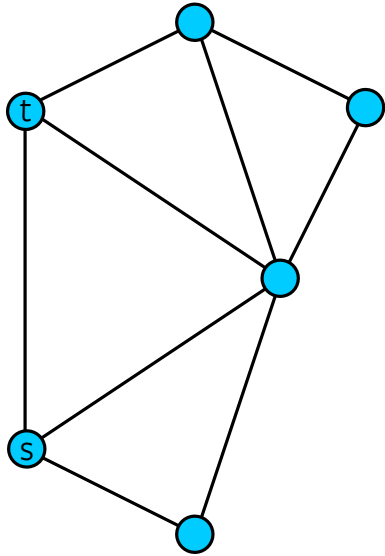
undirected



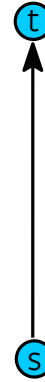
directed acyclic

Outerplanar DAGs

undirected

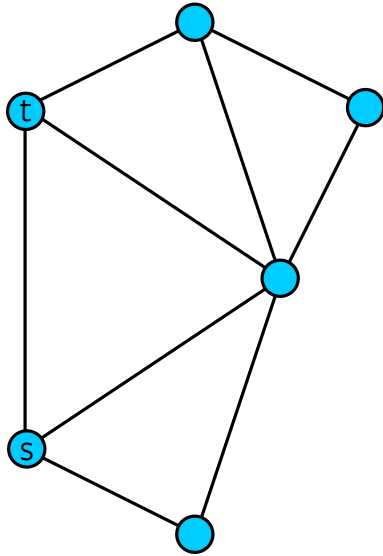


directed acyclic

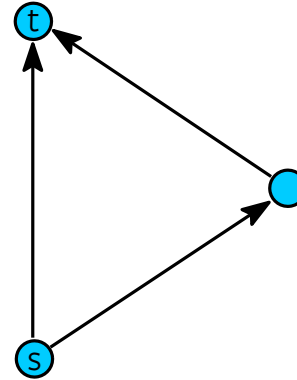


Outerplanar DAGs

undirected

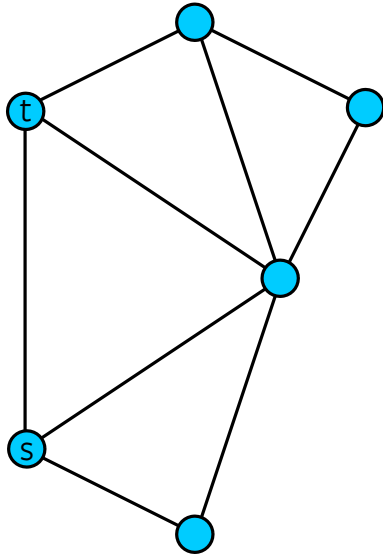


directed acyclic

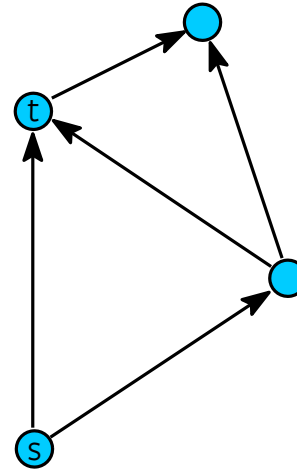


Outerplanar DAGs

undirected

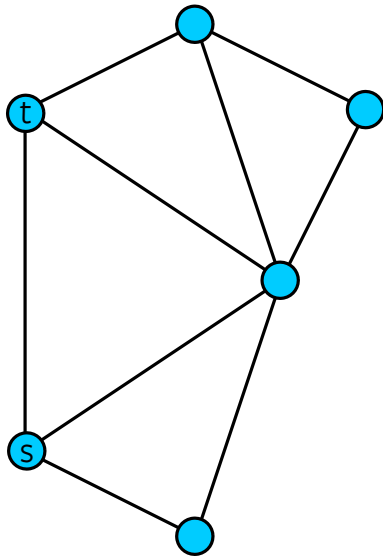


directed acyclic

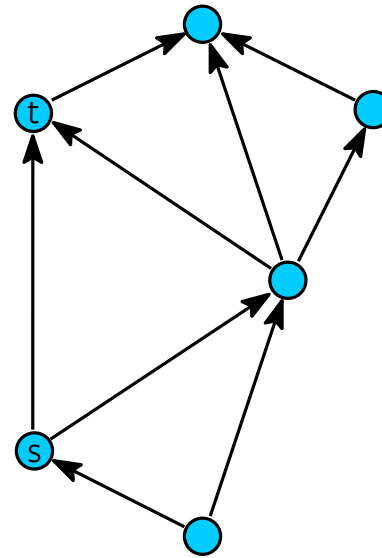


Outerplanar DAGs

