Tree Drawings with Columns

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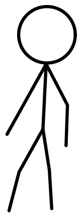
THE UNIVERSITY OF AUCKLAND
Te Whare Wānanga o Tāmaki Makaurau
NEW ZEALAND

Julius-Maximilians-
UNIVERSITÄT
WÜRZBURG

Motivation



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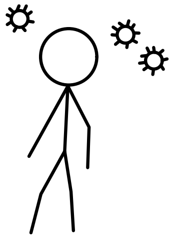

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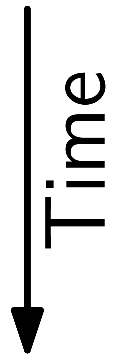

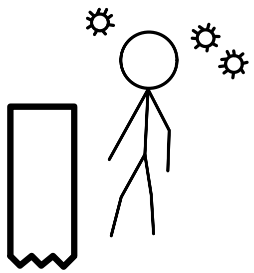

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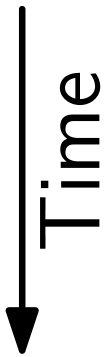

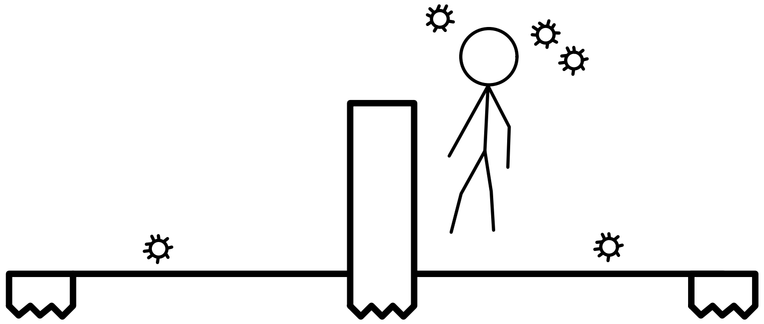

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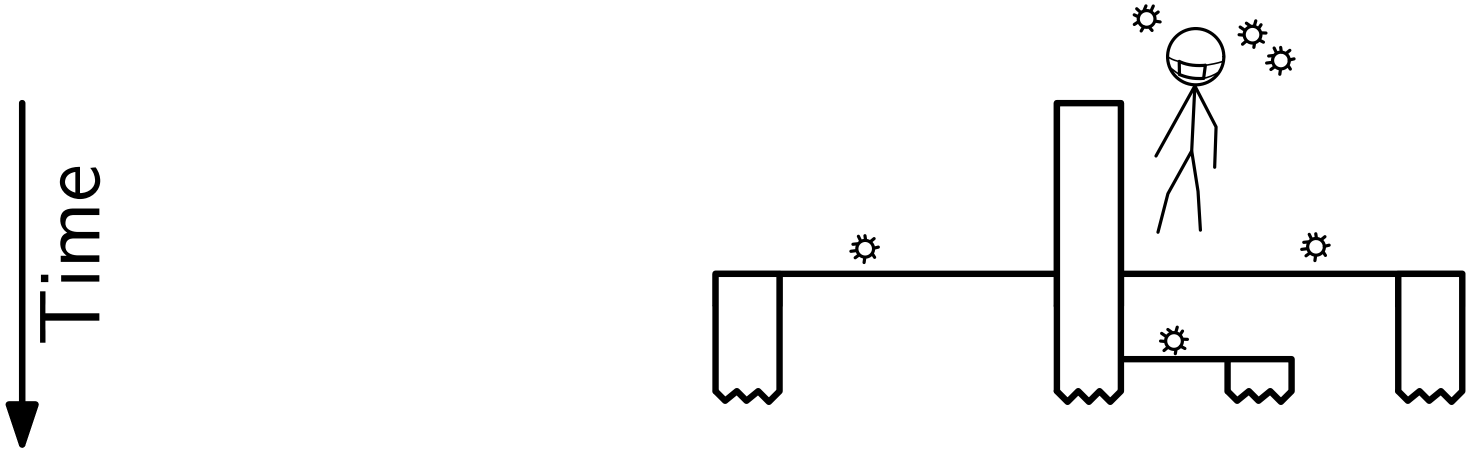

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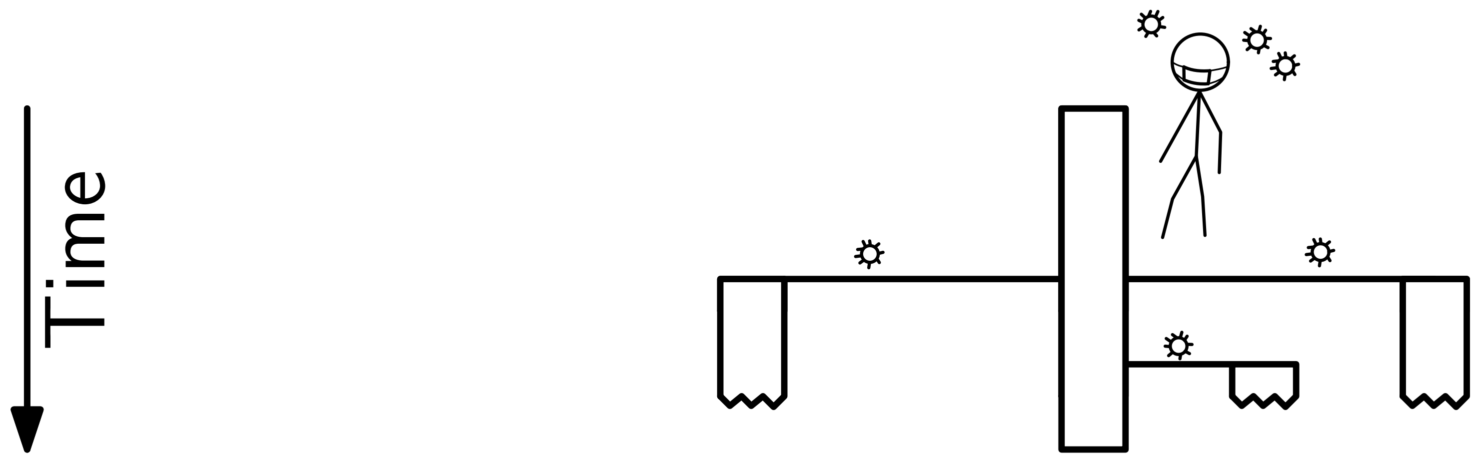

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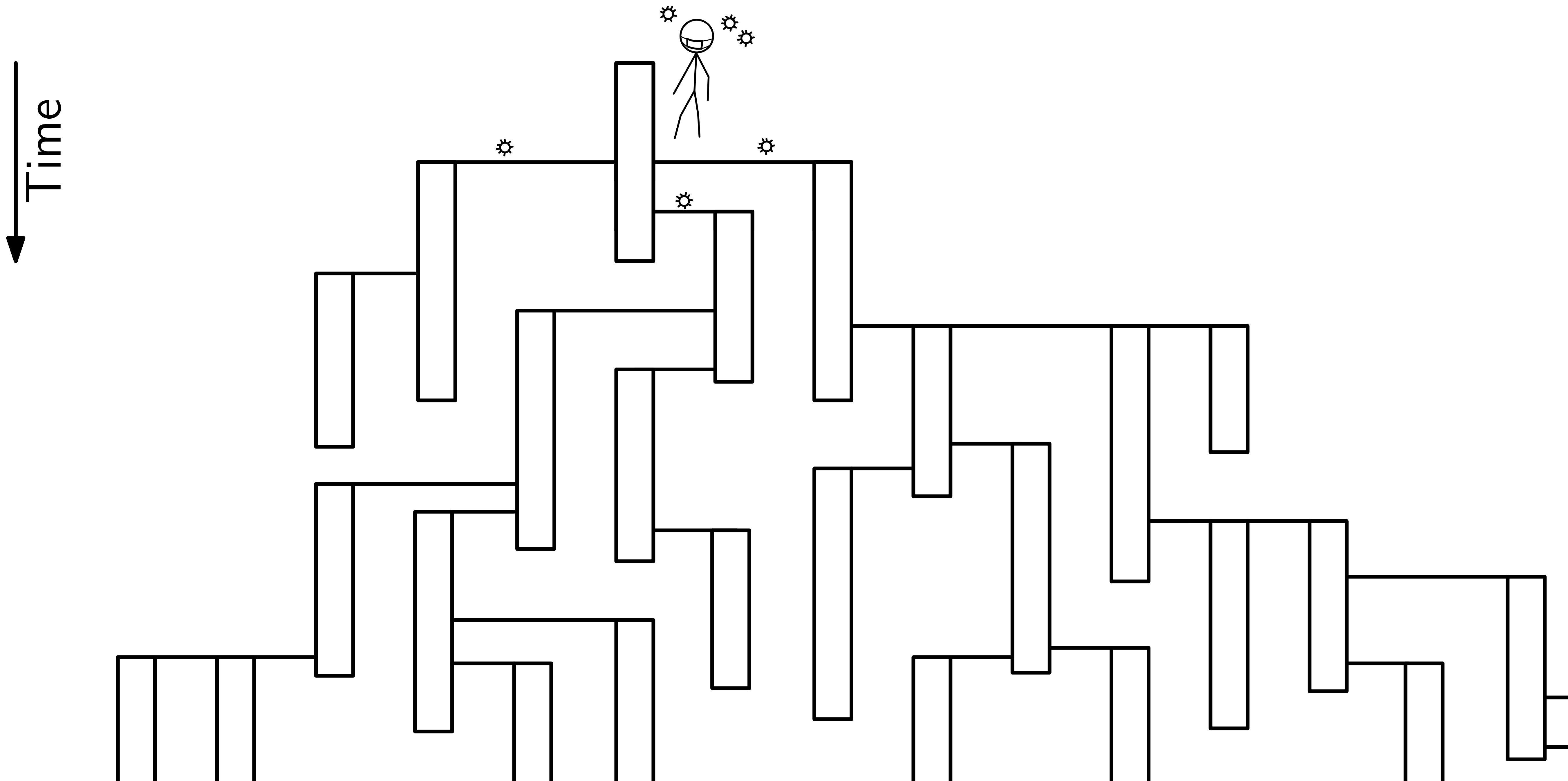

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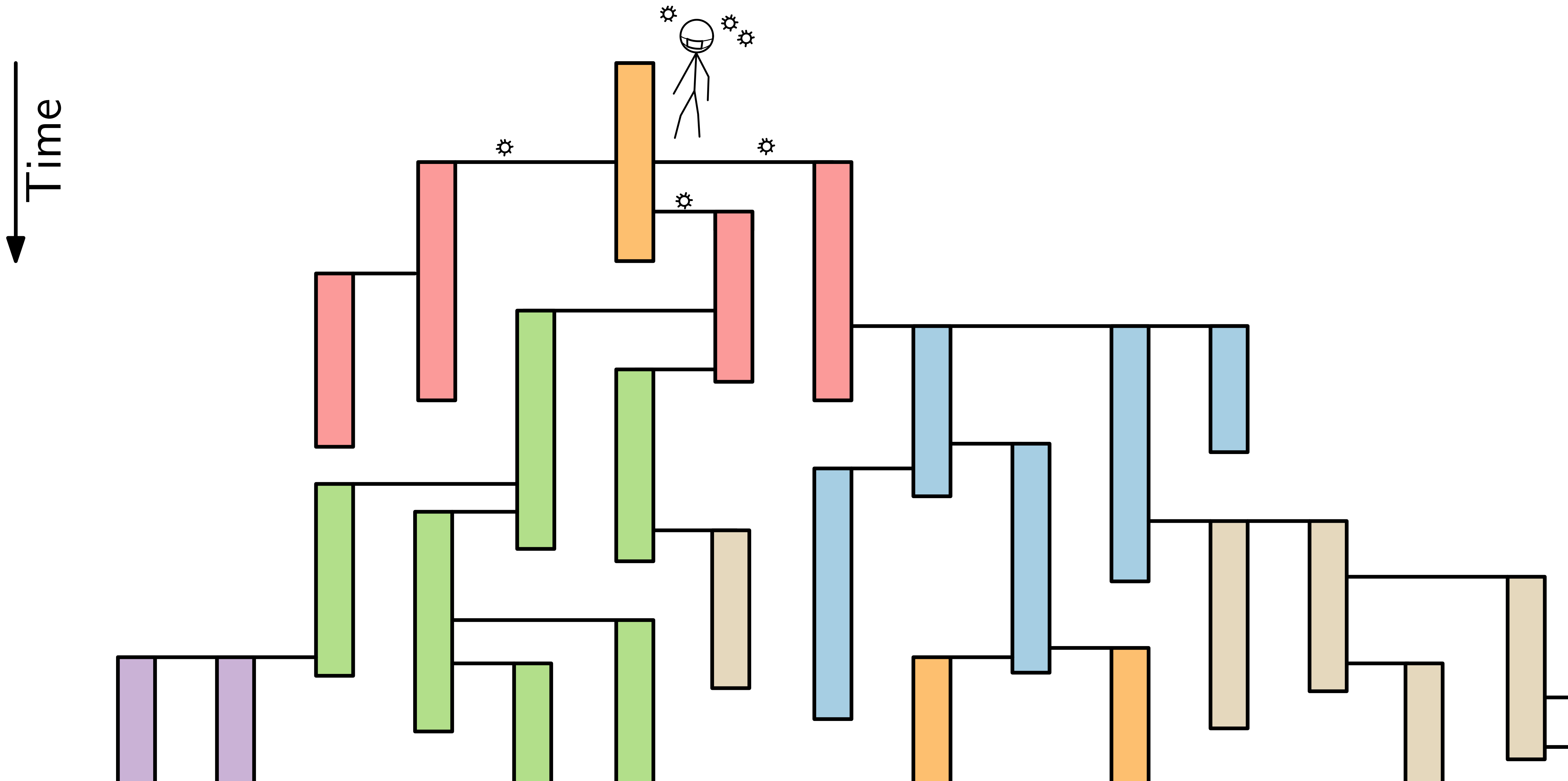

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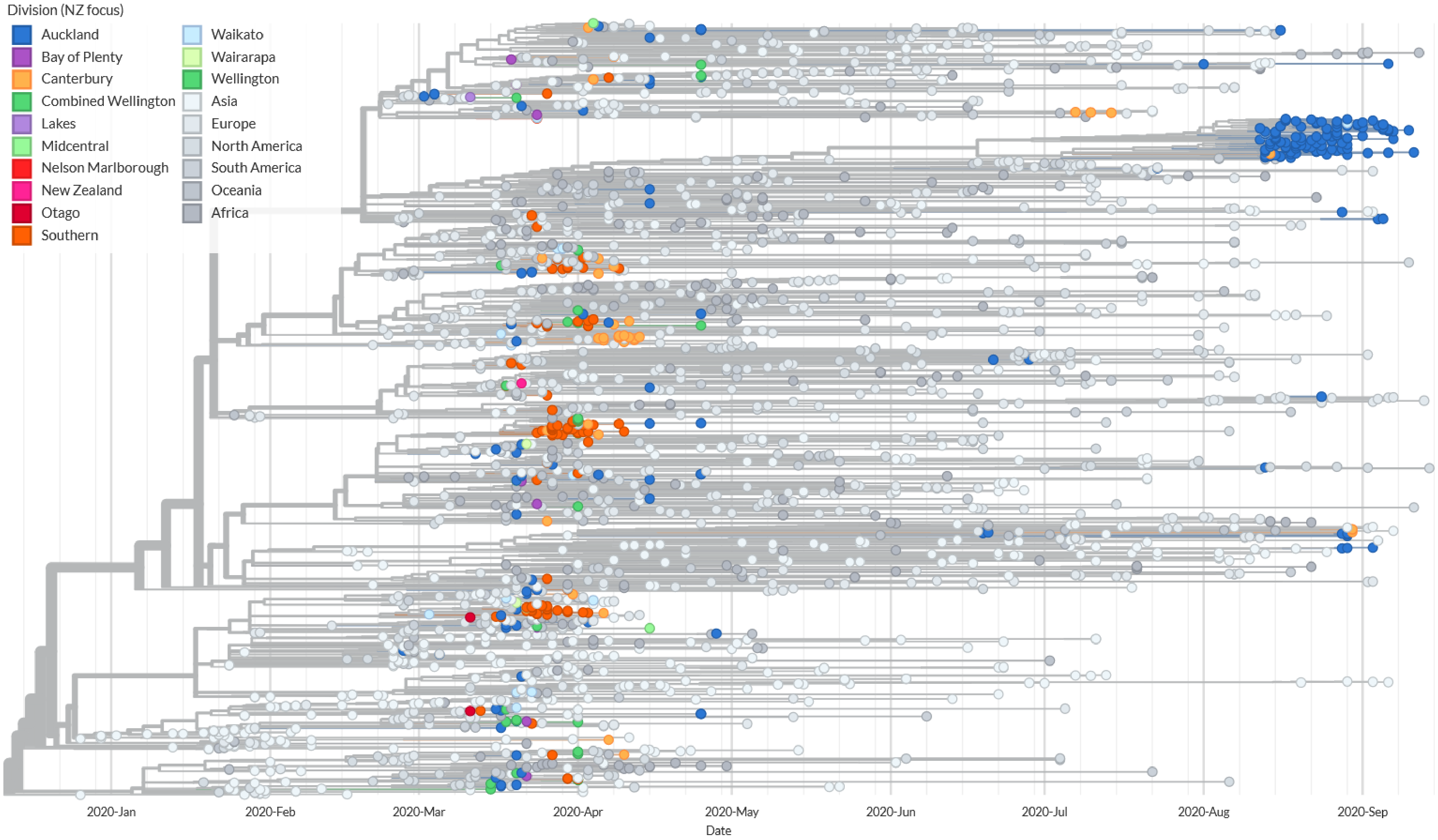
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Bay of Plenty
Canterbury
Combined Wellington
Lakes
Midcentral
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New Zealand
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Southern
Waikato
Wairarapa
Wellington
Asia
Europe
North America
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2020-May
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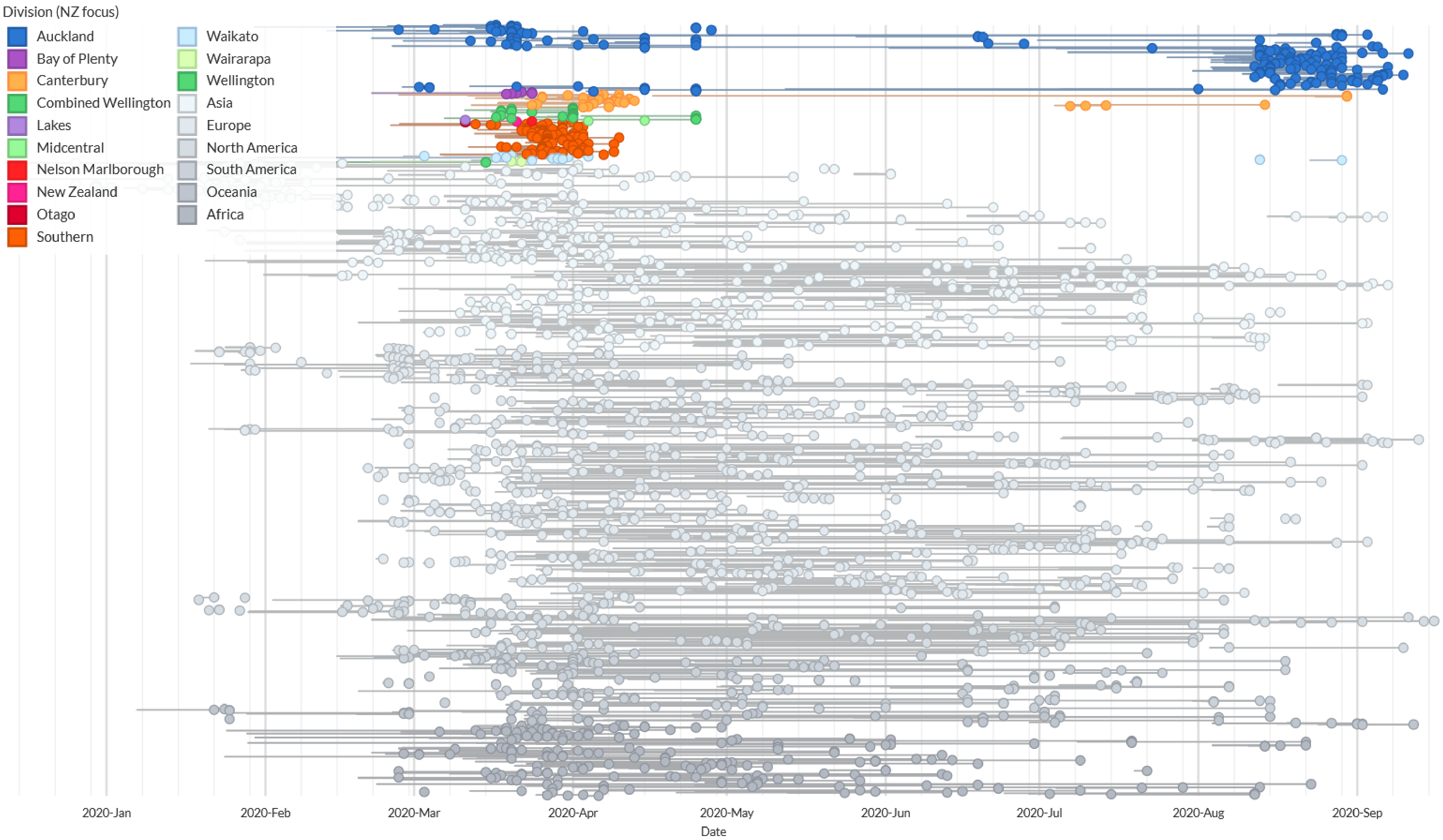
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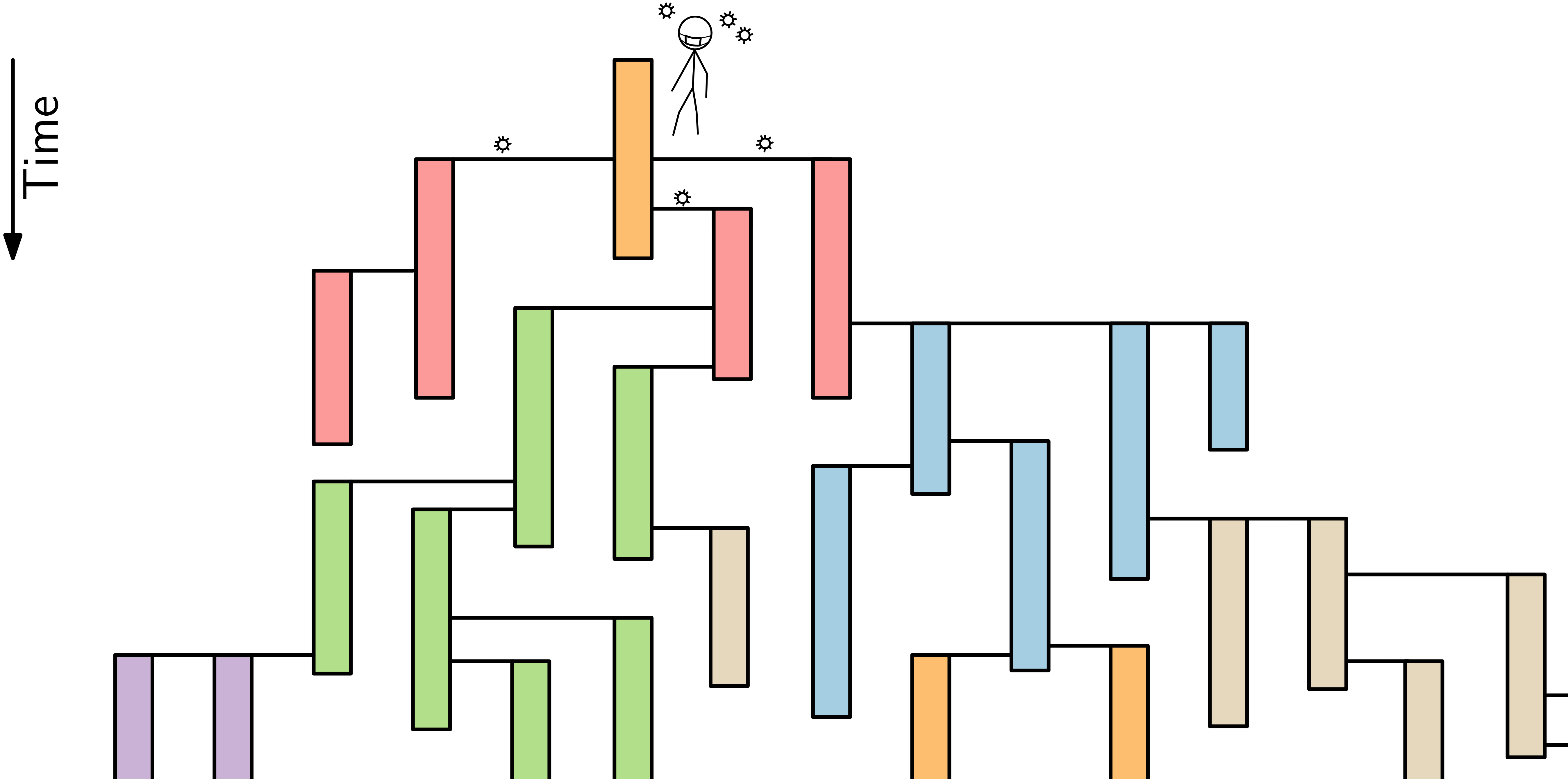

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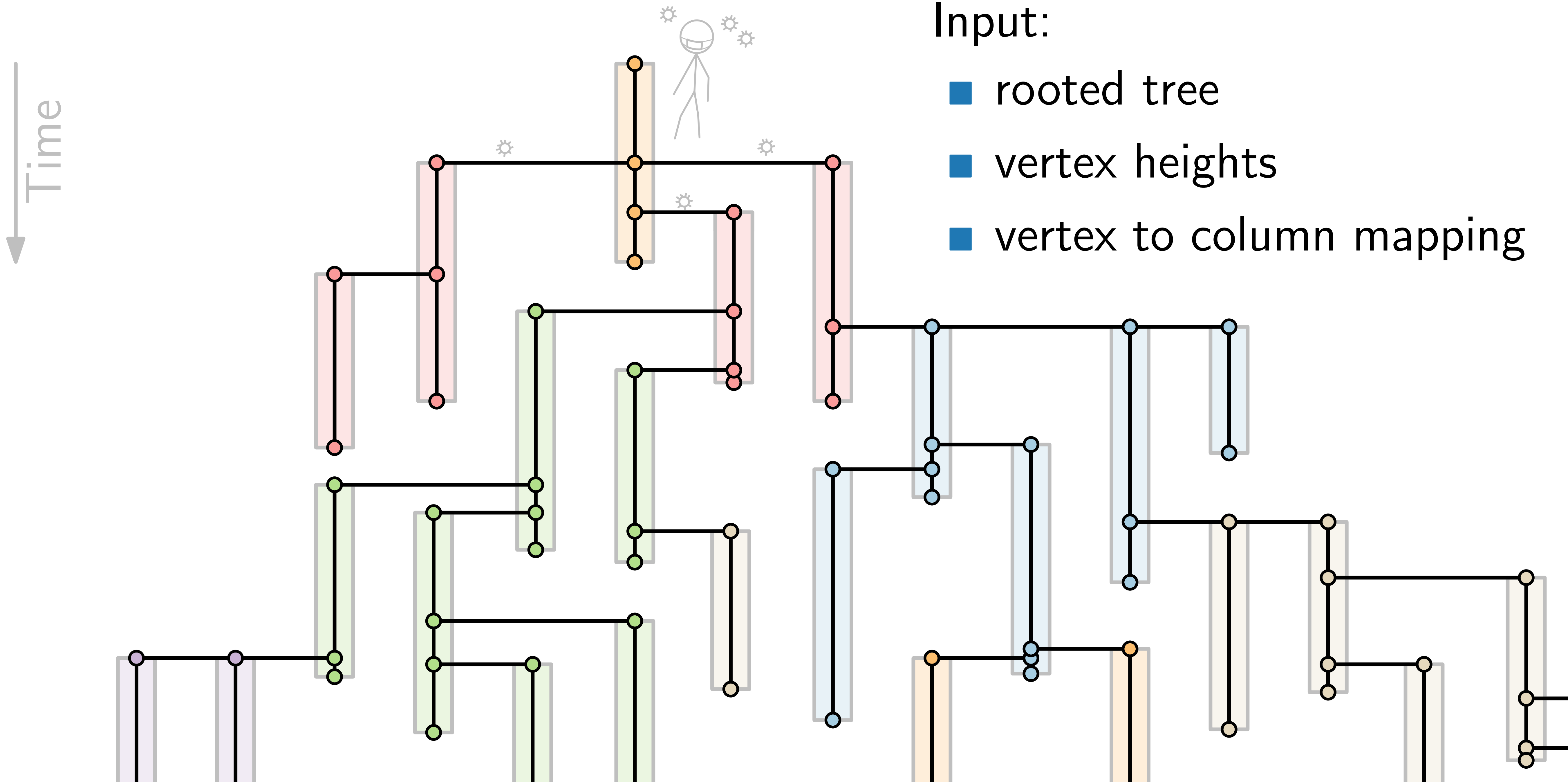


Input:

- rooted tree
- vertex heights
- vertex to column mapping

Overview

Drawing Style

Problem

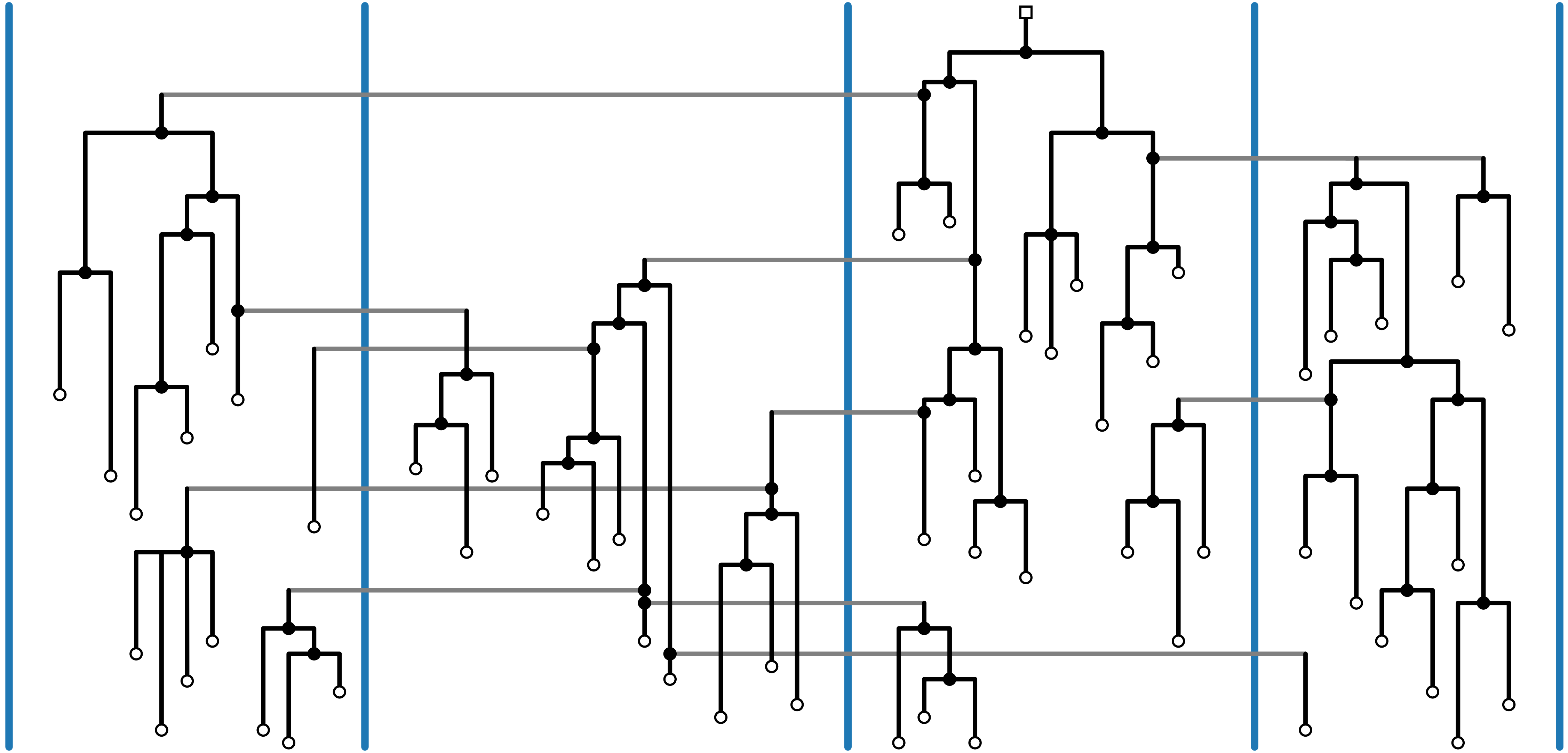
P

NPh

FPT

Drawing Style

- rectangular cladogram
- vertices in their columns at their heights



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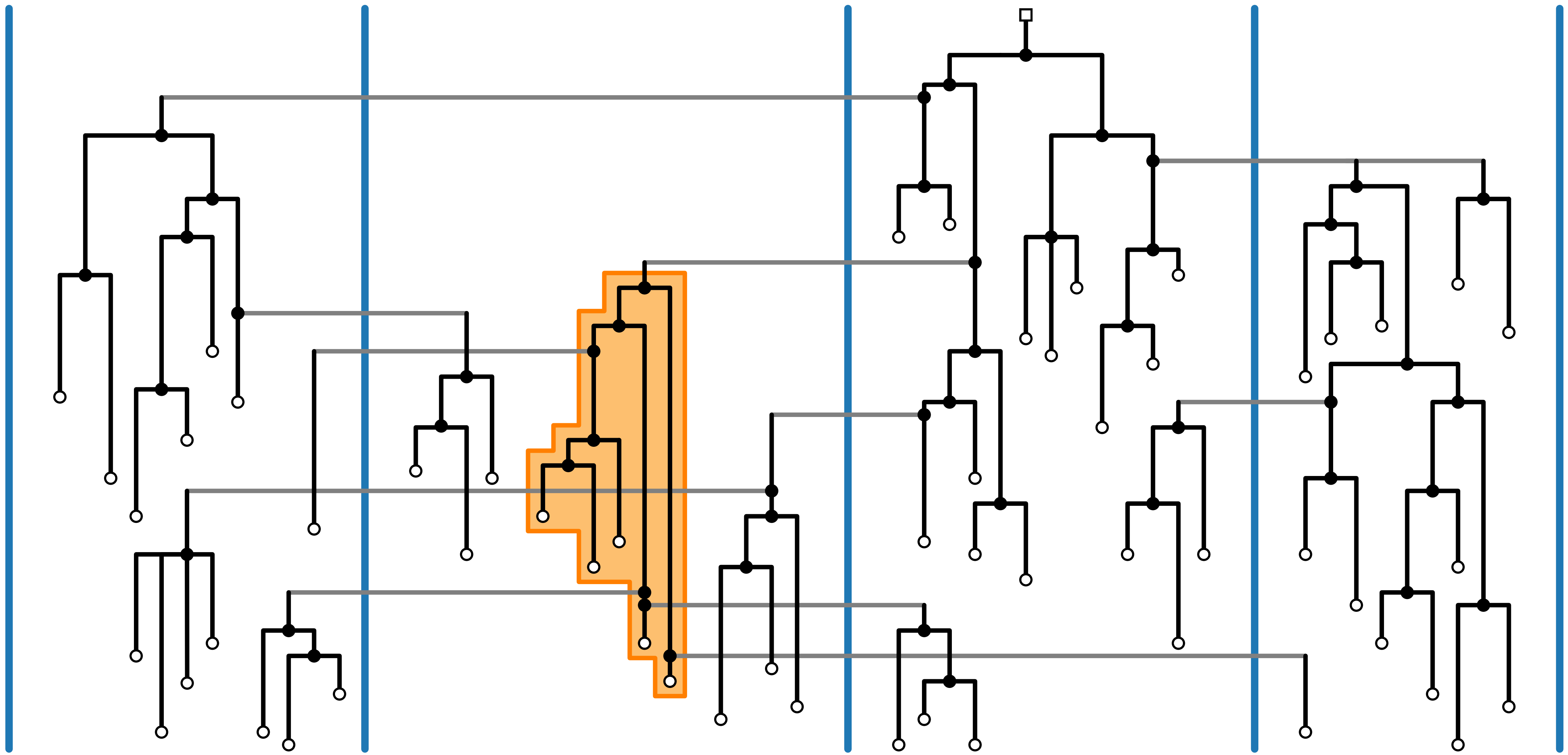

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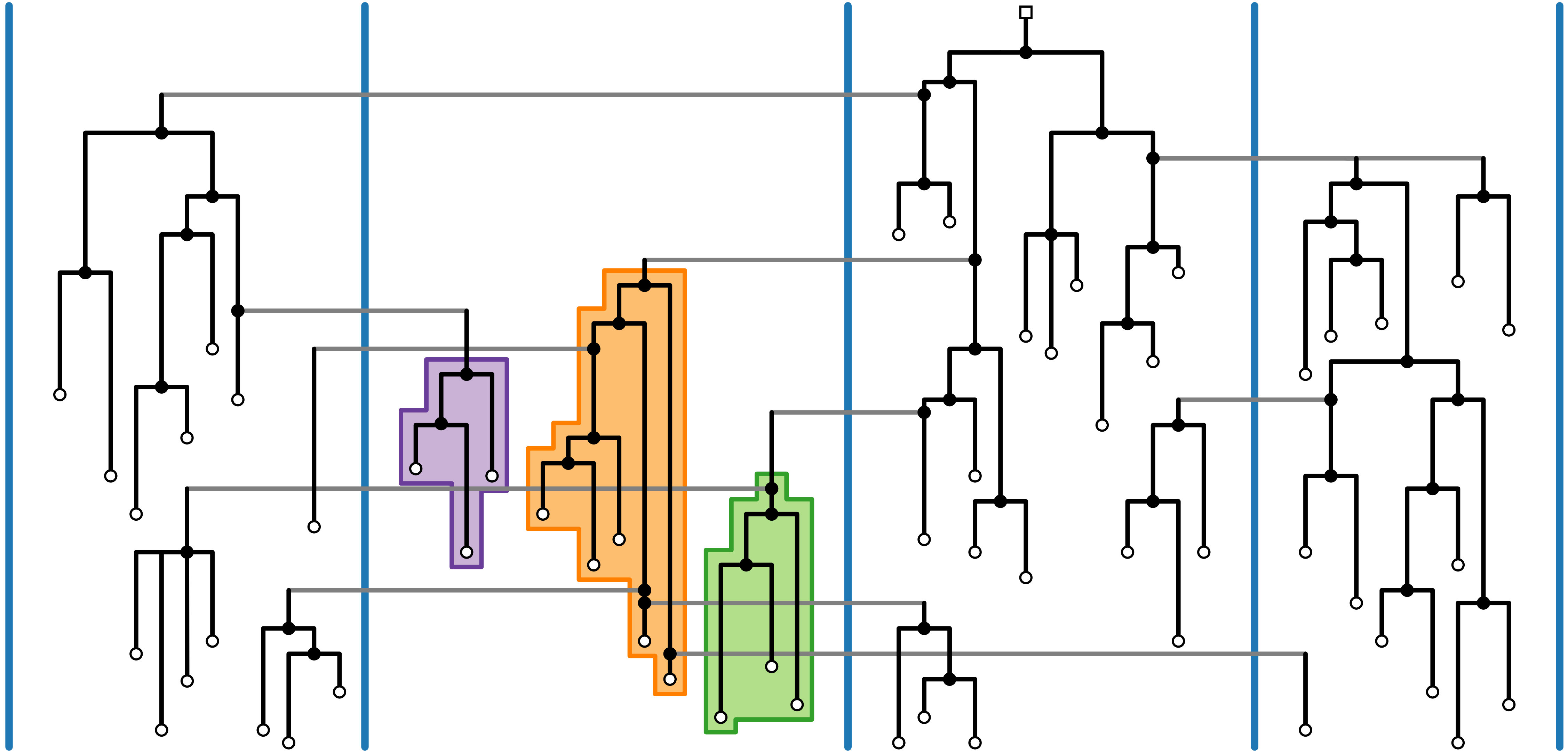
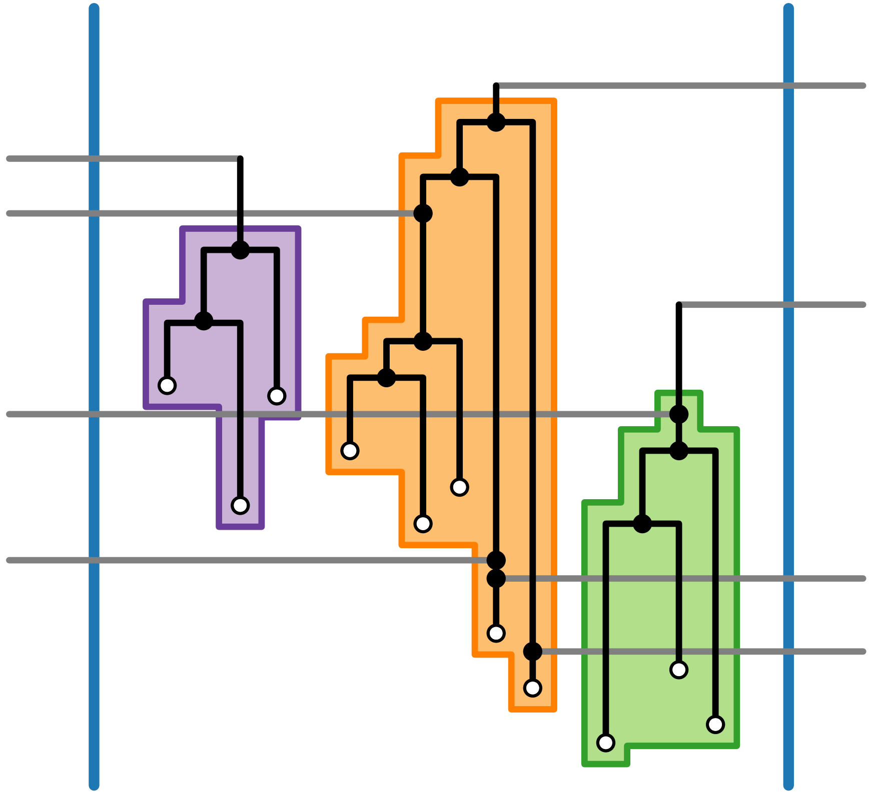


Drawing Style + Variants

- rectangular cladogram
- no two intra-column edges cross
- vertices in their columns at their heights



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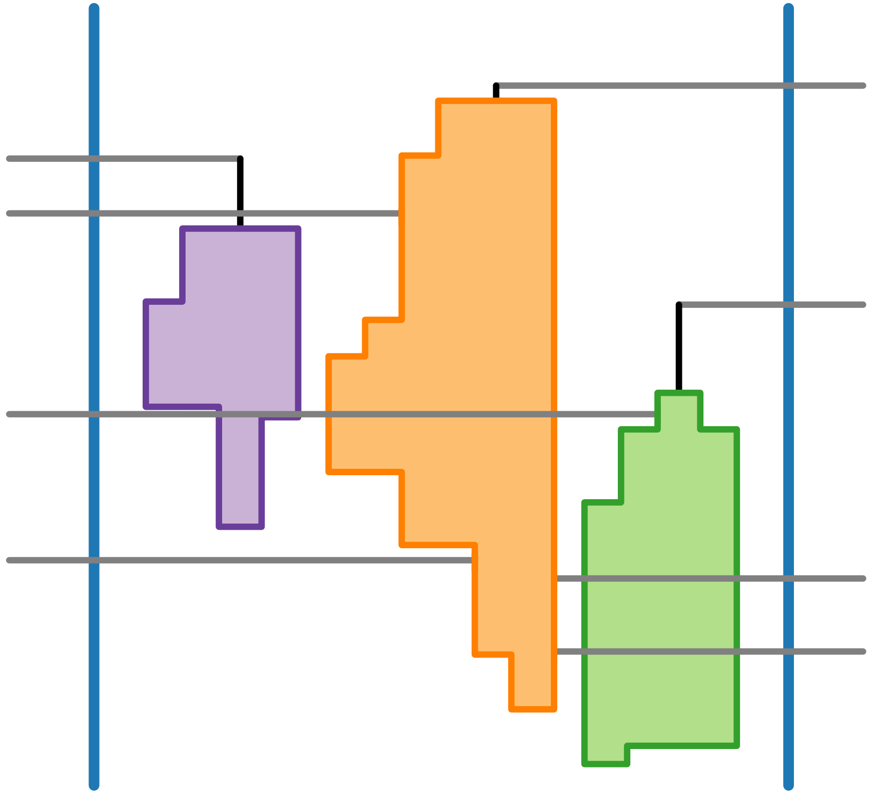

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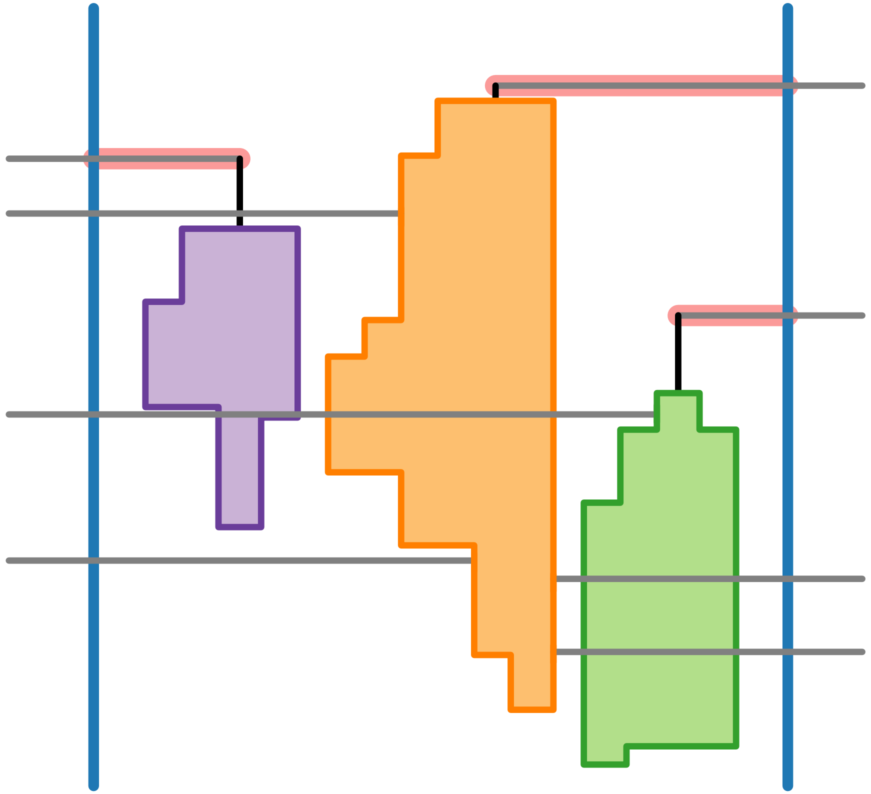
“directly at column border”