undirected

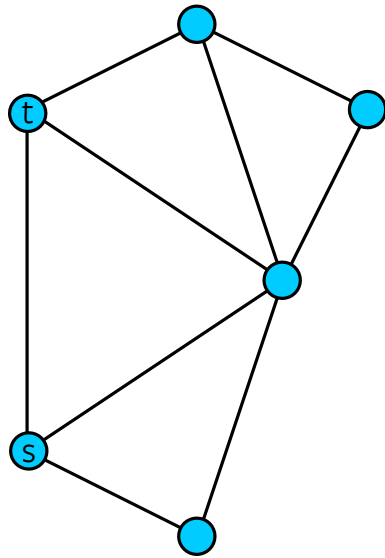


directed acyclic

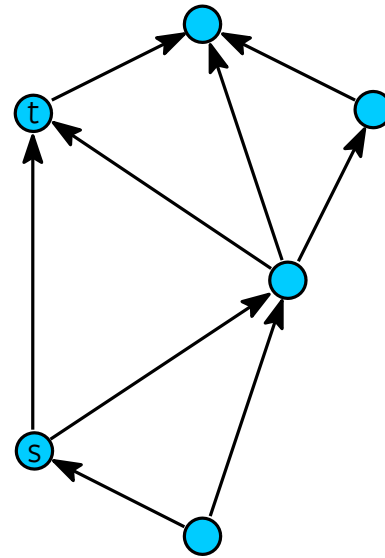


Outerplanar DAGs

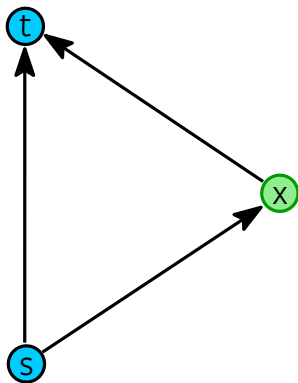
undirected



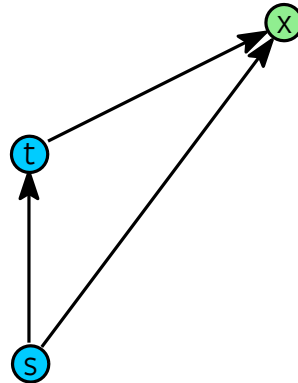
directed acyclic



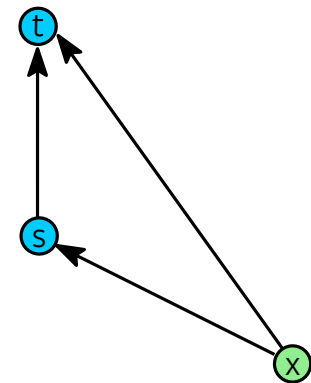
operation 01



operation 02



operation 03

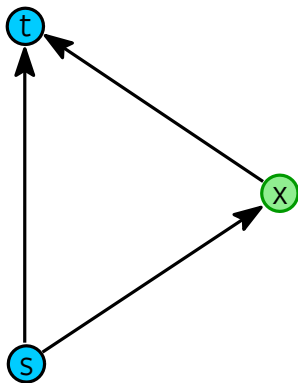


Outerplanar DAGs

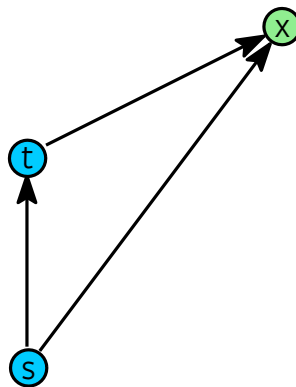
Theorem 1(a,b,c)

Every outerplanar DAG constructed by **two** operations has a constant stack number