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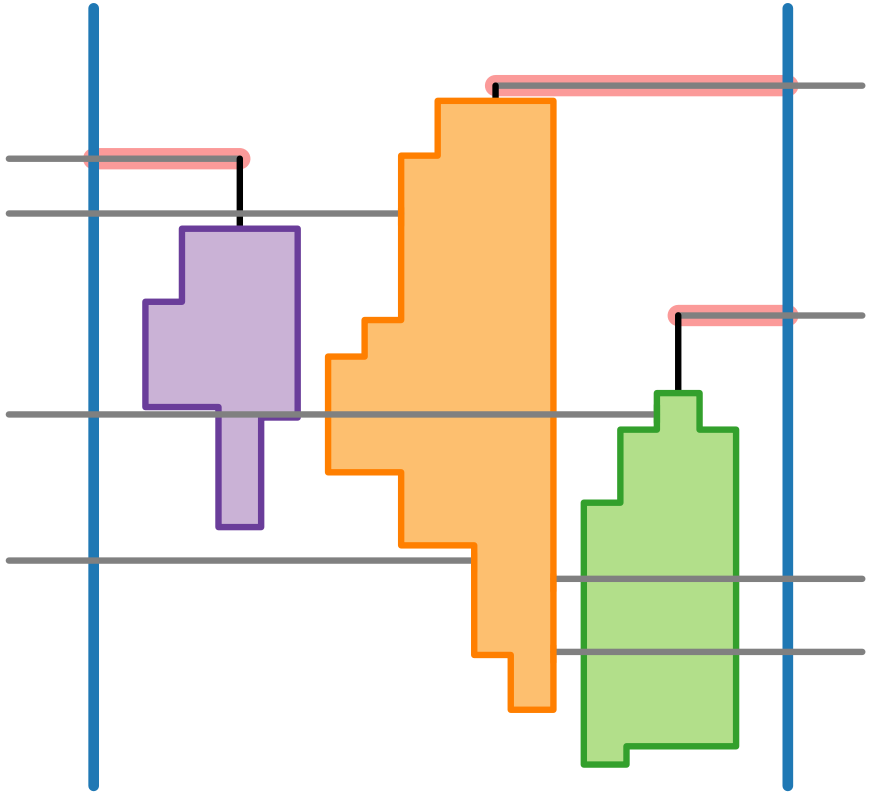
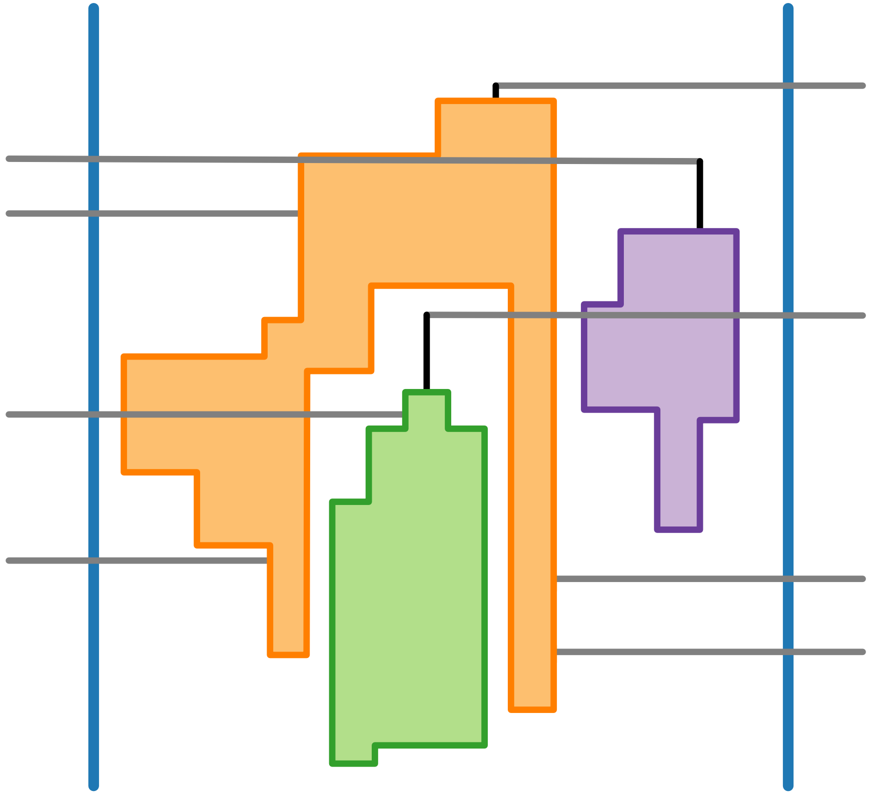


with interleaving



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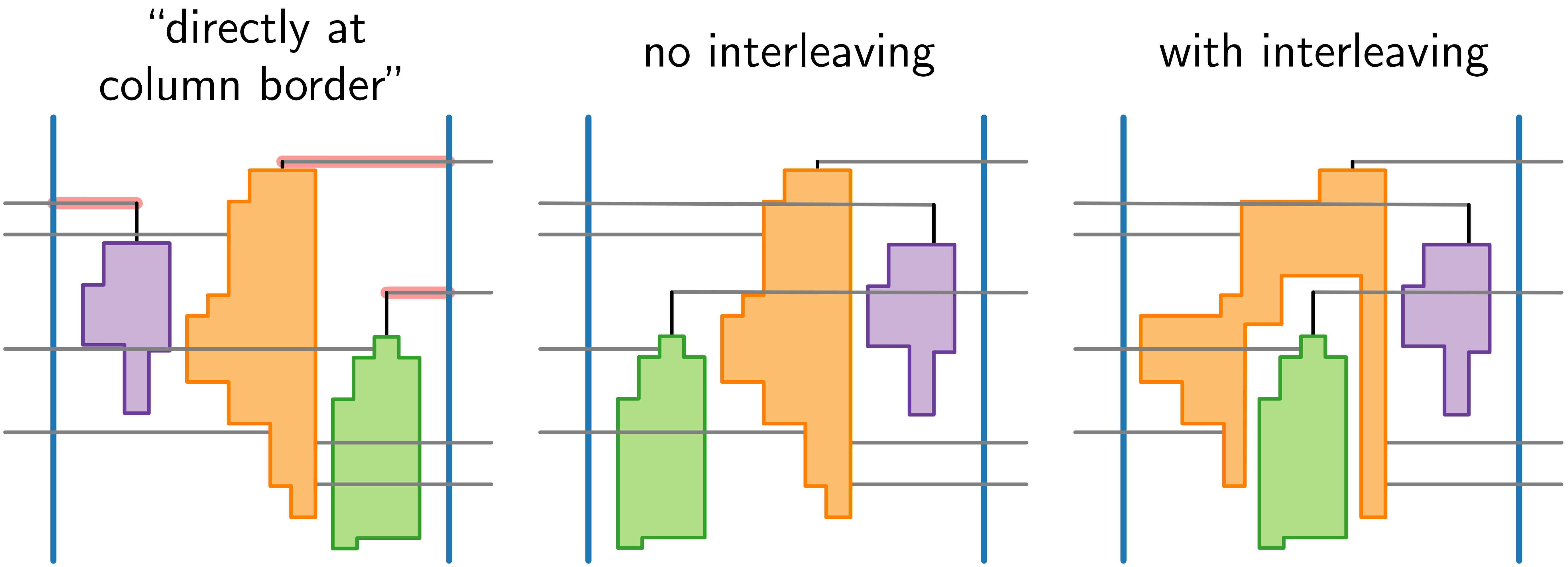

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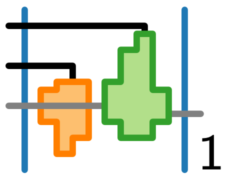

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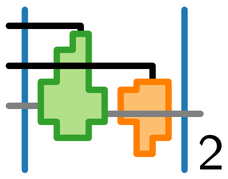

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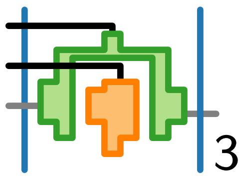

Crossing Minimisation Problem

Find a column tree embedding
with min number of crossings.

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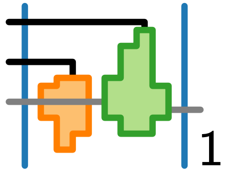
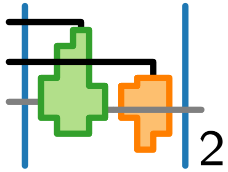
- independent tasks for  and 

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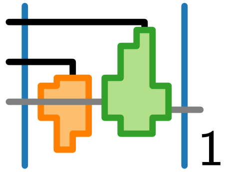
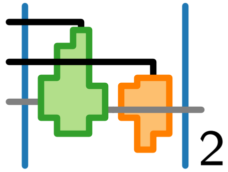
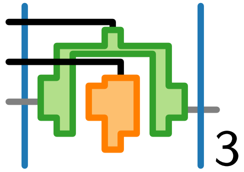
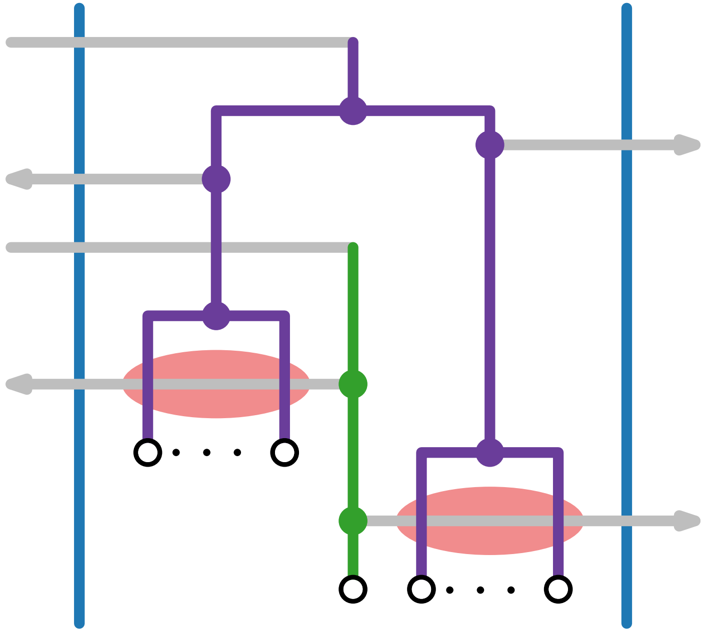
- independent tasks for  1 and  2
- . . . but not so for  3



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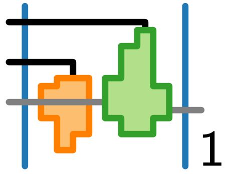
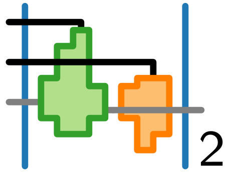
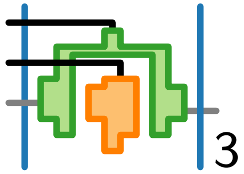
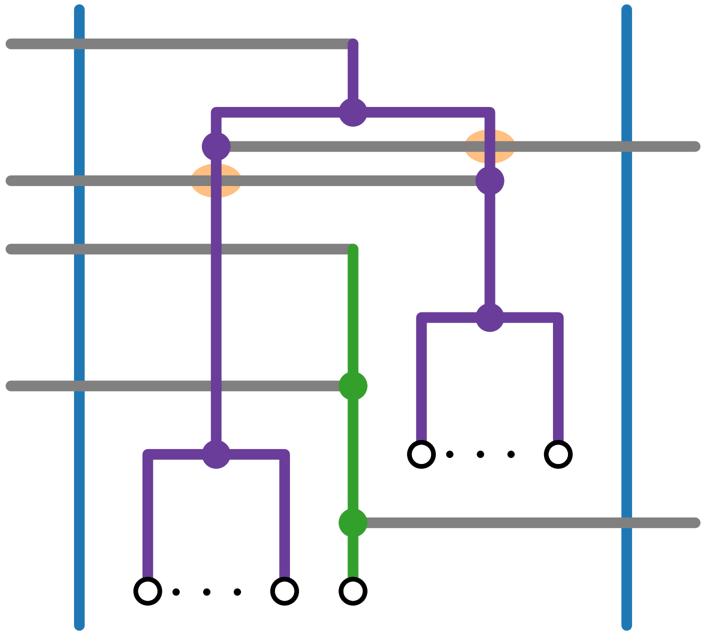
- independent tasks for  and 
- . . . but not so for 



Overview

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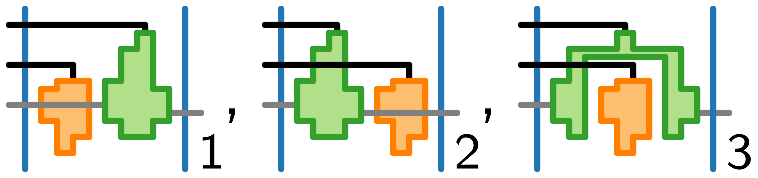

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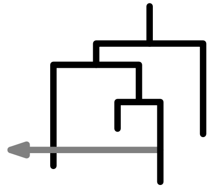

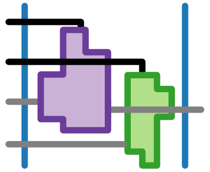

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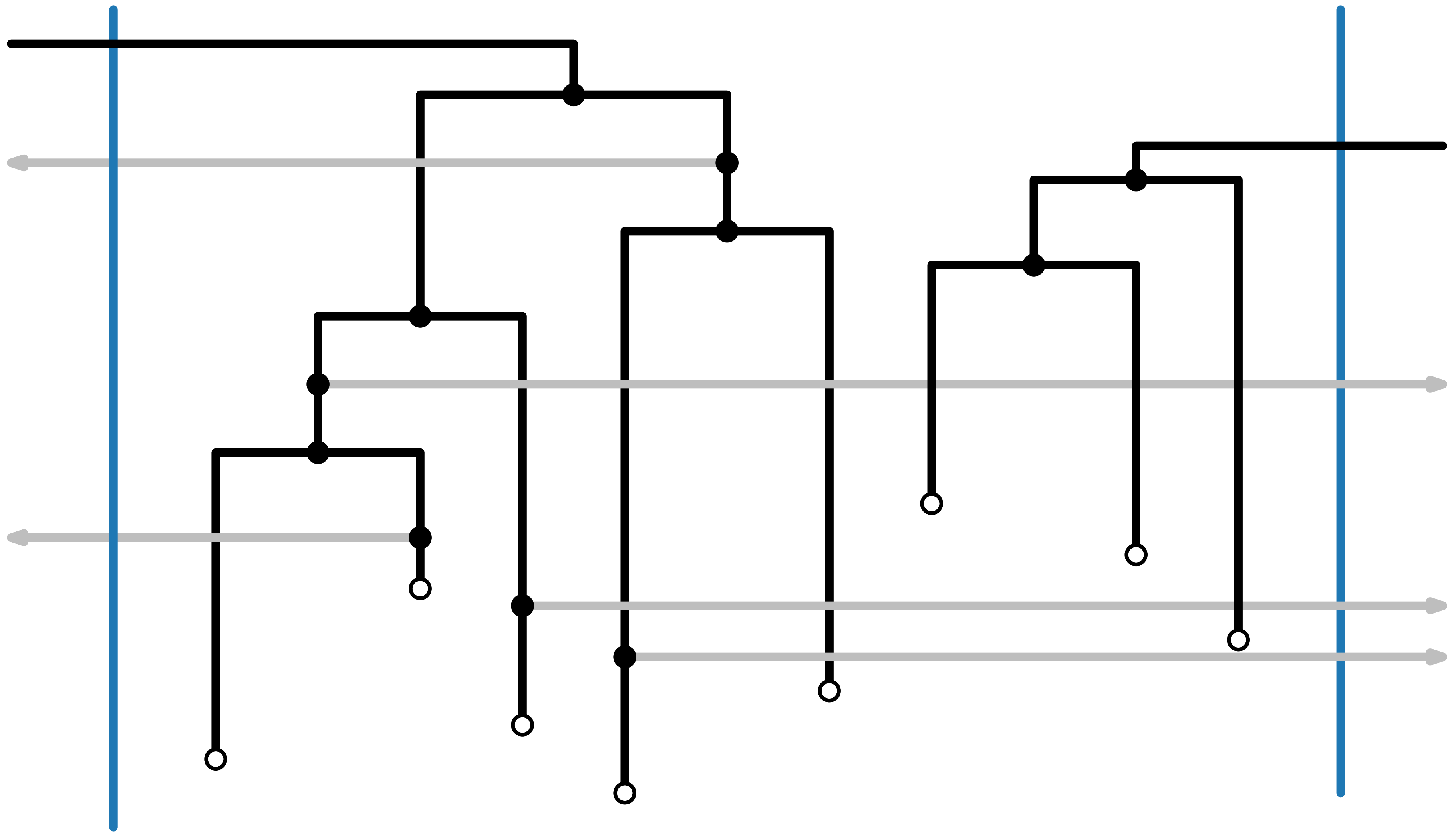
Subtree Embedding Algorithm



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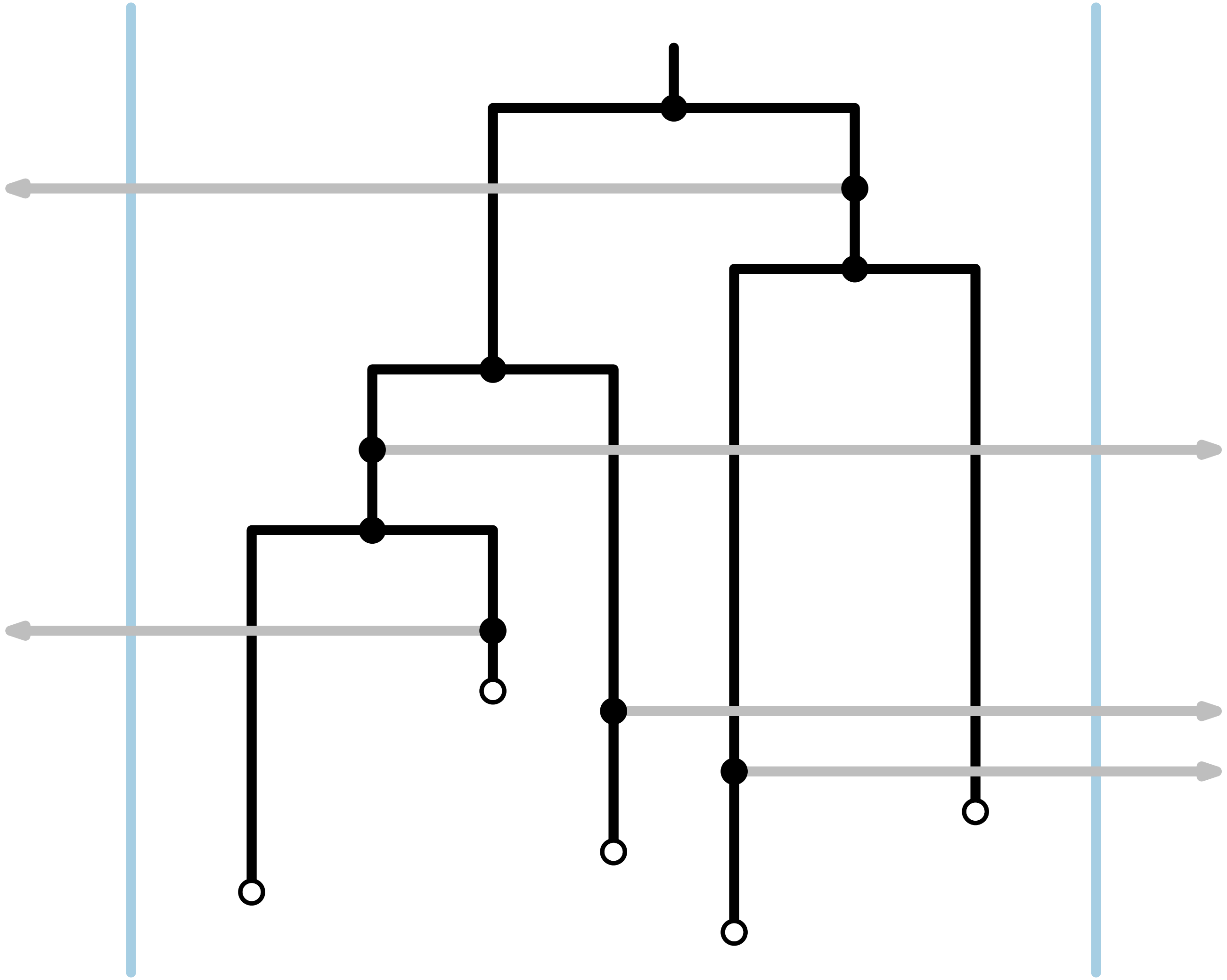

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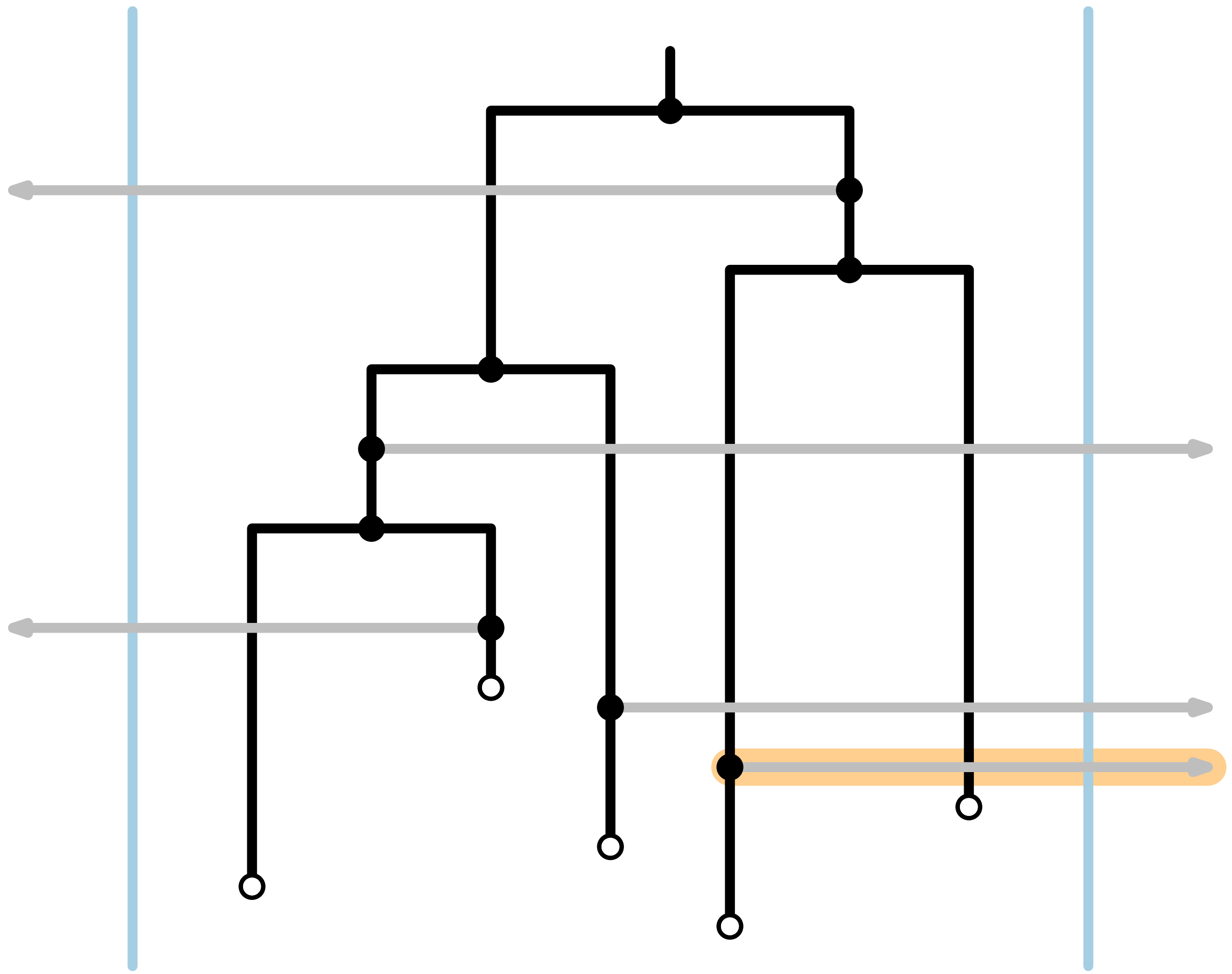

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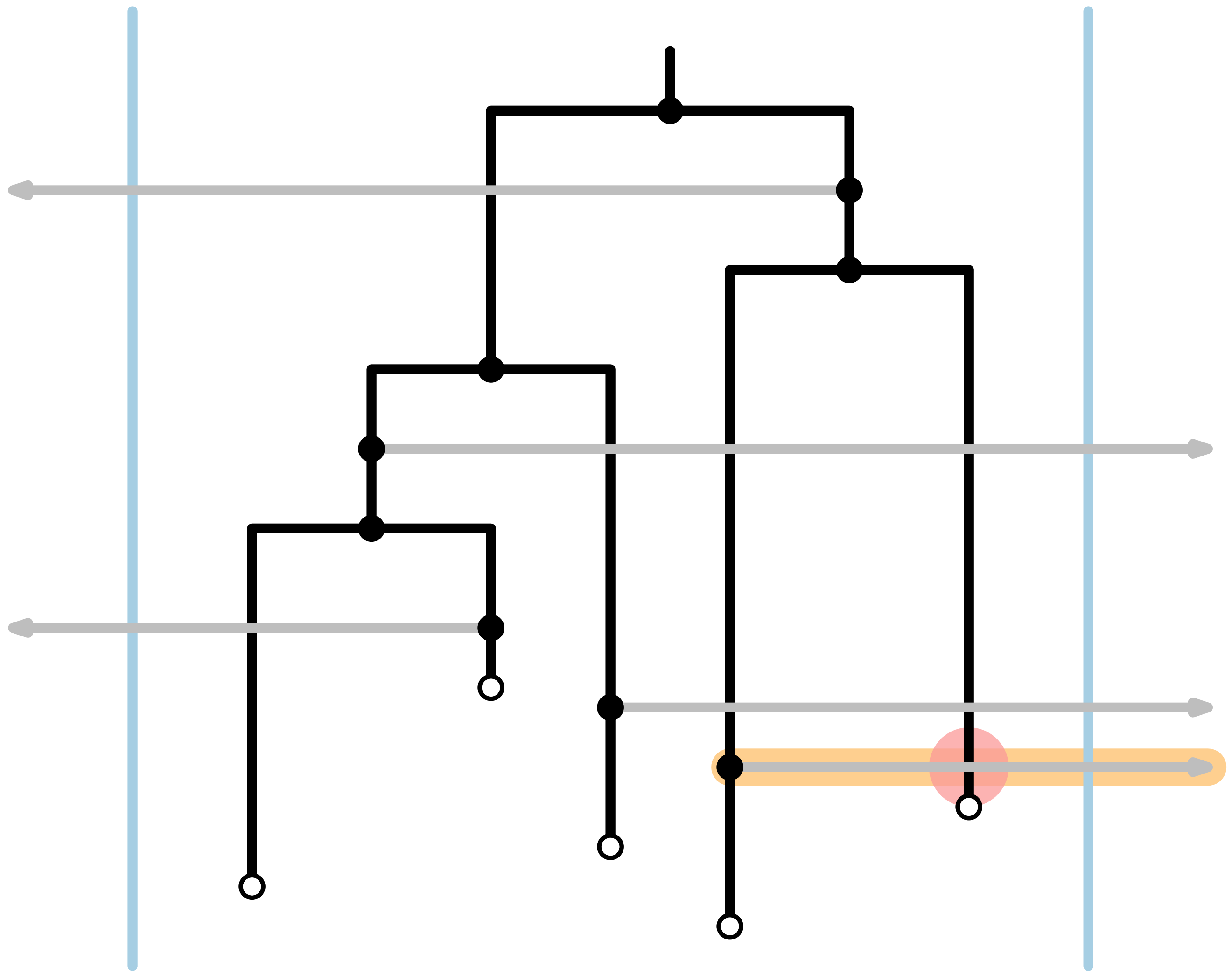