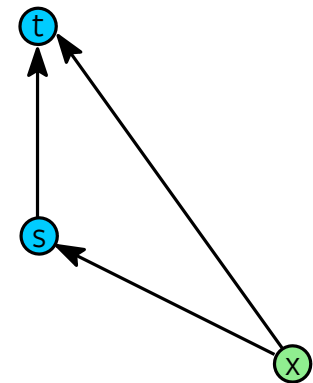
operation 01



operation 02



operation 03



Outerplanar DAGs

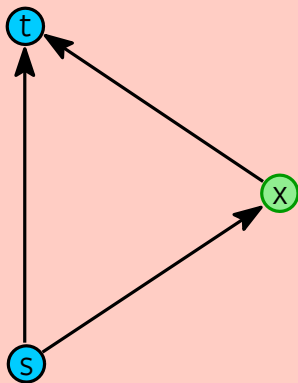
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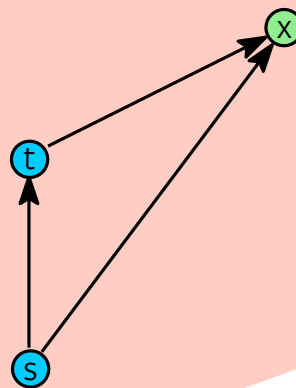
Theorem 1a

Every *single-source* outerplanar DAG is constructed by operations O1 and O2 and admits a 4-stack layout

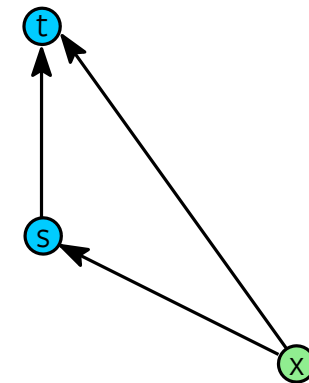
operation 01



operation 02



operation 03



Single-Source oDAGs

Theorem 1a

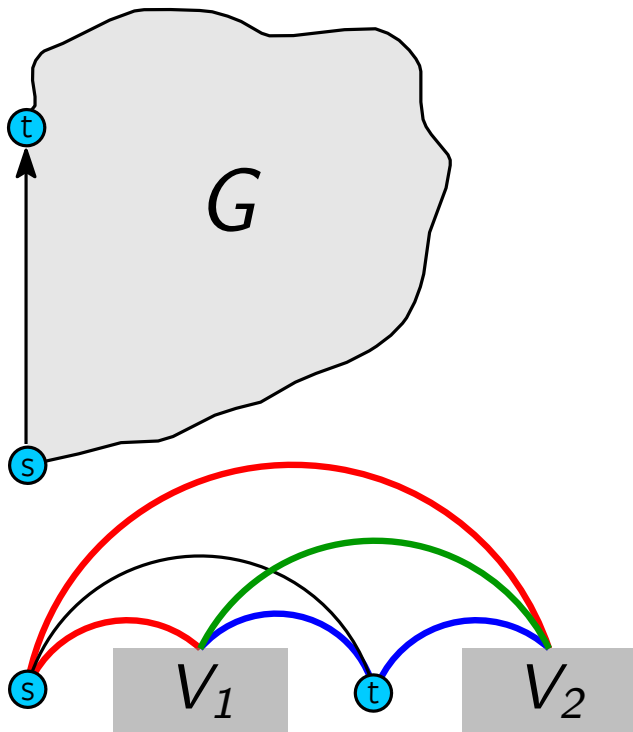
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Single-Source oDAGs

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Every *single-source* outerplanar DAG is constructed by operations O1 and O2 and admits a 4-stack layout

Invariants:



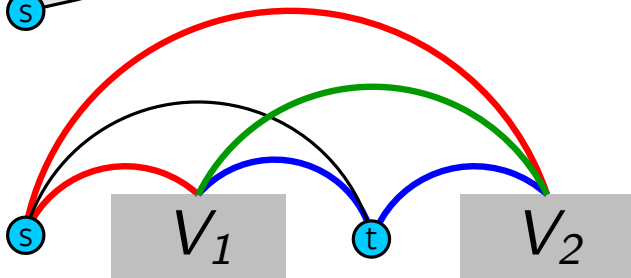
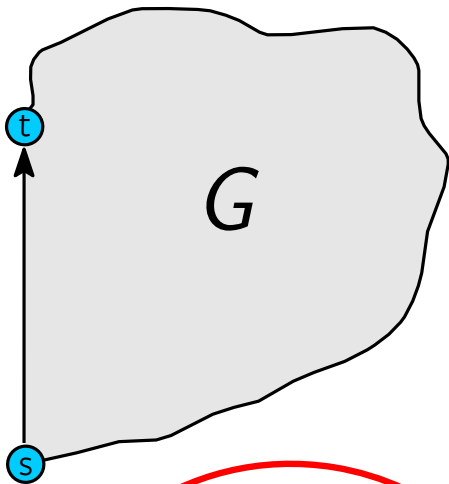
- $E(s \rightarrow V_1, s \rightarrow V_2) \in S_1$
- $E(V_1 \rightarrow t, t \rightarrow V_2) \in S_2$
- $E(V_1 \rightarrow V_2) \in S_3$

Single-Source oDAGs

Theorem 1a

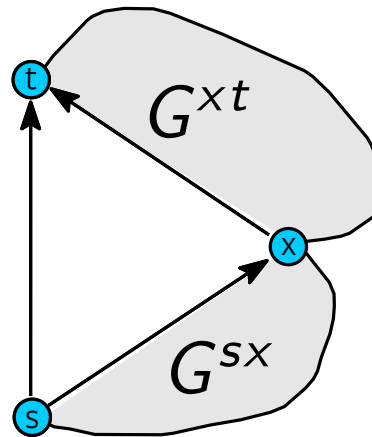
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operation O1

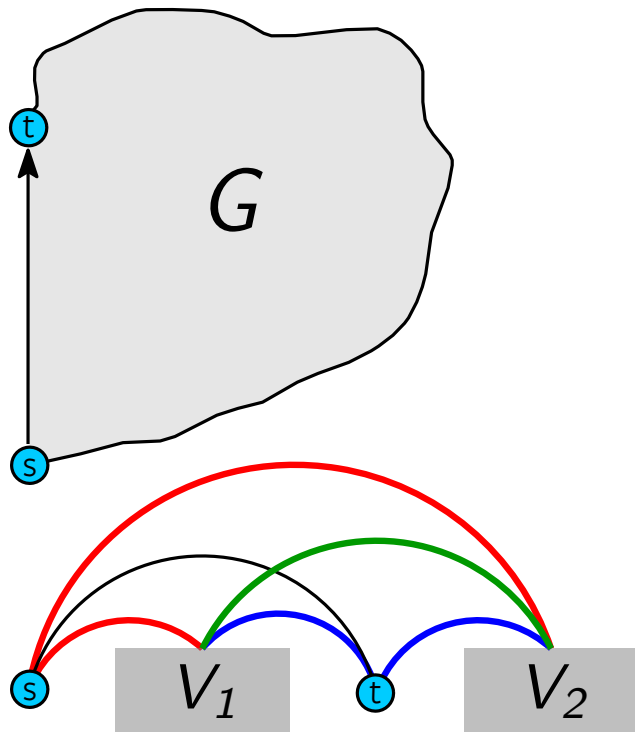


Single-Source oDAGs

Theorem 1a

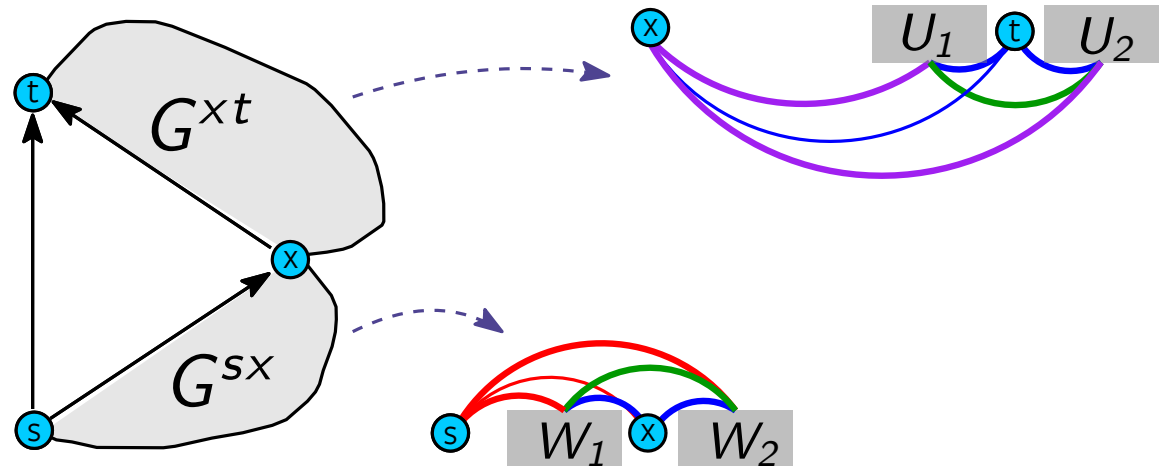
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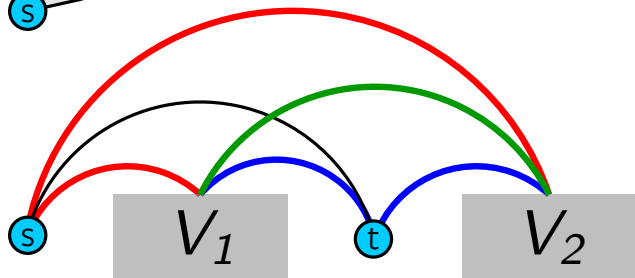
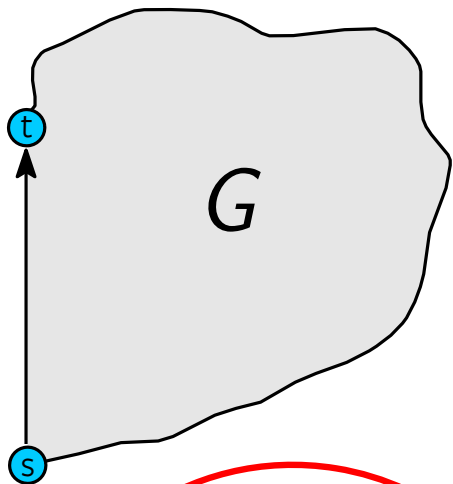