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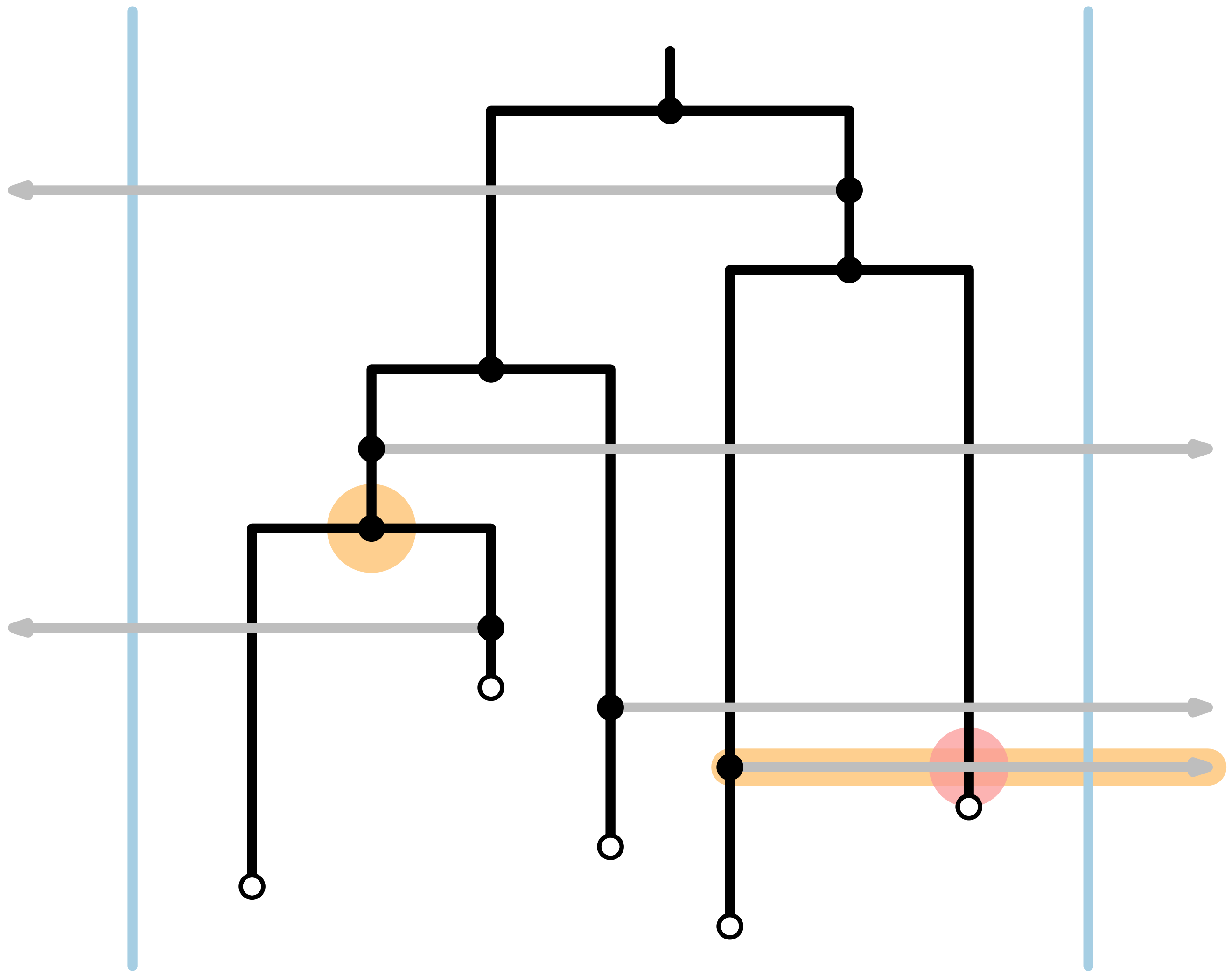

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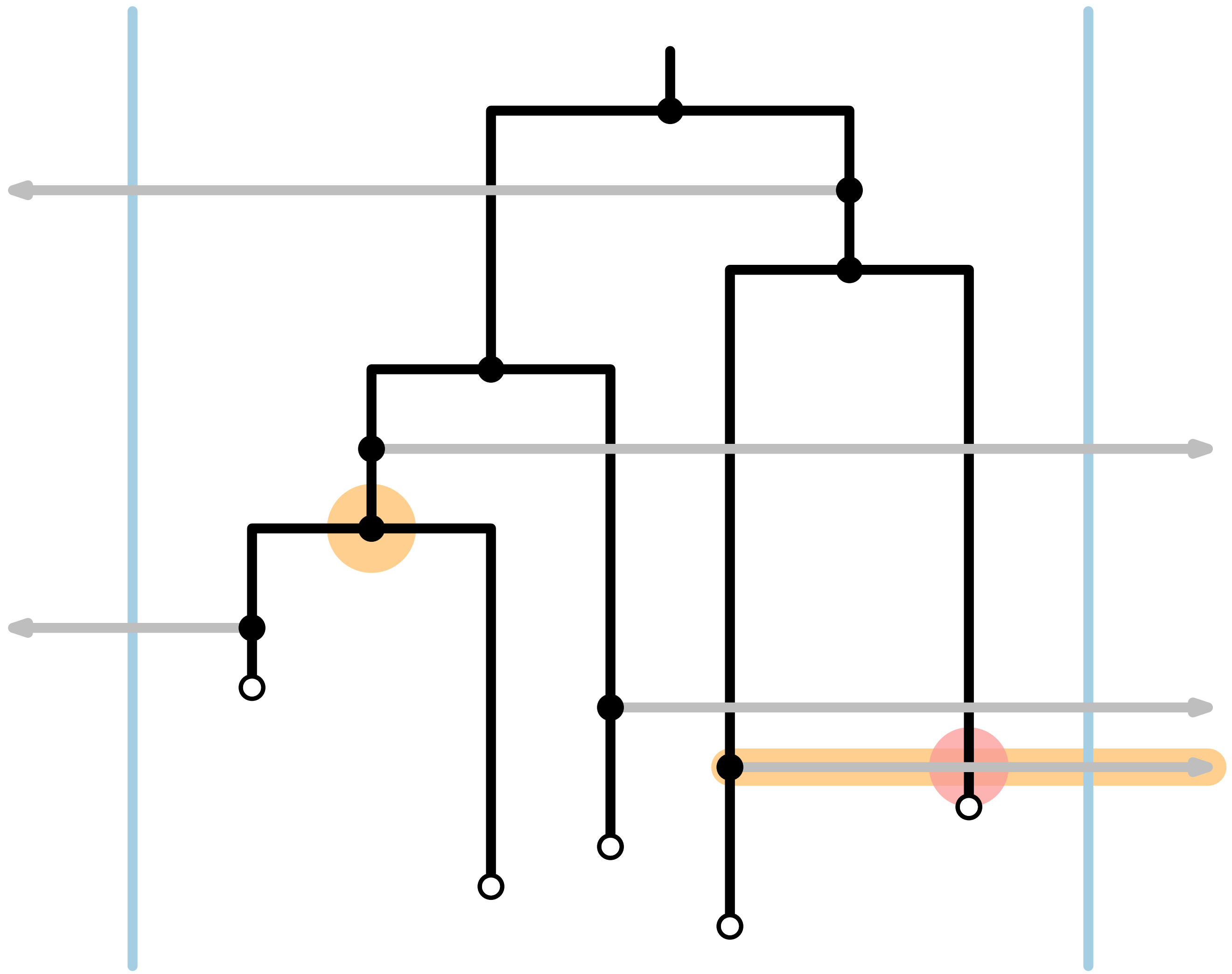

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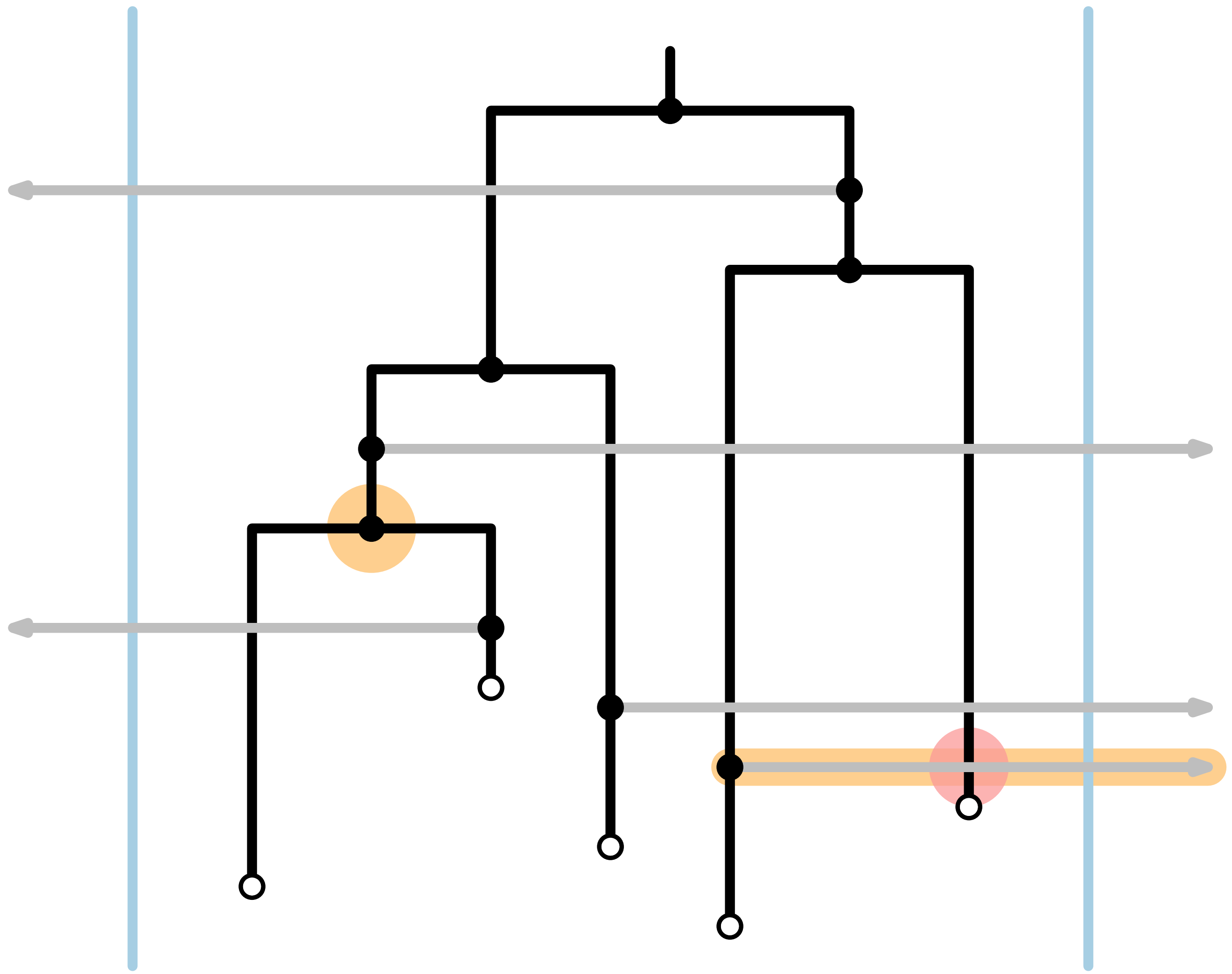

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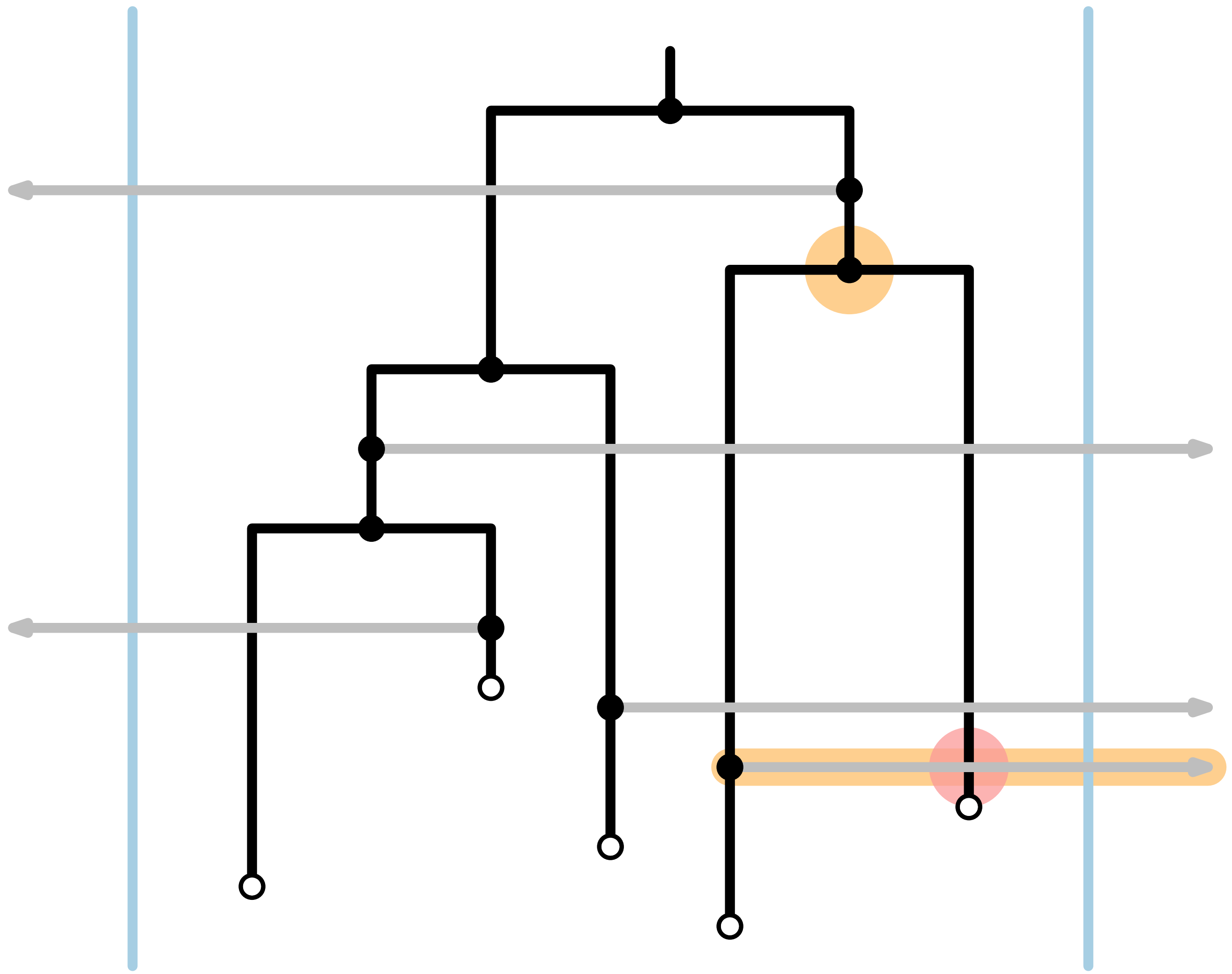

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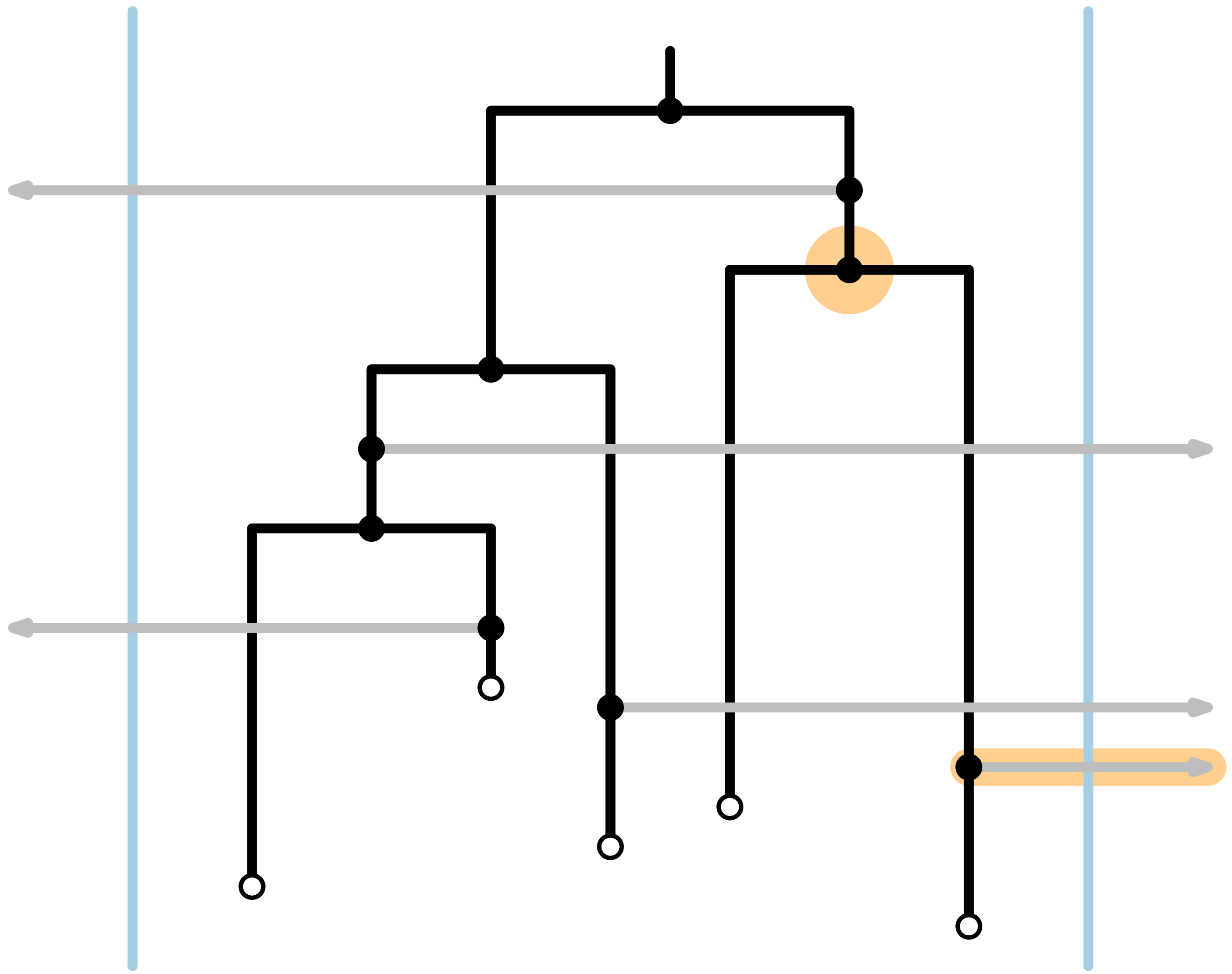

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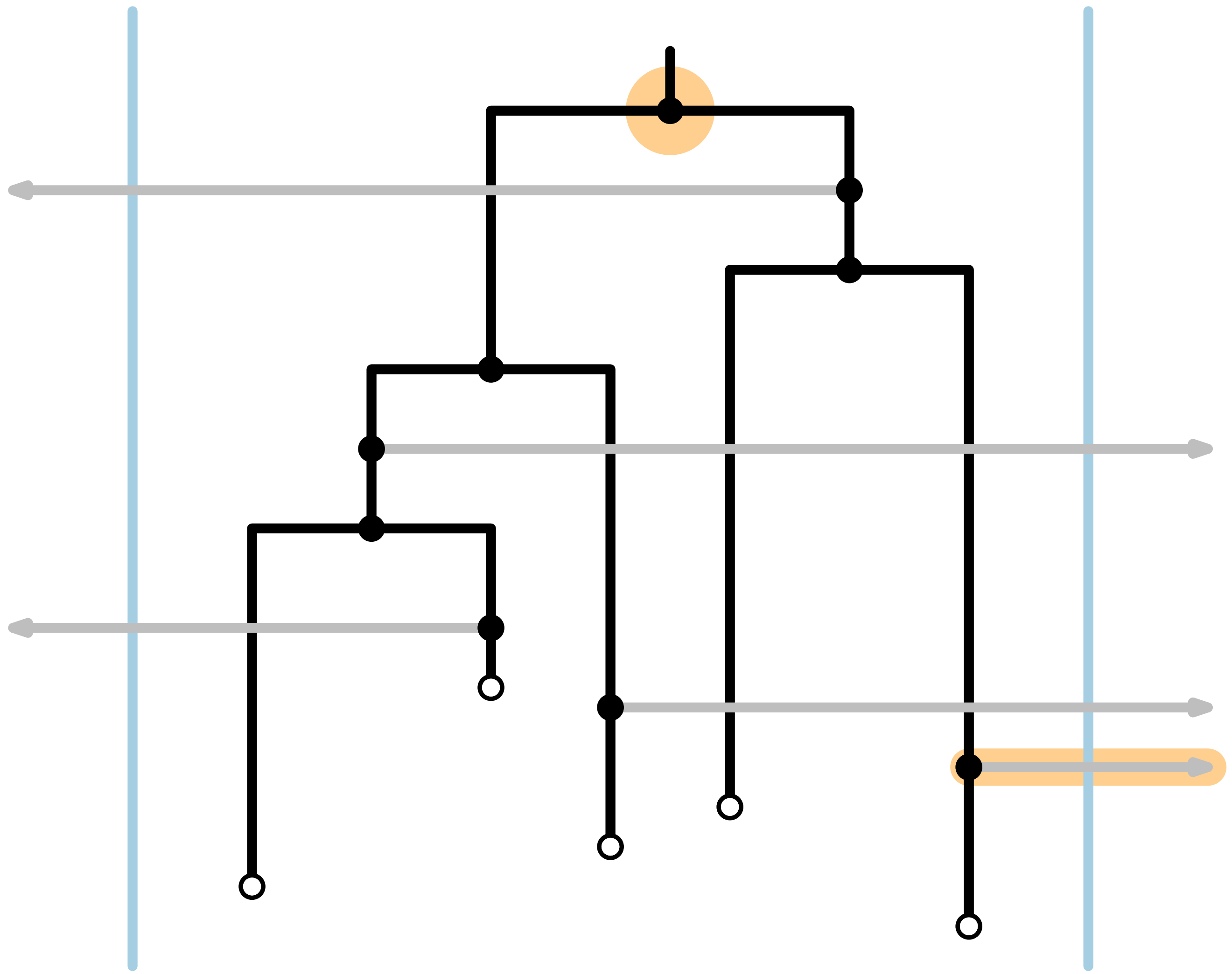