Single-Source oDAGs

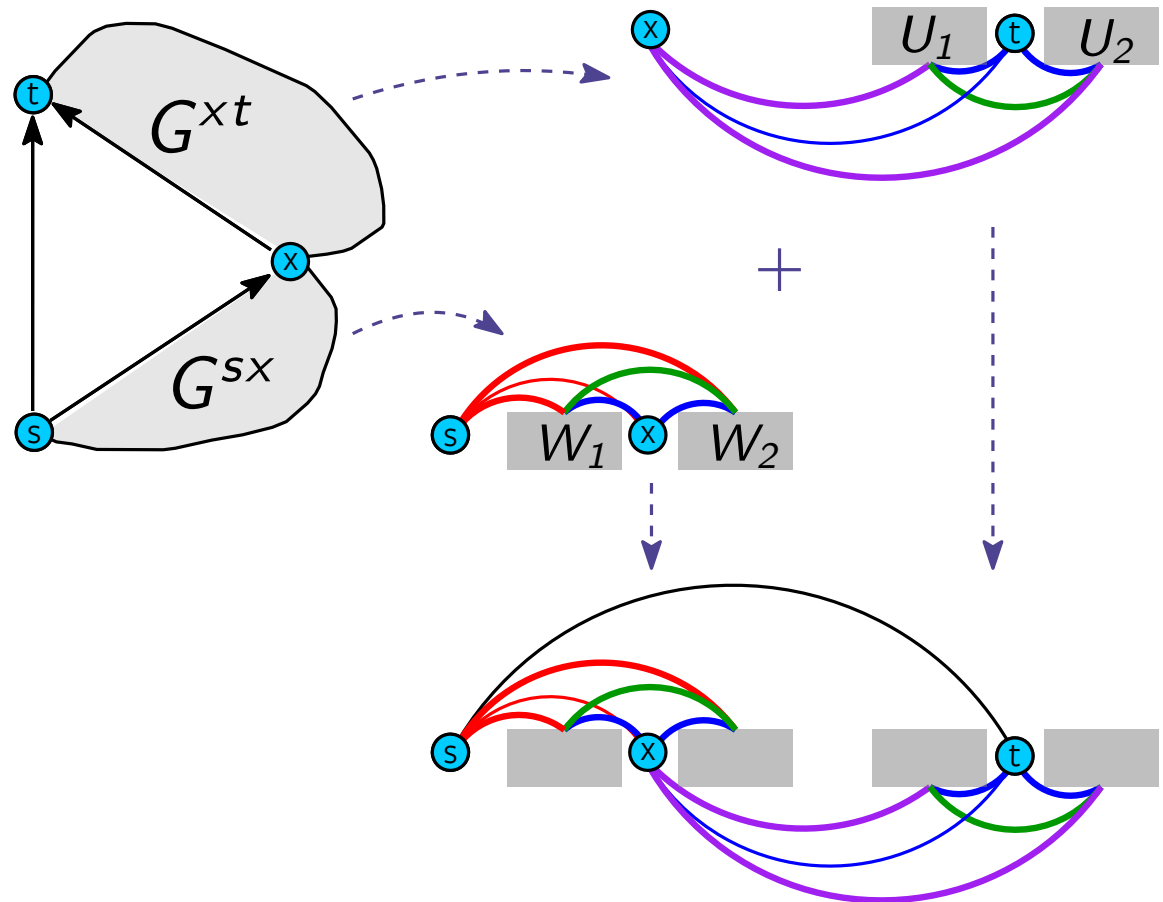
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Wrapping up

Theorem 1(a,b,c)

Every outerplanar DAG constructed by **two** operations has a constant stack number

Theorem 2

Every upward planar 3-tree admits an order with twist size is at most 5

Theorem 3

Lower bounds:

- single-source single-sink oDAG ≥ 3 stacks
- upward oDAG ≥ 4 stacks
- upward planar 3-tree DAG ≥ 5 stacks

Open Problems

Problem 1

Is the stack number of *outerplanar* DAGs bounded by a constant?