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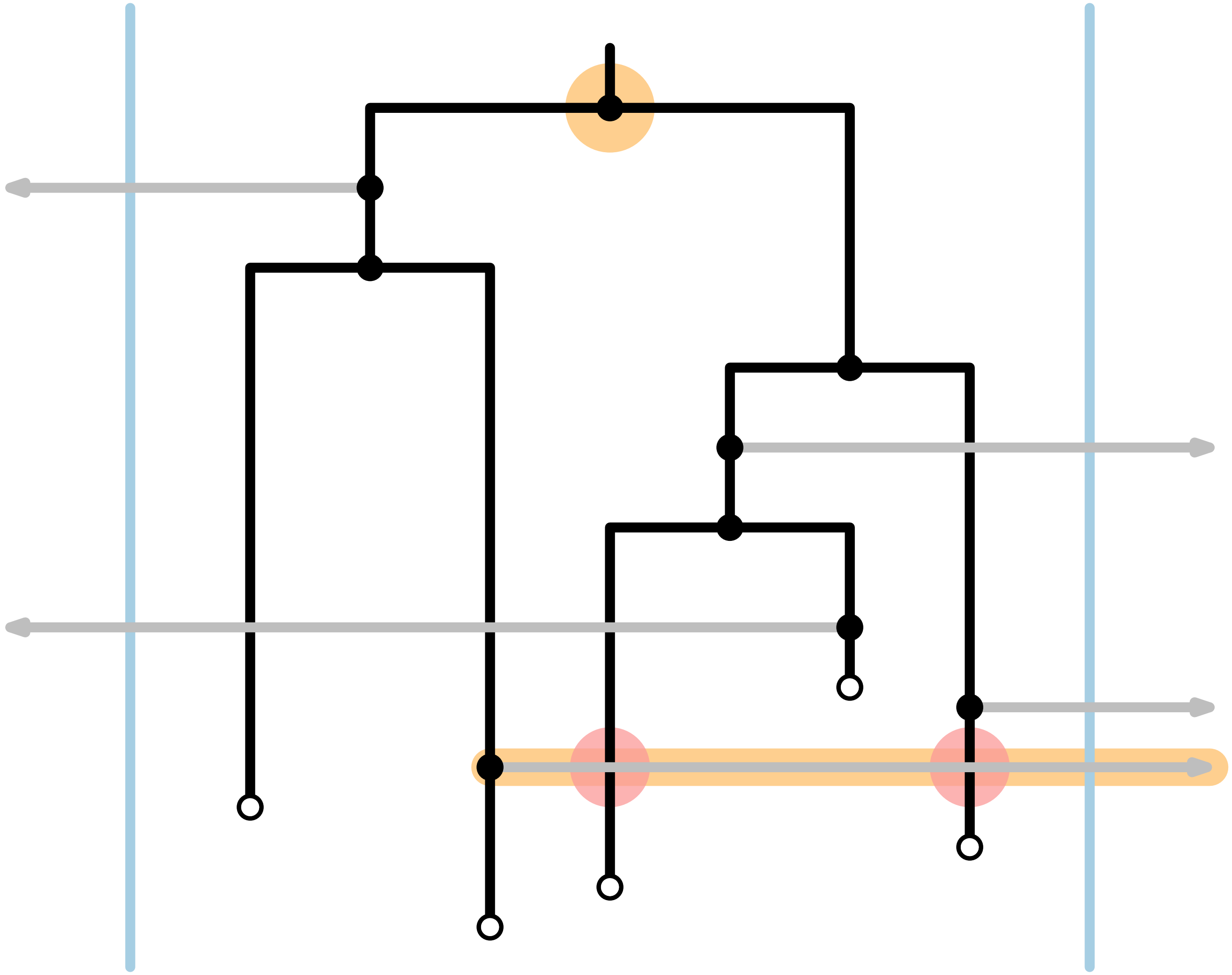

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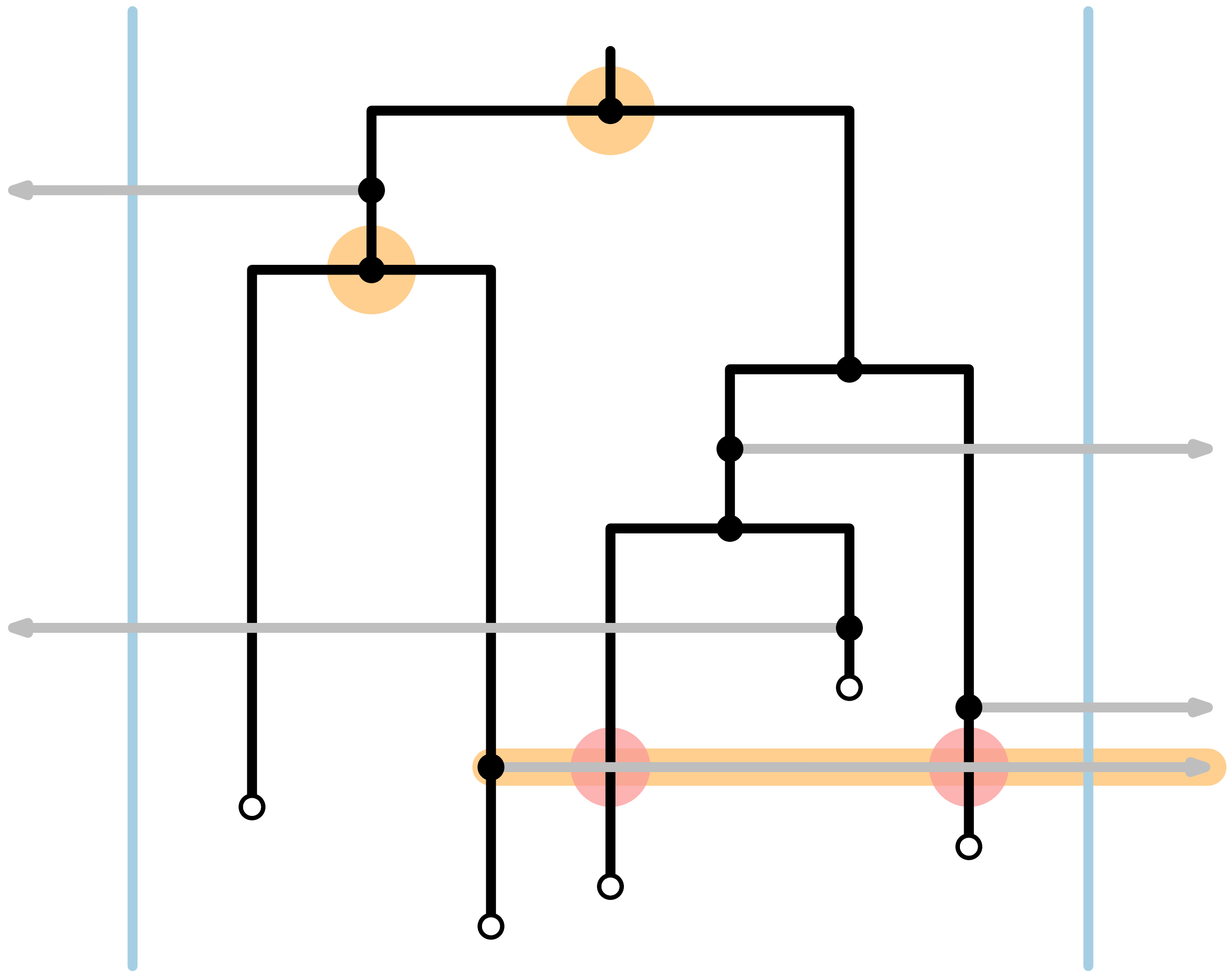
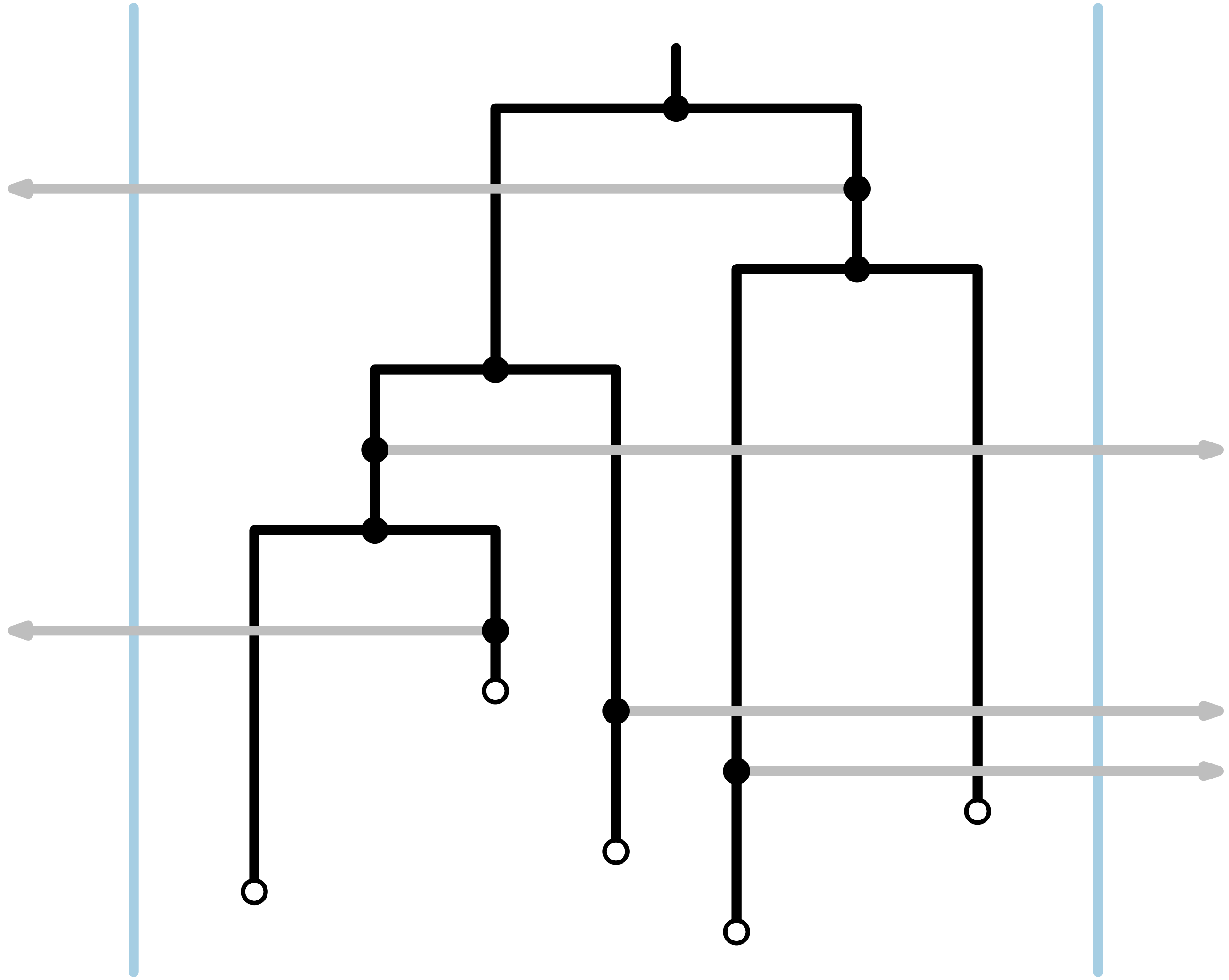


- only rotations of ancestors matter

Subtree Embedding Algorithm



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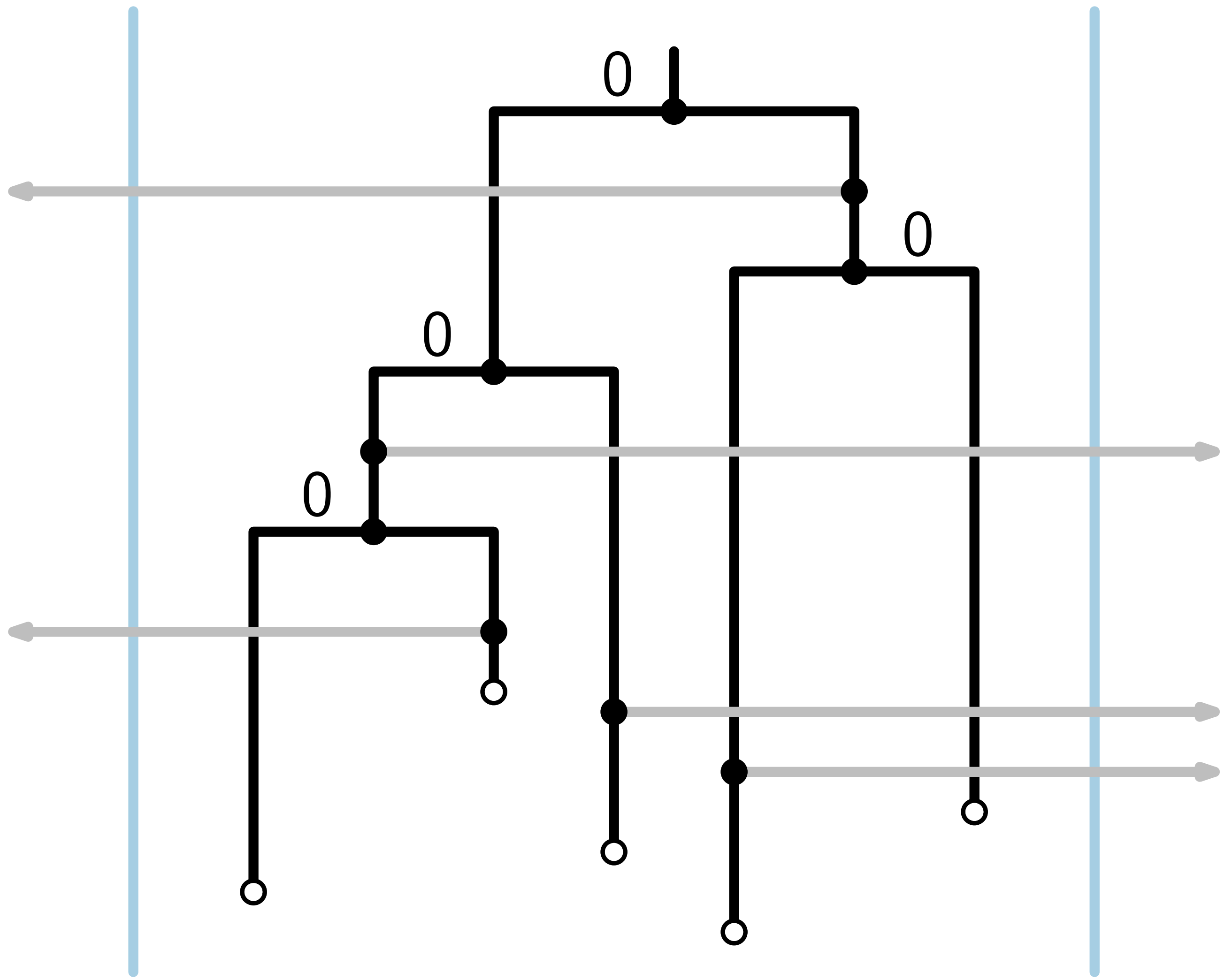

- only rotations of ancestors matter
- store crossing counts

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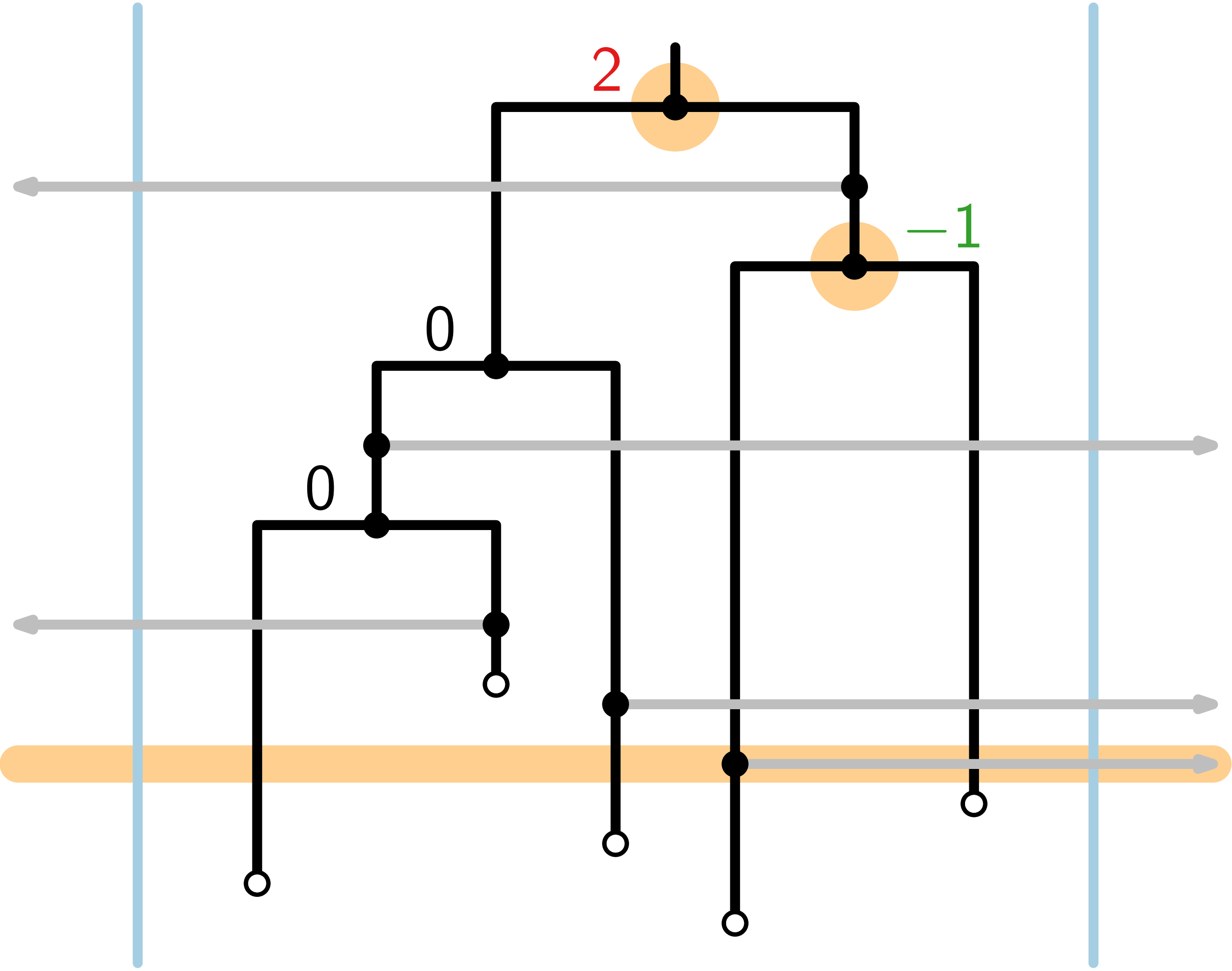

- only rotations of ancestors matter
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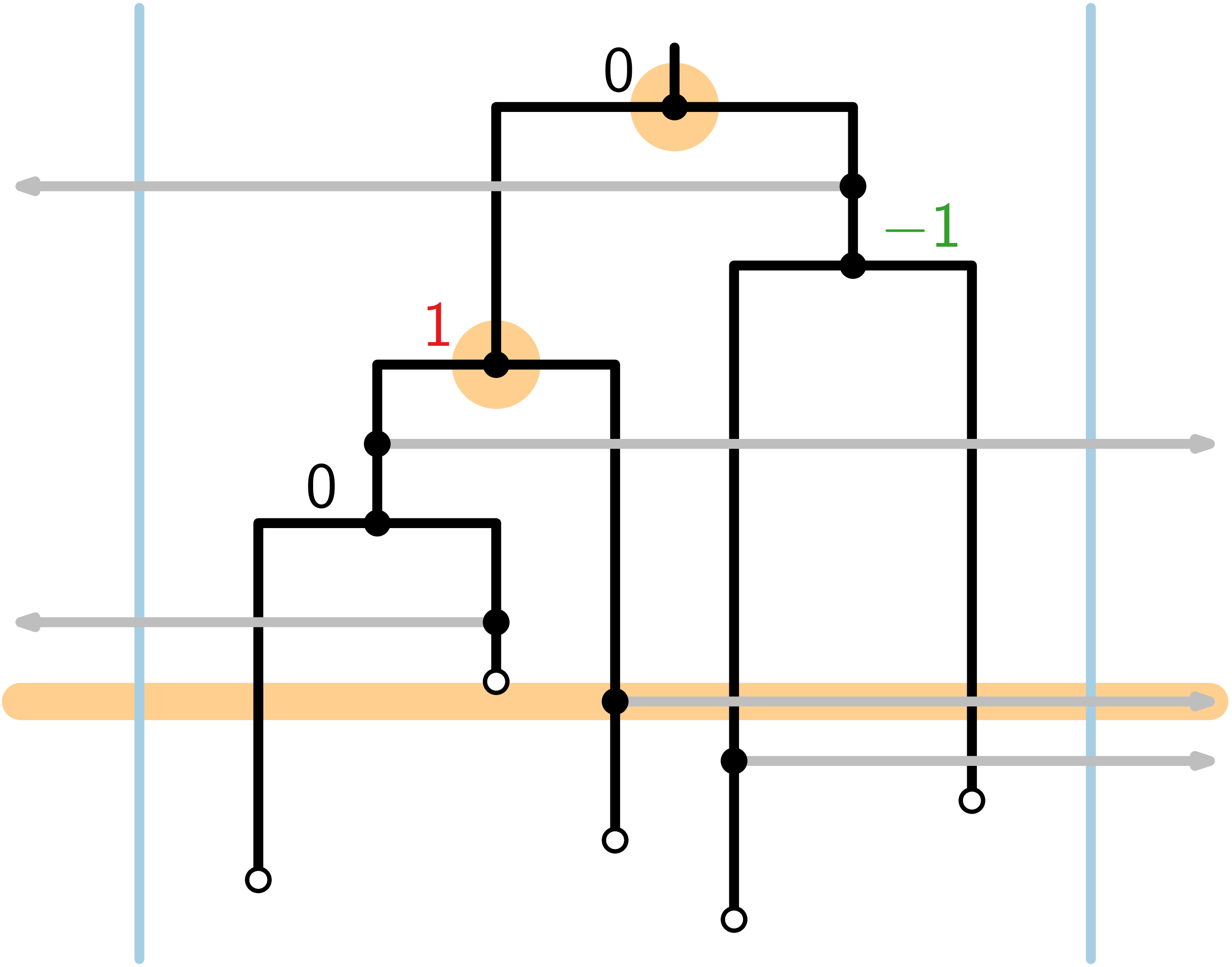

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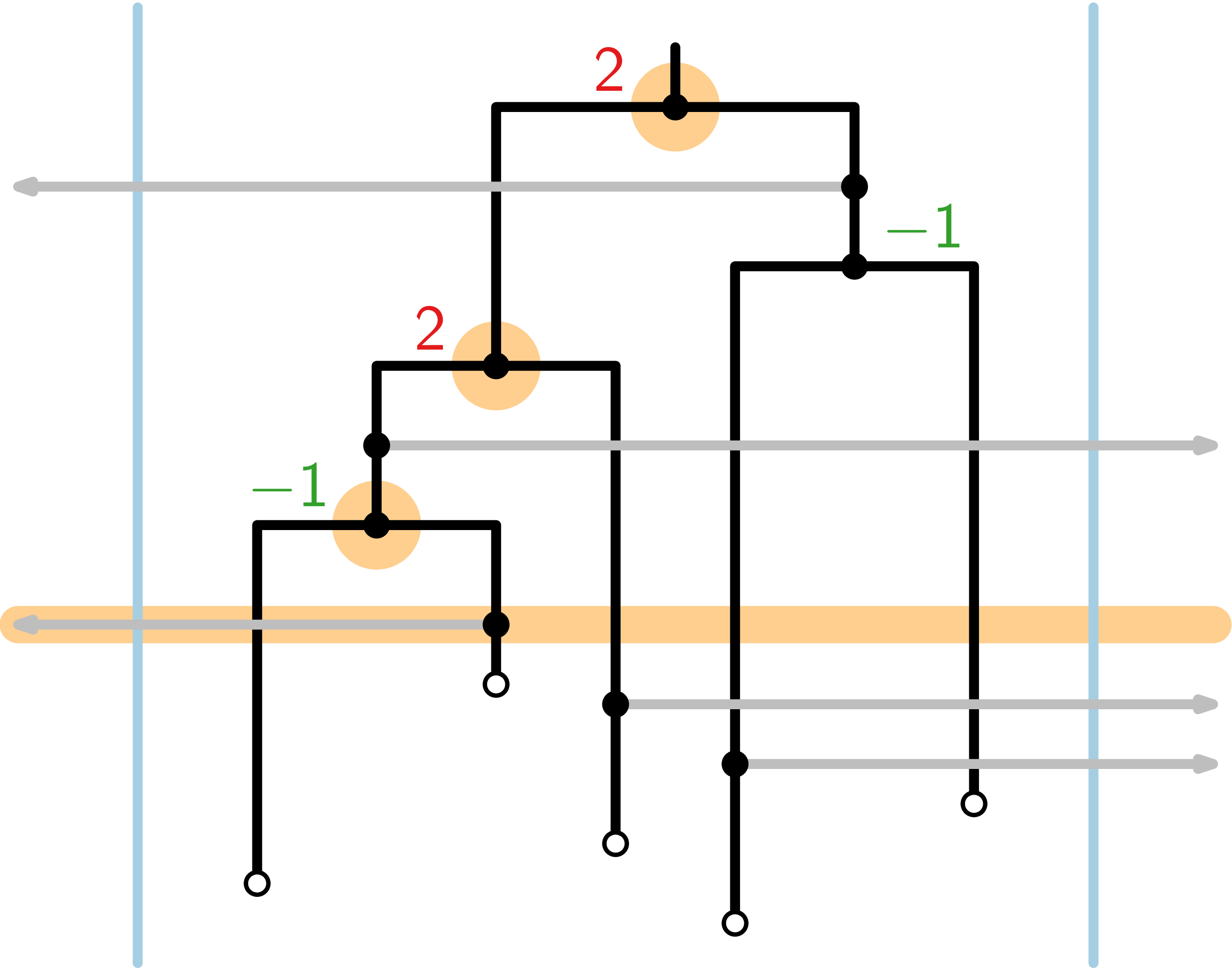

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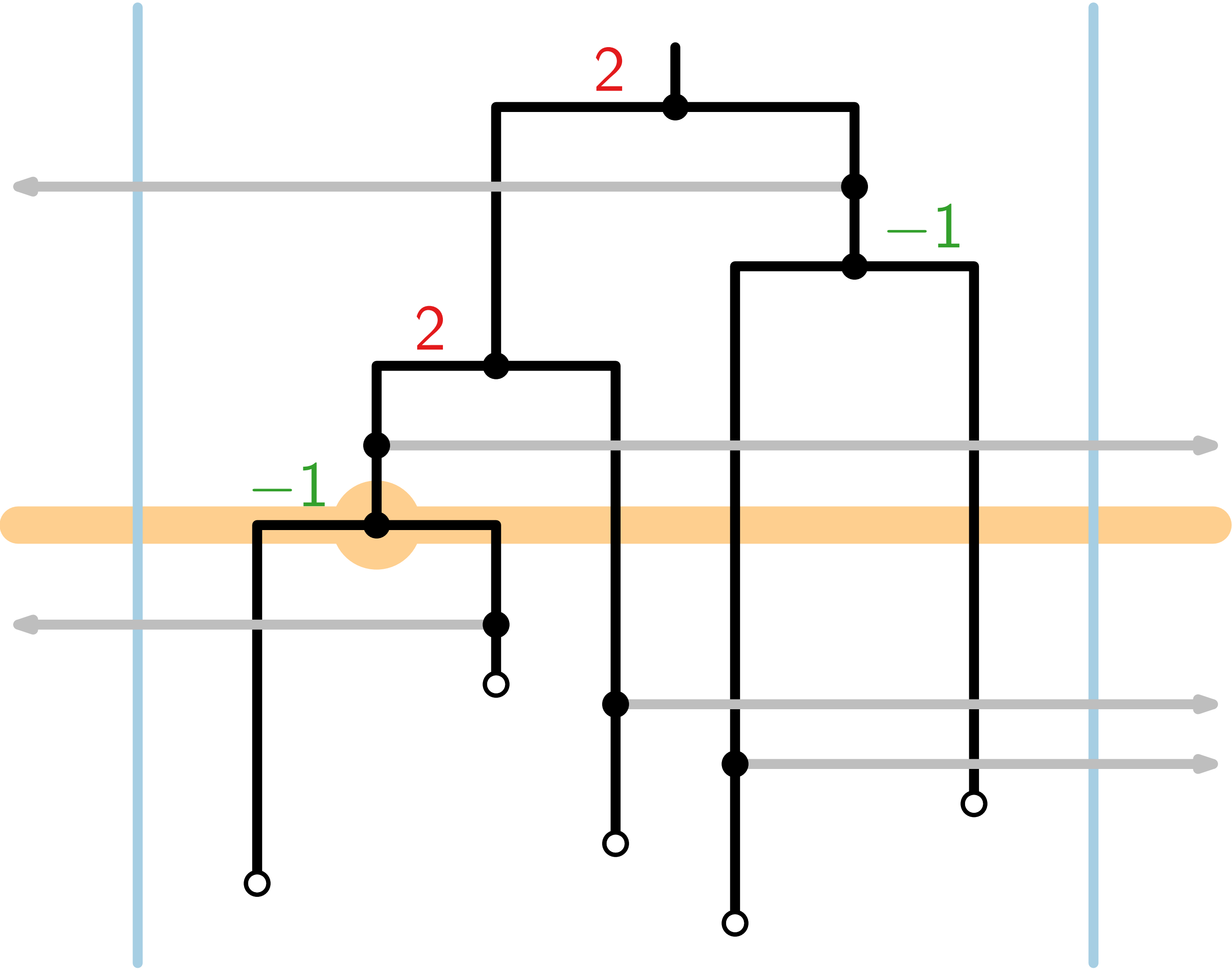

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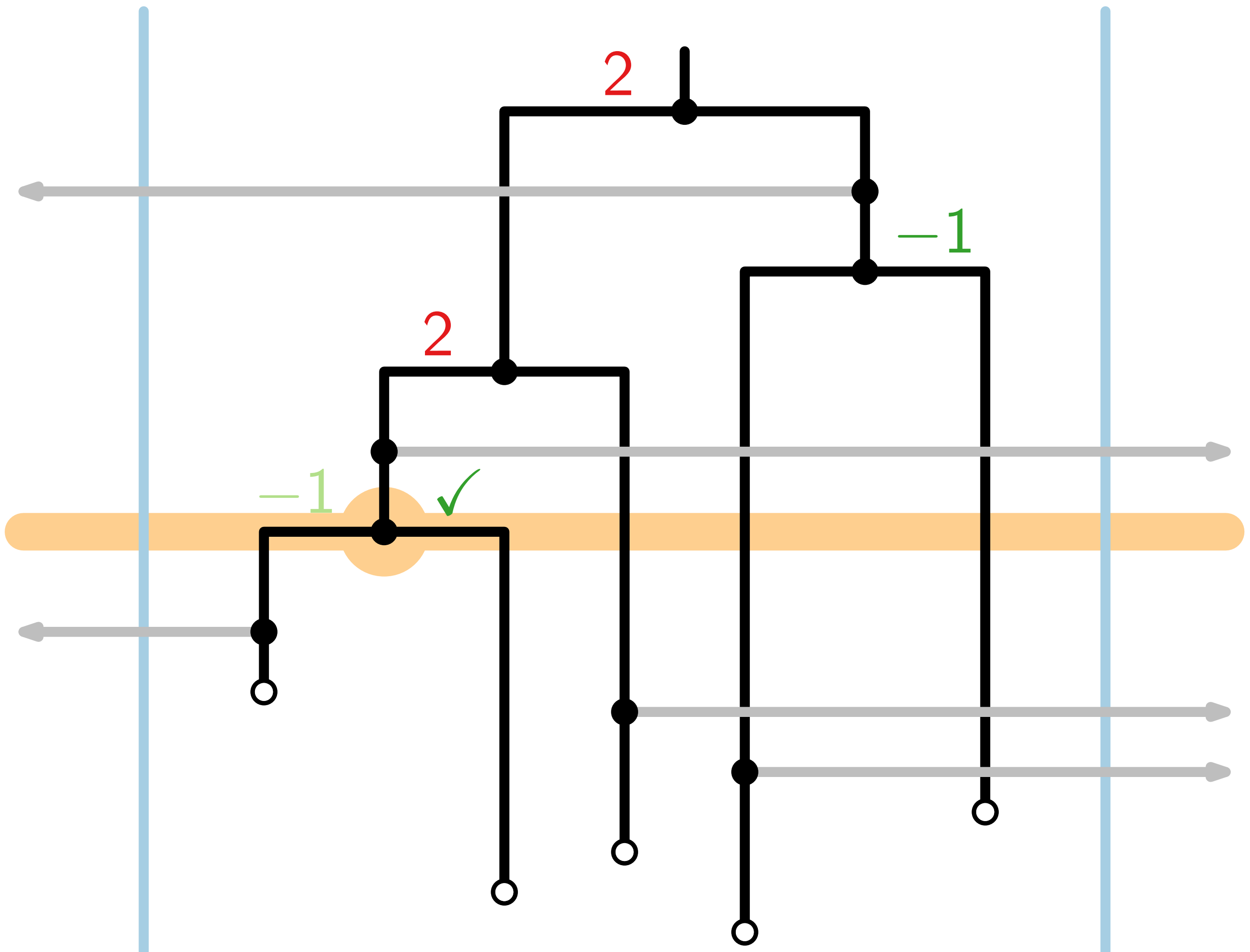

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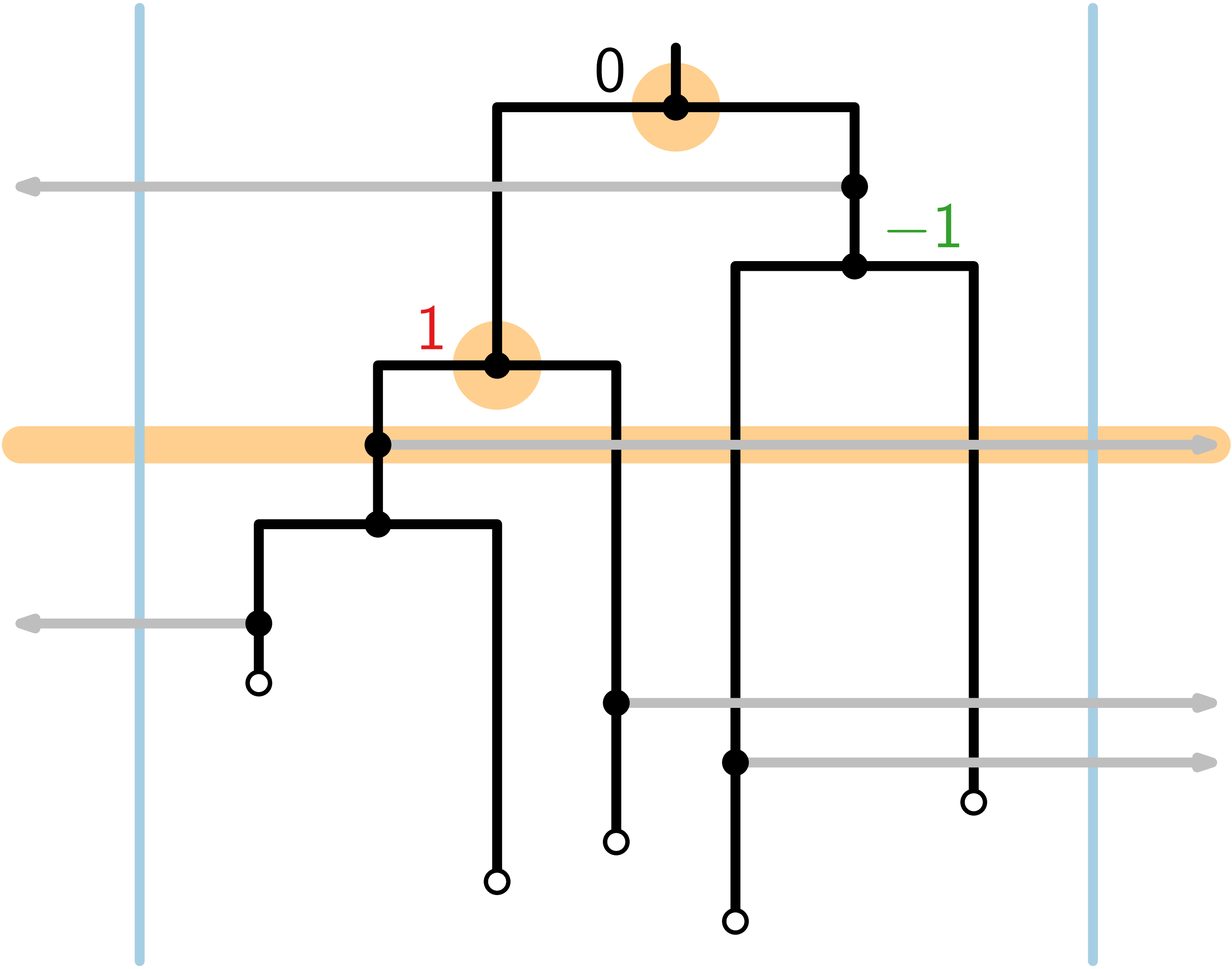

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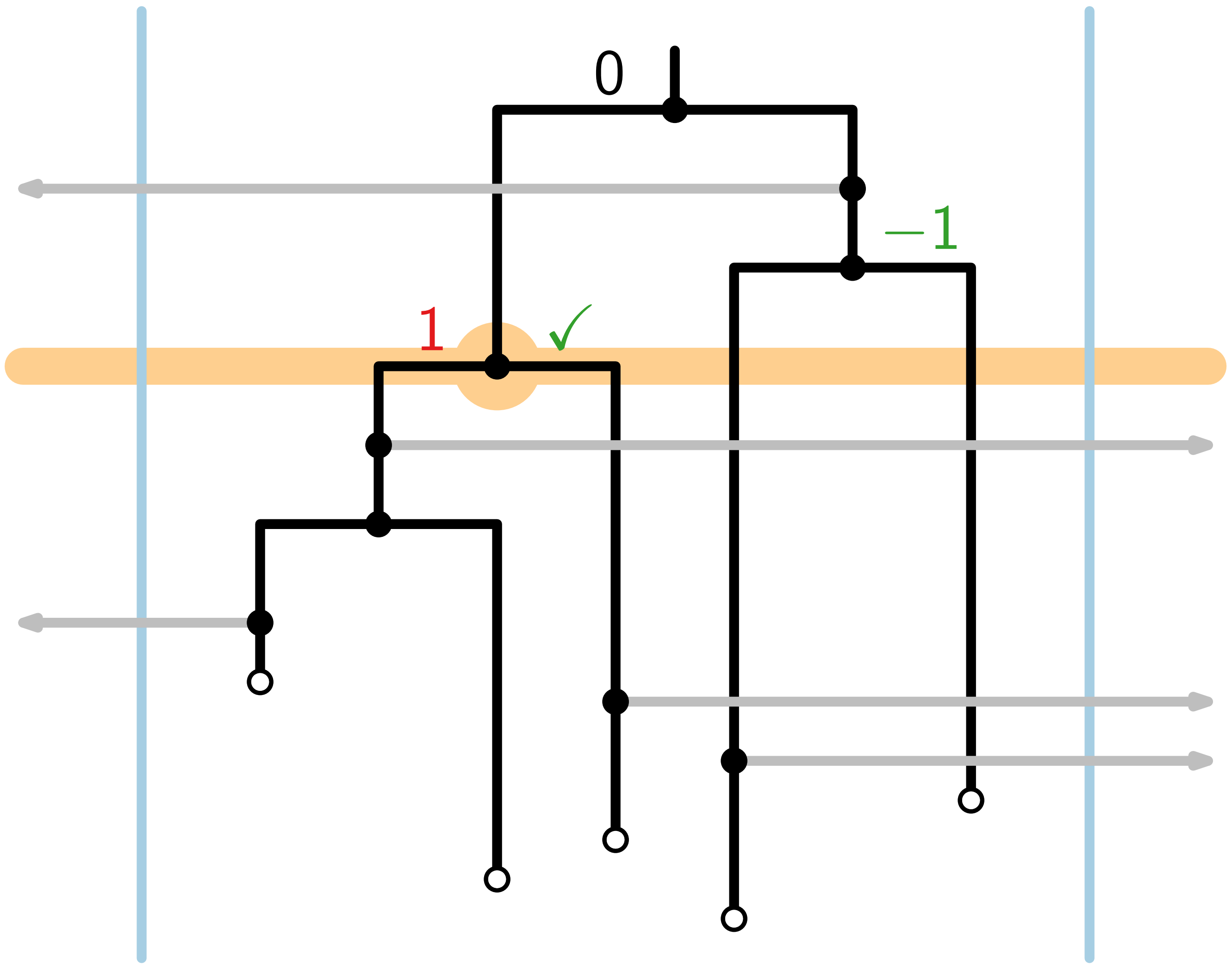

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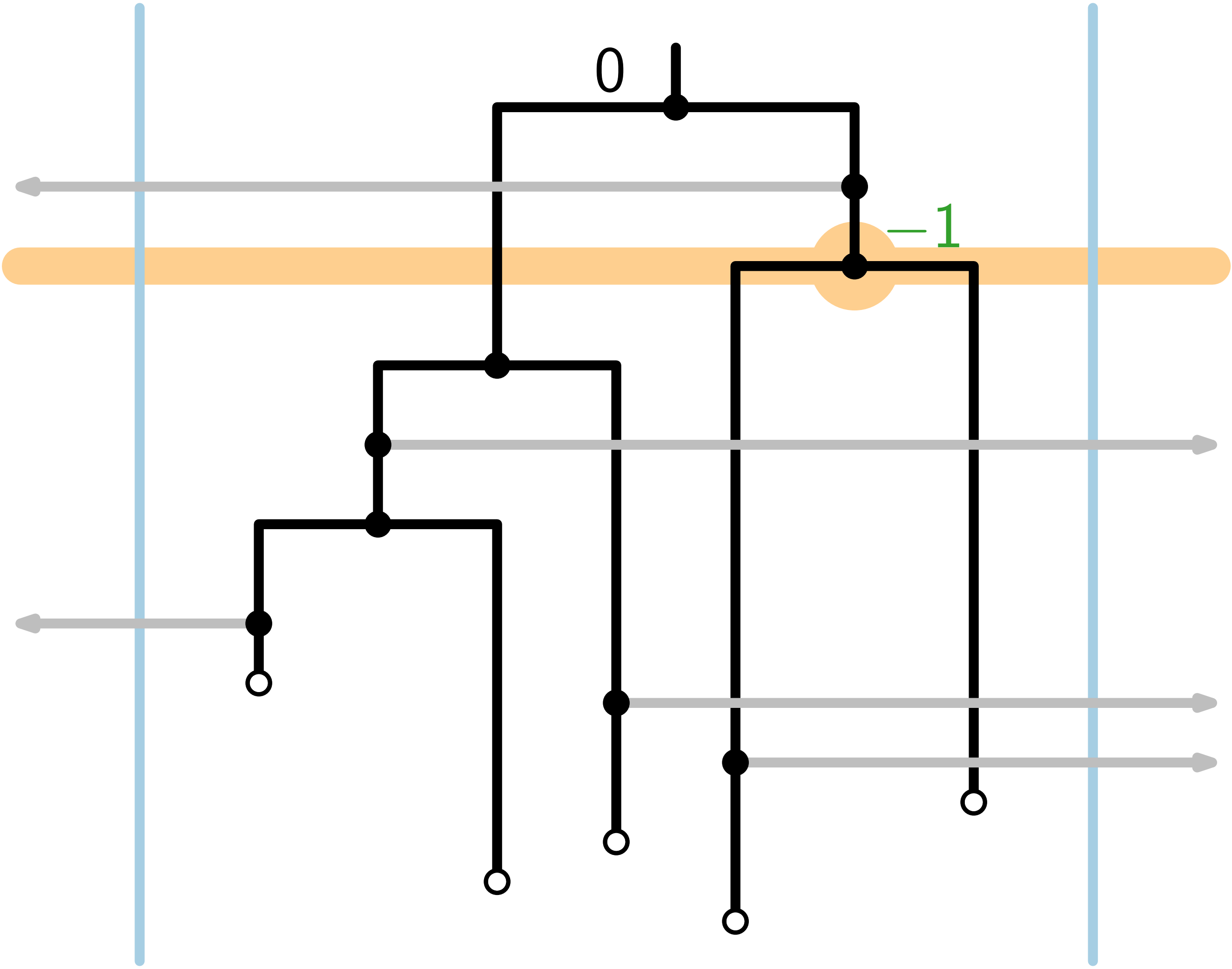

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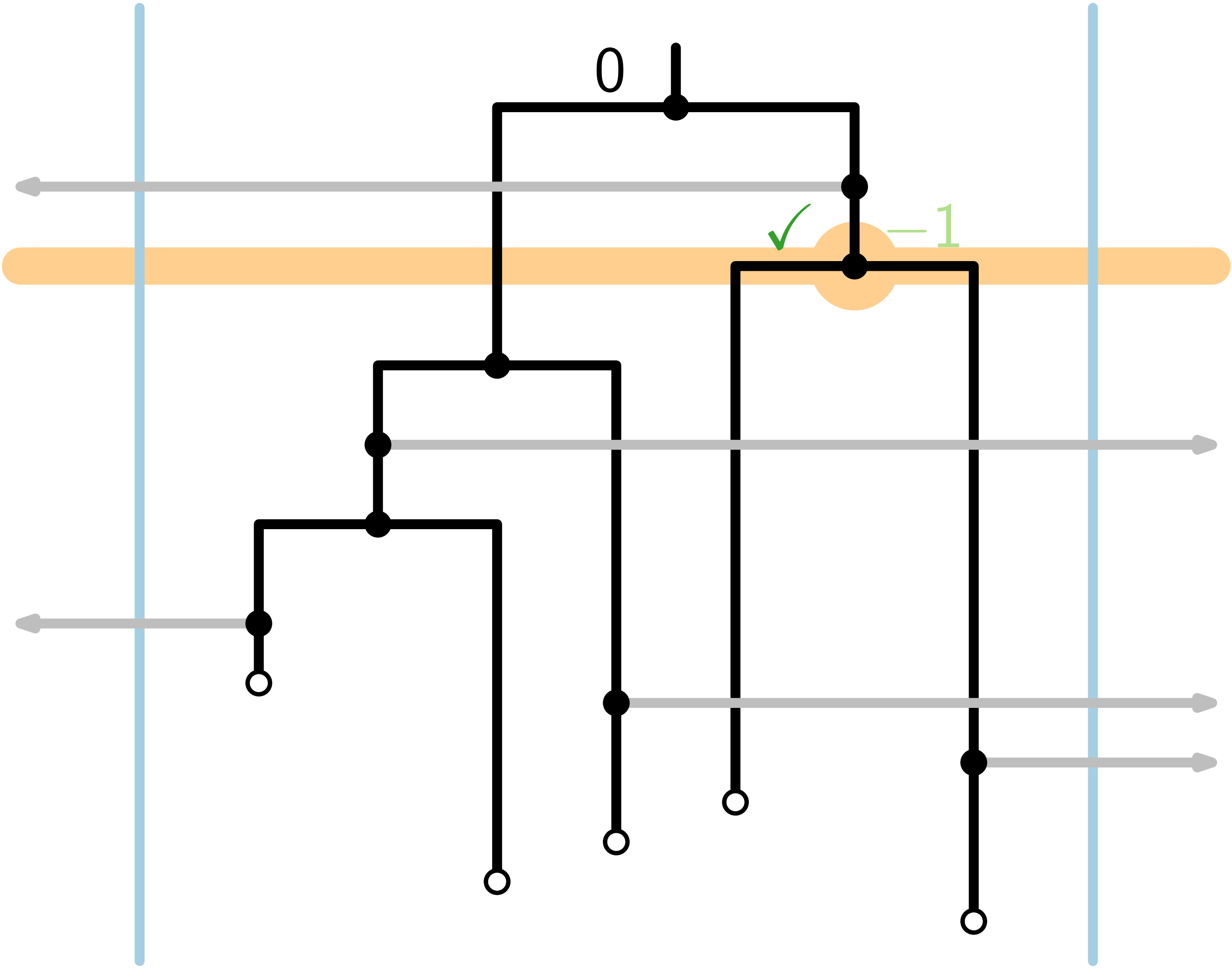

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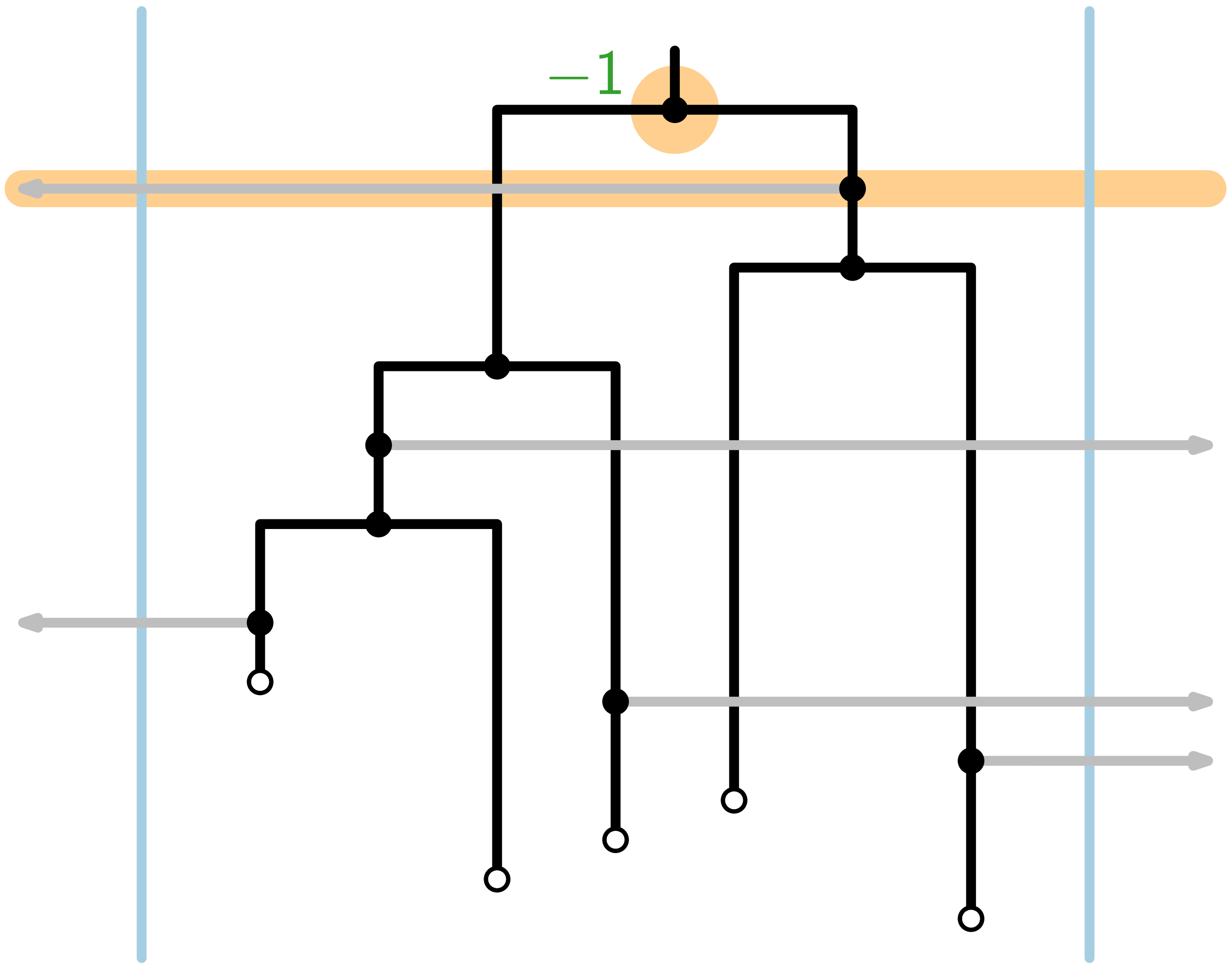