Problem 2

Is the stack number of *2-tree* DAGs bounded by a constant?

Open Problems

Problem 1

Is the stack number of *outerplanar* DAGs bounded by a constant?



Problem 2

Is the stack number of *2-tree* DAGs bounded by a constant?



[JMU @ FOCS'23]

Open Problems

Problem 1

Is the stack number of *outerplanar* DAGs bounded by a constant?



Problem 2

Is the stack number of *2-tree* DAGs bounded by a constant?



[JMU @ FOCS'23]

Problem 3

What is the stack number of *outerplanar* DAGs?

$[4 \leq ? \leq 24776]$

Problem 4 [Heath and Pemmaraju, 1999]:

Is the stack number of *upward planar* DAGs bounded by a constant? $O\left(n^{\frac{2}{3}}\right)$

Open Problems

SP

Is the stack number of upward planar graphs bounded by a constant?



Yes, the stack number of upward planar graphs is bounded by a constant. The exact value of this constant has been an active area of research in graph theory, and researchers have made significant progress in understanding the stack number of upward planar graphs.

The best-known upper bound for the stack number of upward planar graphs is 3. In other words, any upward planar graph can be embedded in such a way that it can be represented as a collection of at most three layers or pages, where each edge is either horizontal or strictly increasing in the vertical direction.

This result is an important milestone in the study of upward planar graphs and their stack numbers. It demonstrates that, unlike general planar graphs where the stack number can be unbounded, upward planar graphs have a bounded stack number.

Open Problems

SP

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This result is an important milestone in the study of upward planar graphs and their stack numbers. It demonstrates that, unlike general planar graphs where the stack number can be unbounded, upward planar graphs have a bounded stack number.

SP

Are you sure???



I apologize for any confusion in my previous responses. I made an error in my previous messages. The stack number of upward planar graphs is not necessarily bounded by a constant like 3. Upward planar graphs can have unbounded stack numbers.

Open Problems

Problem 1

Is the stack number of *outerplanar* DAGs bounded by a constant?



Problem 2

Is the stack number of *2-tree* DAGs bounded by a constant?



[JMU @ FOCS'23]

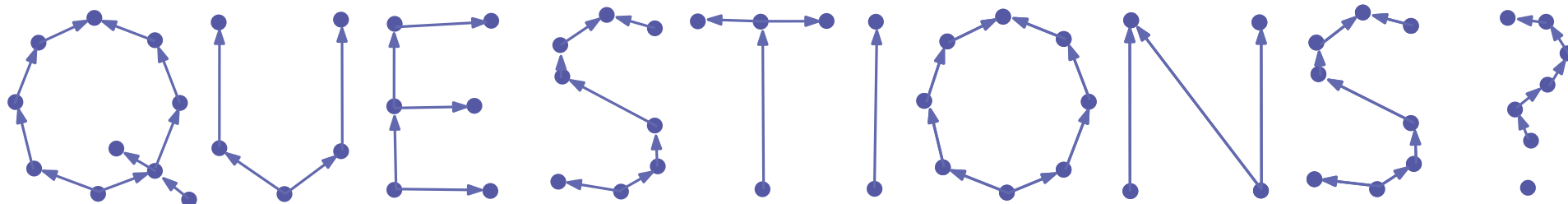
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What is the stack number of *outerplanar* DAGs?

$[4 \leq ? \leq 24776]$

Problem 4 [Heath and Pemmaraju, 1999]:

Is the stack number of *upward planar* DAGs bounded by a constant? $O\left(n^{\frac{2}{3}}\right)$



SP

Can you recommend one hidden gem to visit in Palermo?



Certainly! Palermo, the capital of Sicily, is a city filled with history, culture, and many hidden gems waiting to be discovered. One such hidden gem is the "Orto Botanico di Palermo" or the Botanical Gardens of Palermo. While it may not be entirely hidden, it is often less crowded than some of the more famous attractions in the city.