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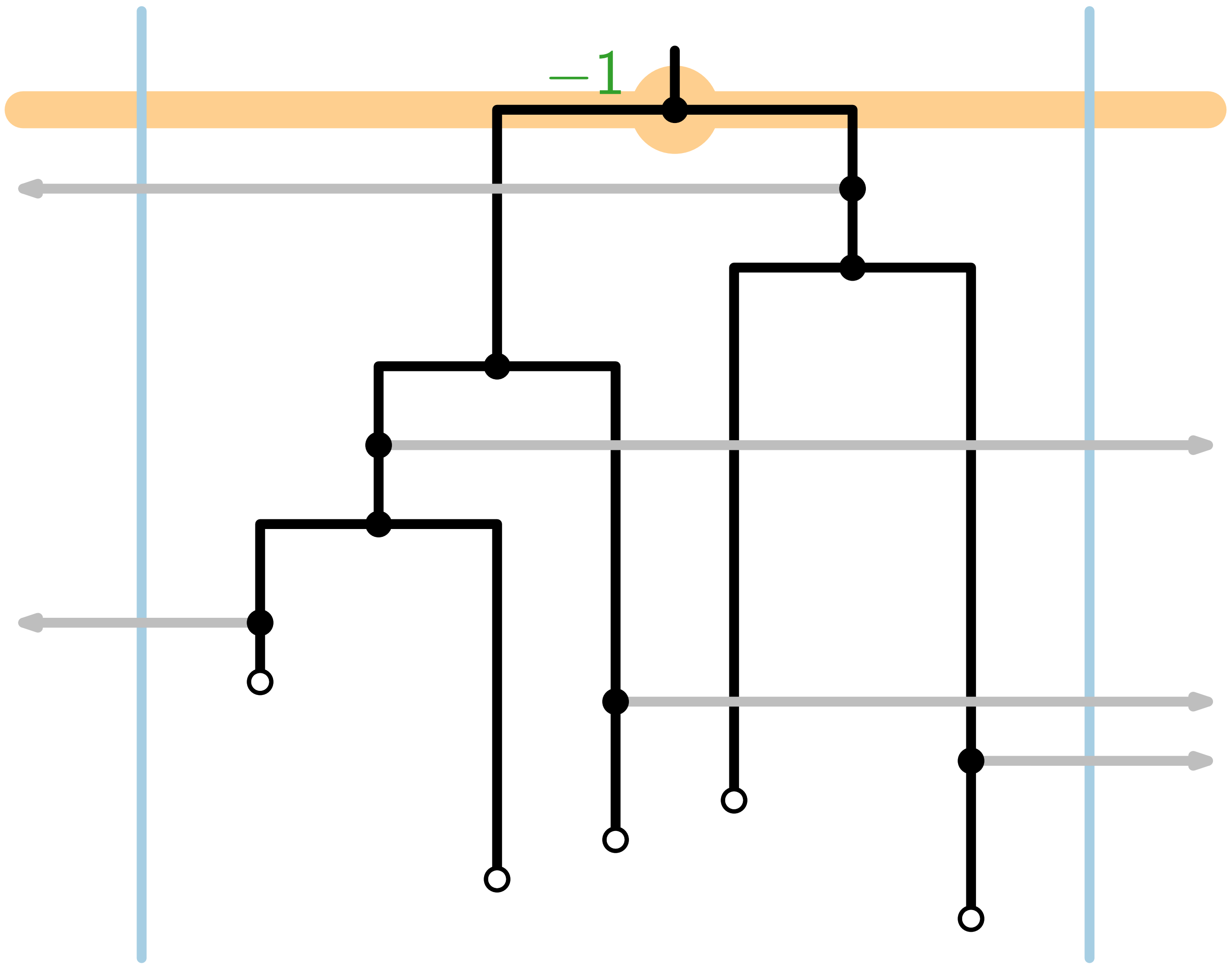

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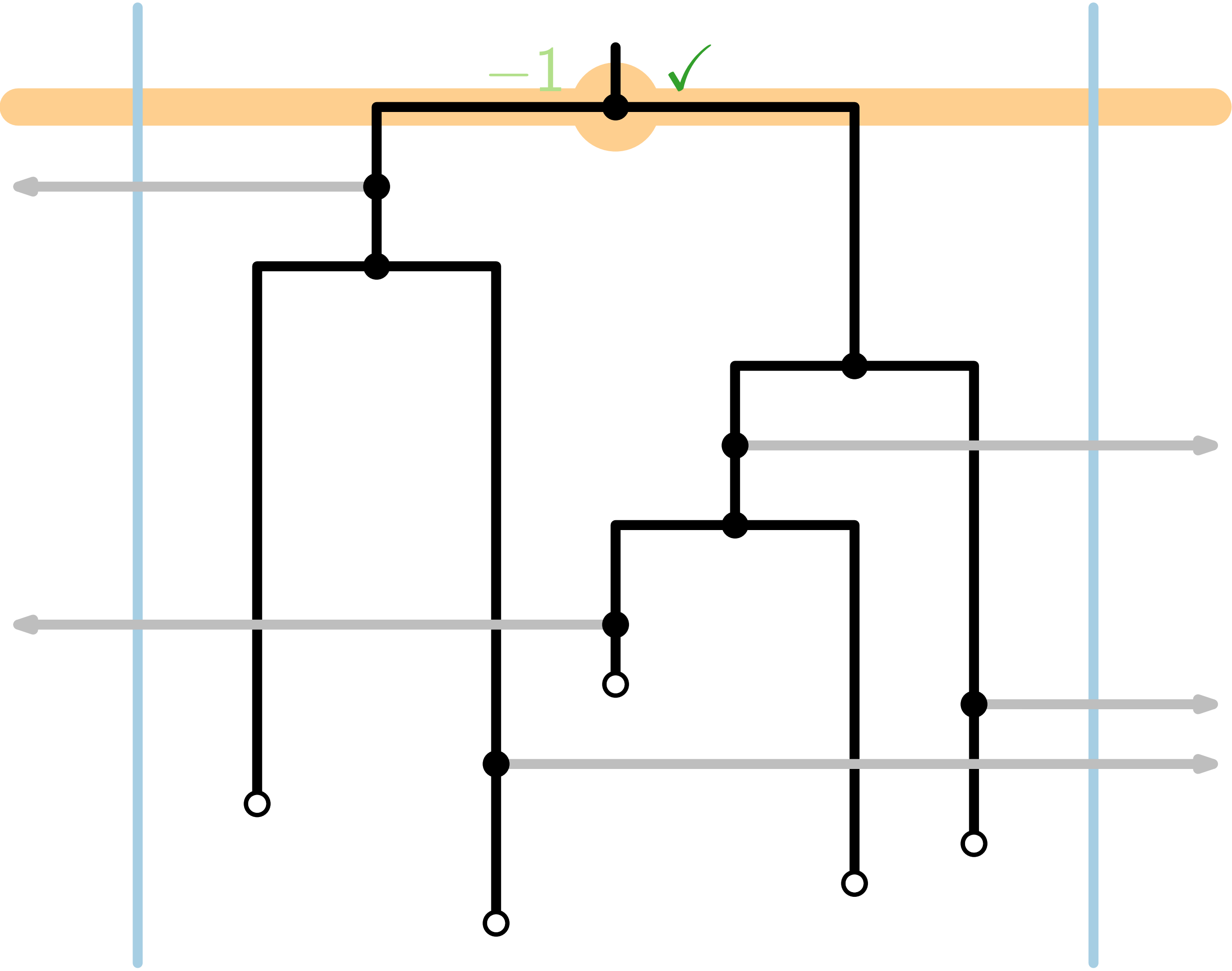
Subtree Embedding Algorithm



- only rotations of ancestors matter
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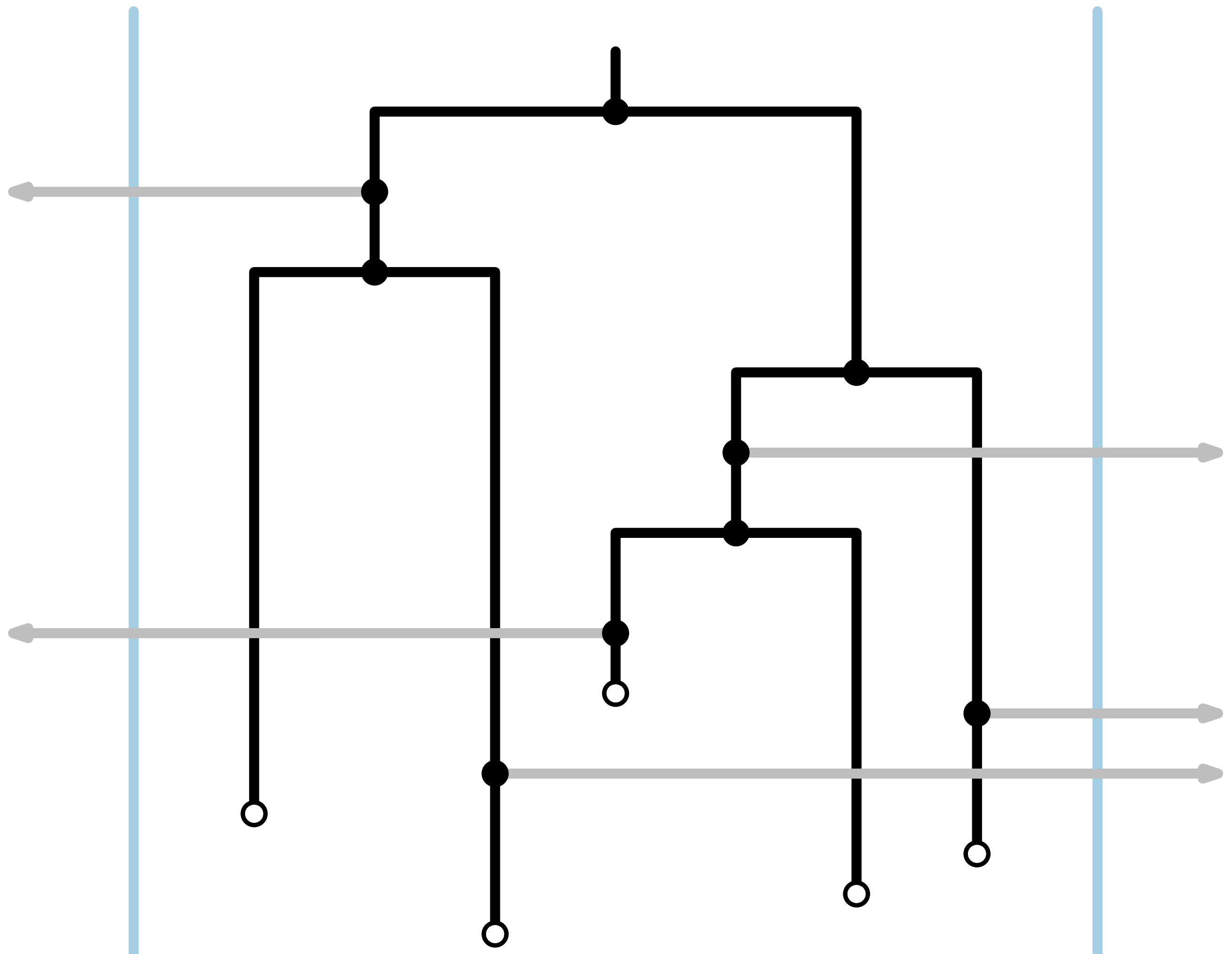
Subtree Embedding Algorithm



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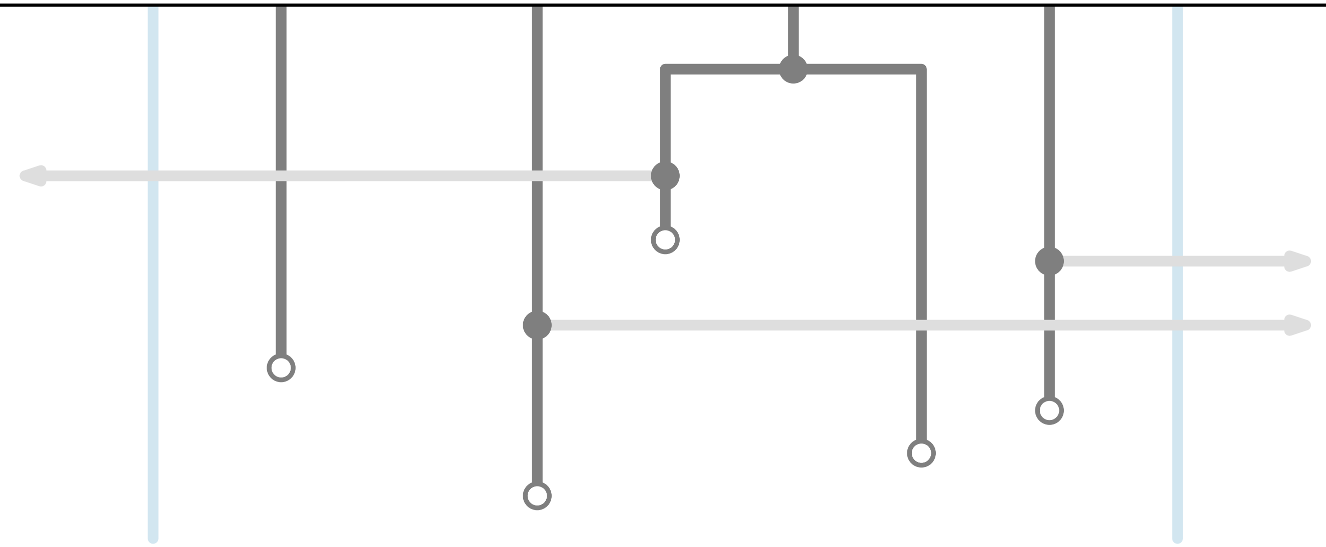
Subtree Embedding Algorithm

Lemma.
Can find optimal (binary) subtree embedding in $\mathcal{O}(n^2)$ time;



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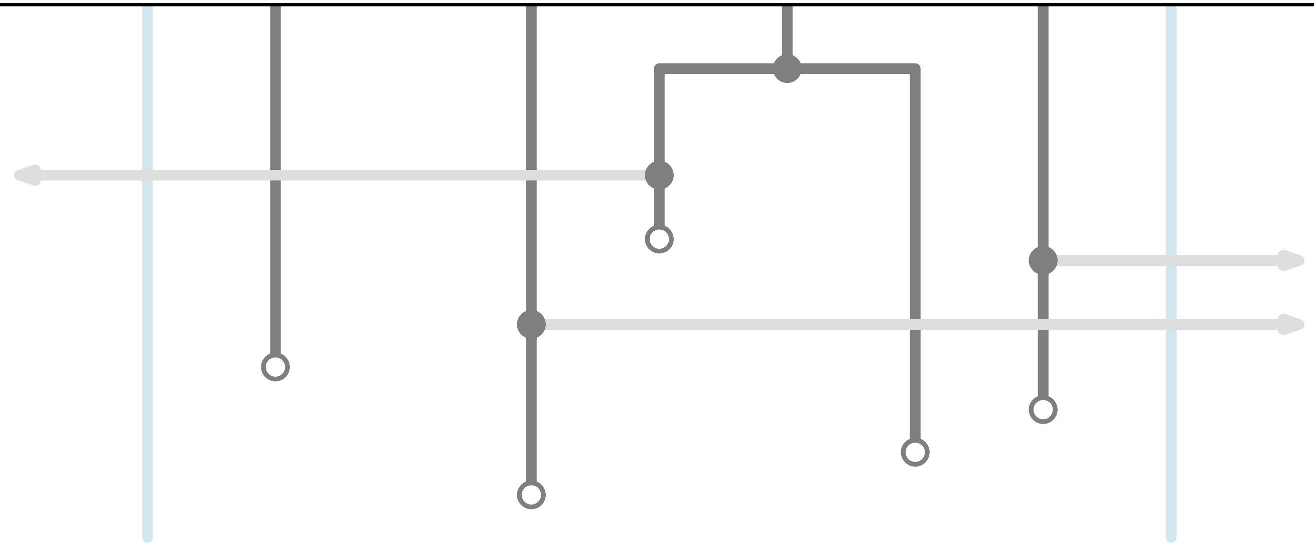
Subtree Embedding Algorithm

Lemma.
Can find optimal (binary) subtree embedding in $\mathcal{O}(n^2)$ time;

...or if with max degree Δ, then in $\mathcal{O}(\Delta!\Delta n^2)$ time.



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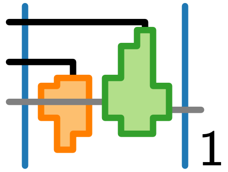
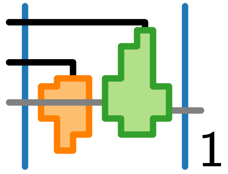
Algorithm for 

- use **subtree embedding** algo for each column subtree

Algorithm for 

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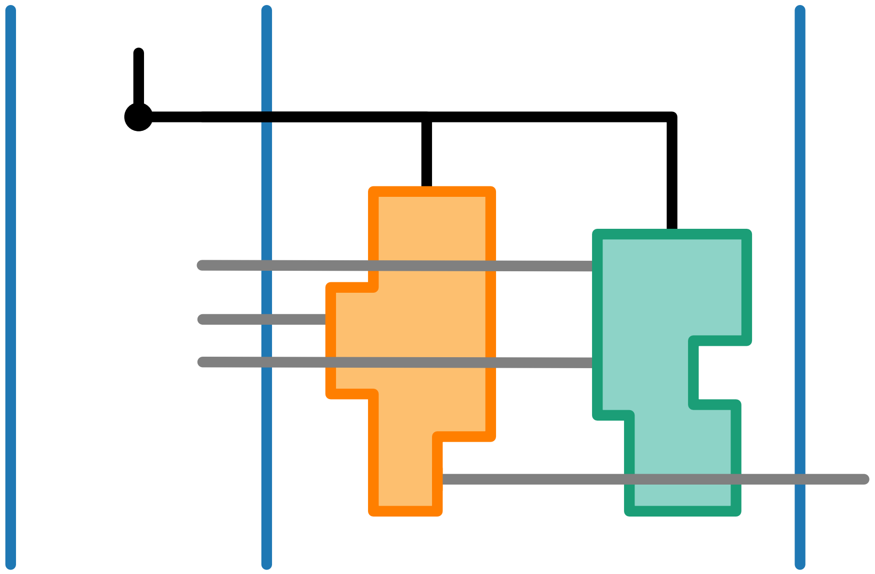
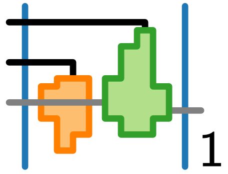
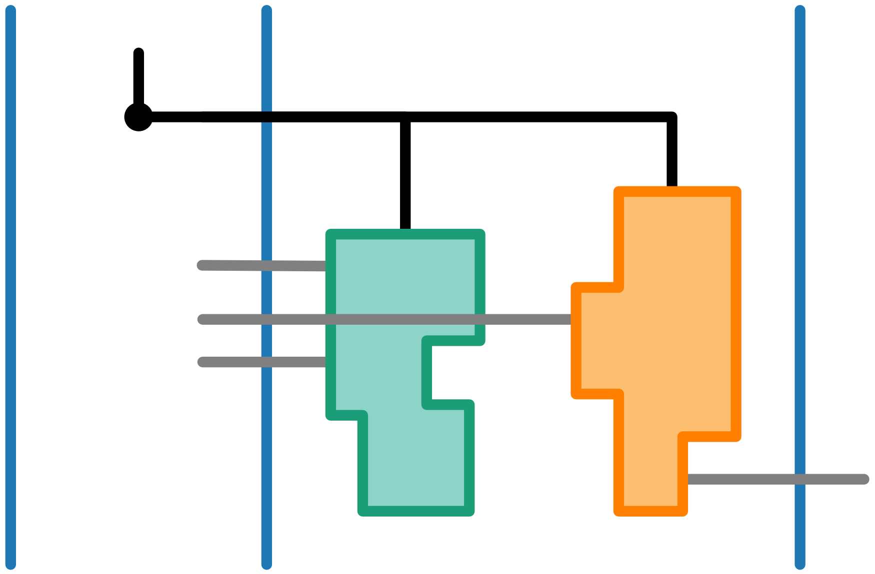
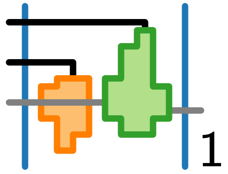
- for **subtree arrangement**,



Algorithm for 

- use **subtree embedding** algo for each column subtree
- for **subtree arrangement**, try both (all) orders of column subtrees with same parent



Algorithm for 

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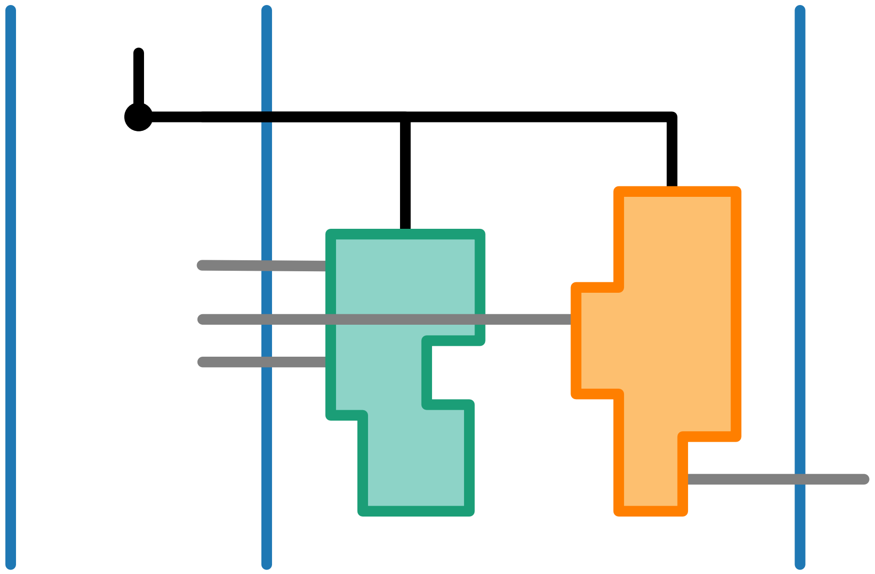
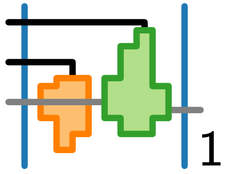
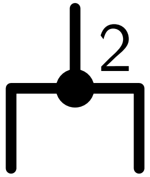
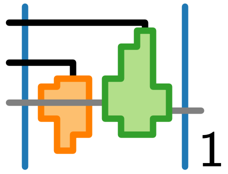


Theorem.

 +  → $\mathcal{O}(n^2)$-time algorithm

Algorithm for 

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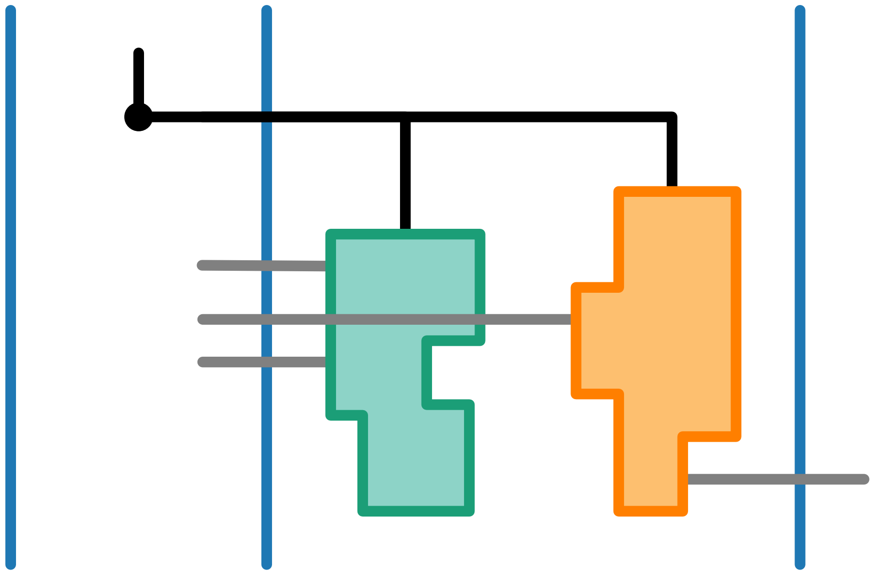

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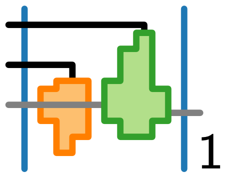
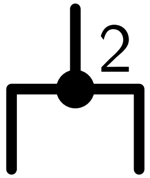
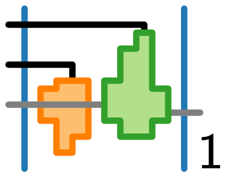
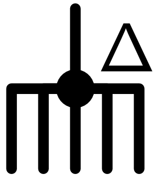
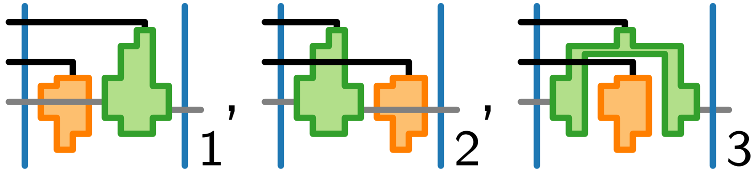
 $+$  → $\mathcal{O}(n^2)$-time algorithm

 $+$  → $\mathcal{O}(\Delta!\Delta n^2)$-time algorithm

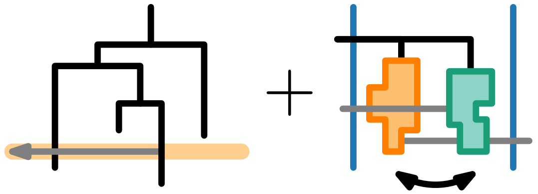
Overview

Drawing Style



Crossing Minimisation

P

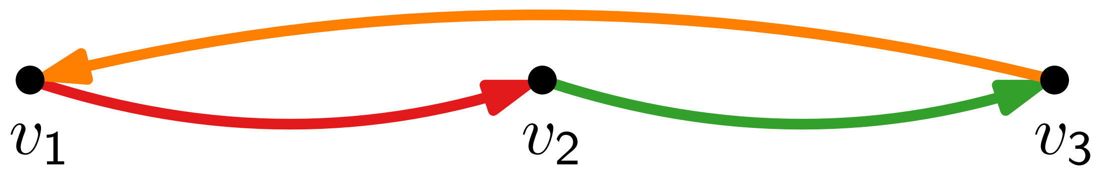


NP

FPT

NP-hardness

Feedback Arc Set
instance



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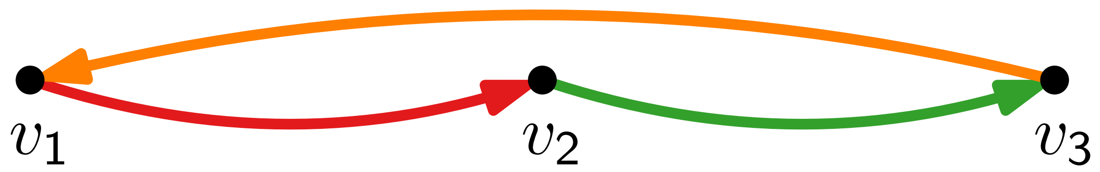

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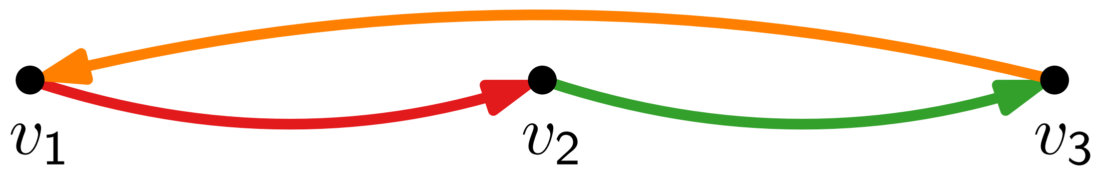

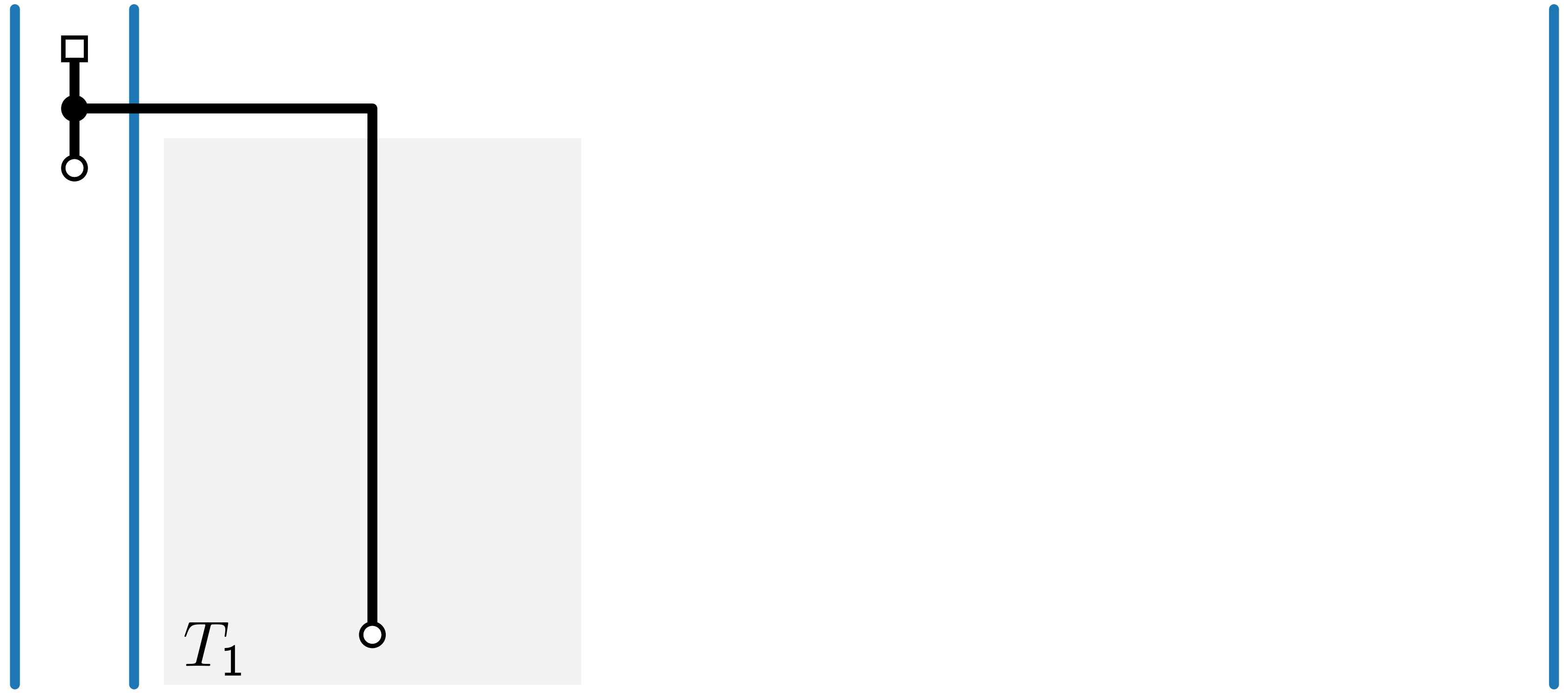

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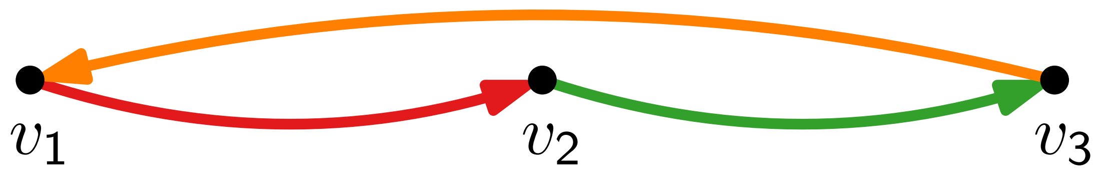

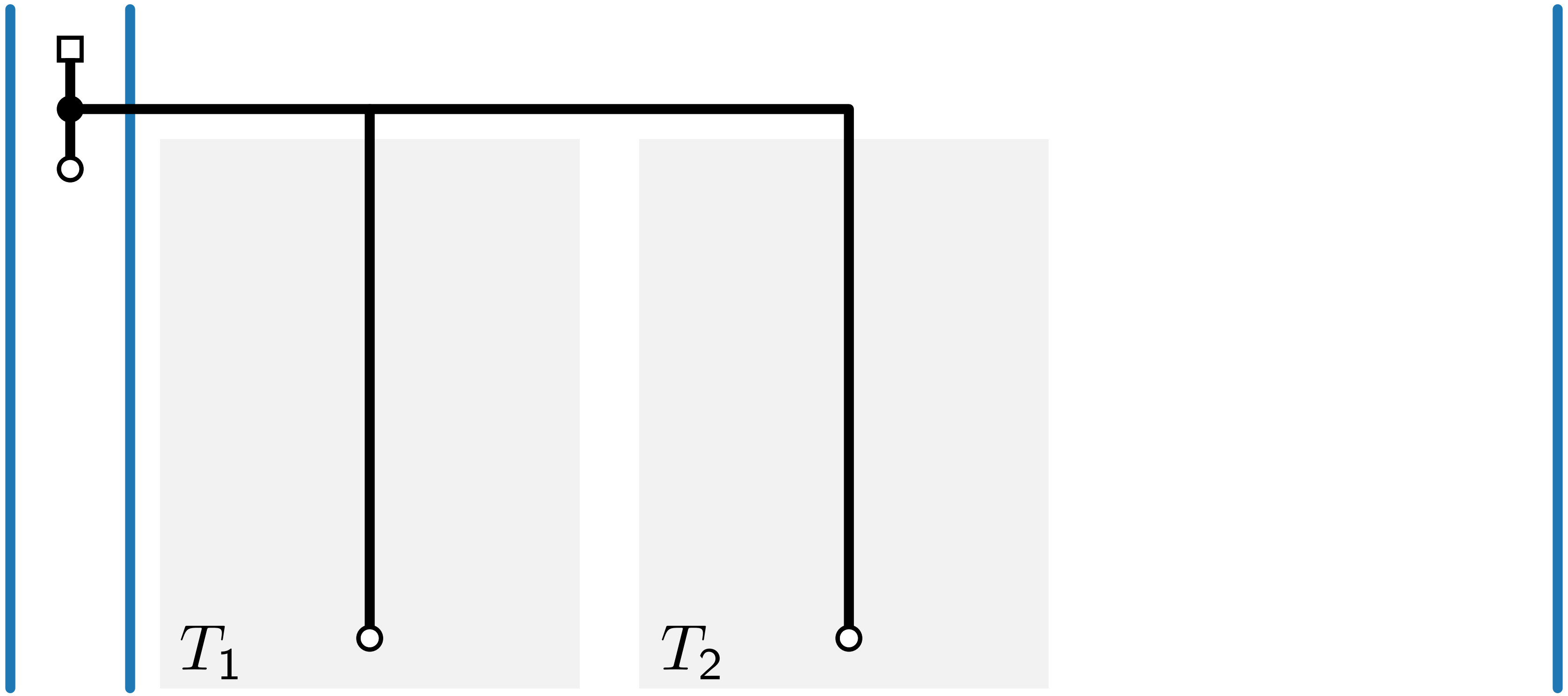

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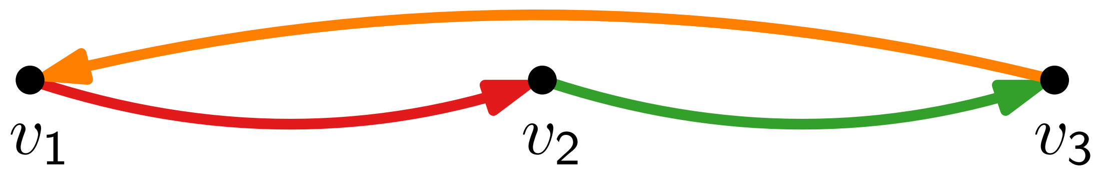

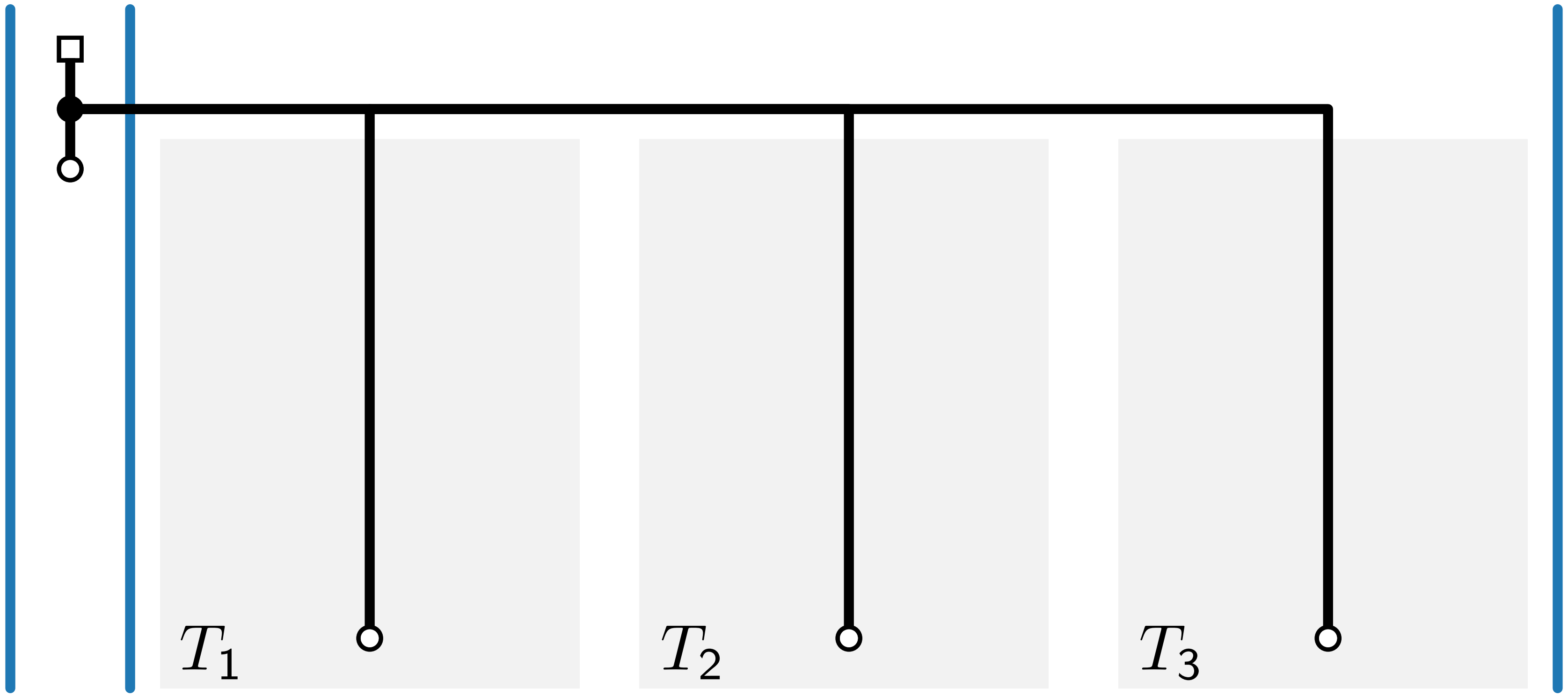

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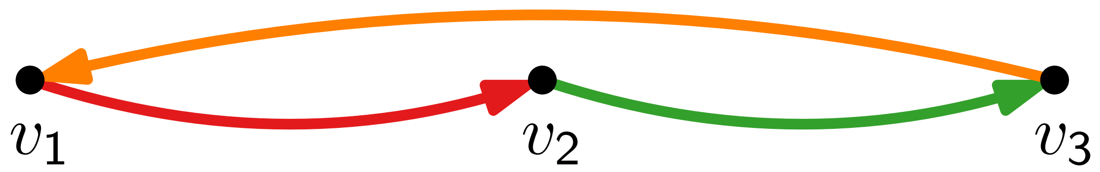

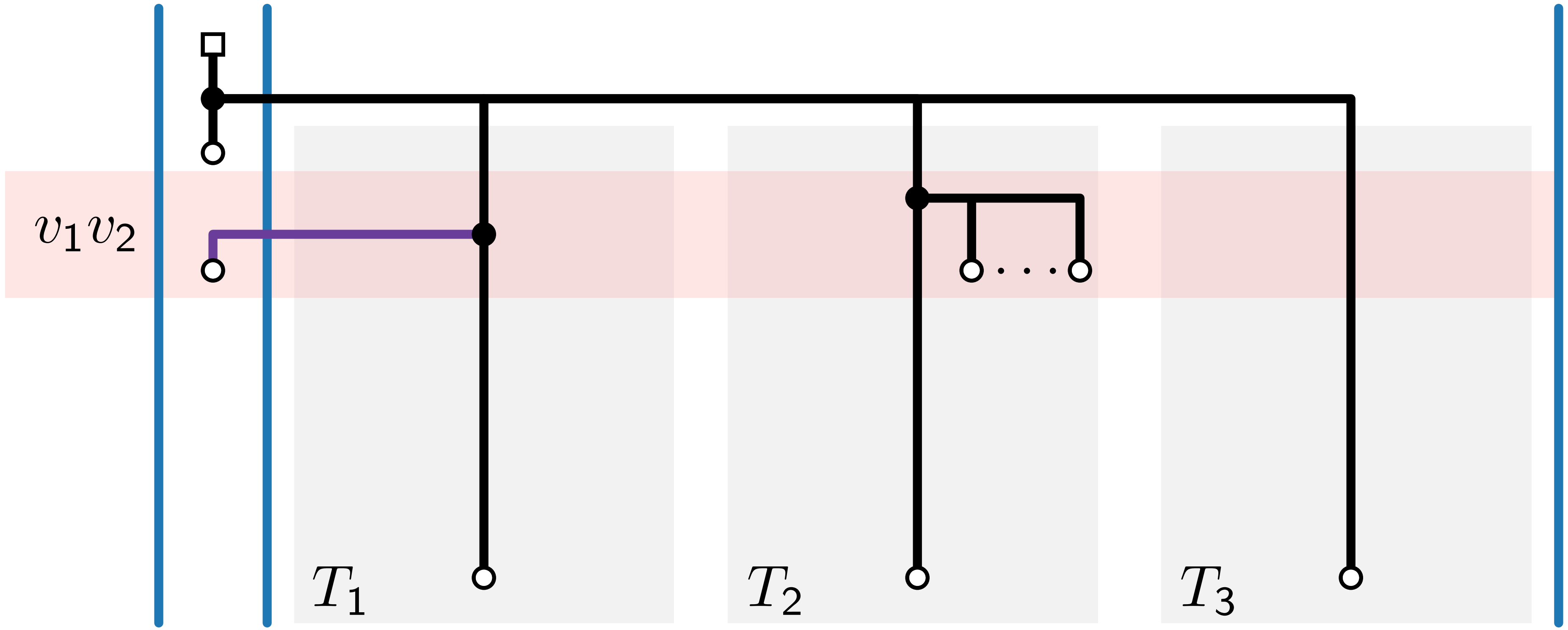

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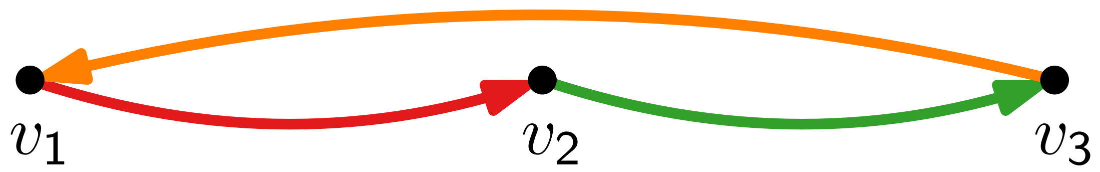

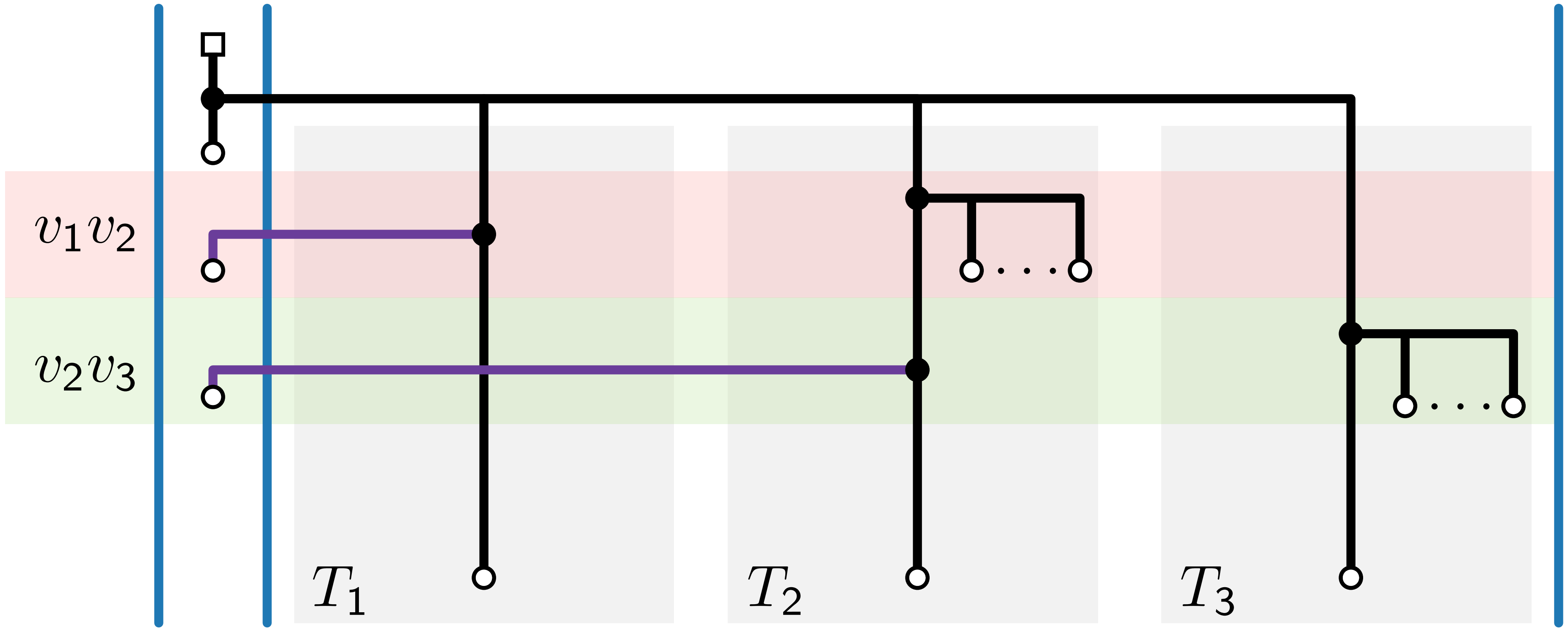

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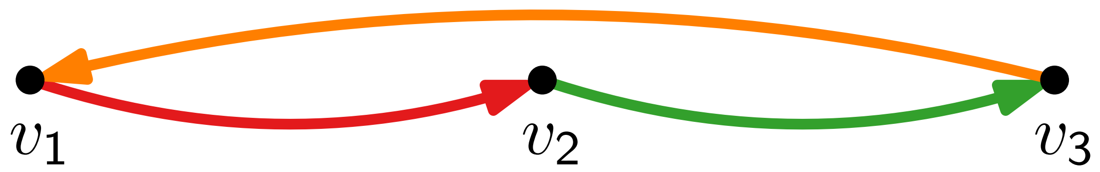

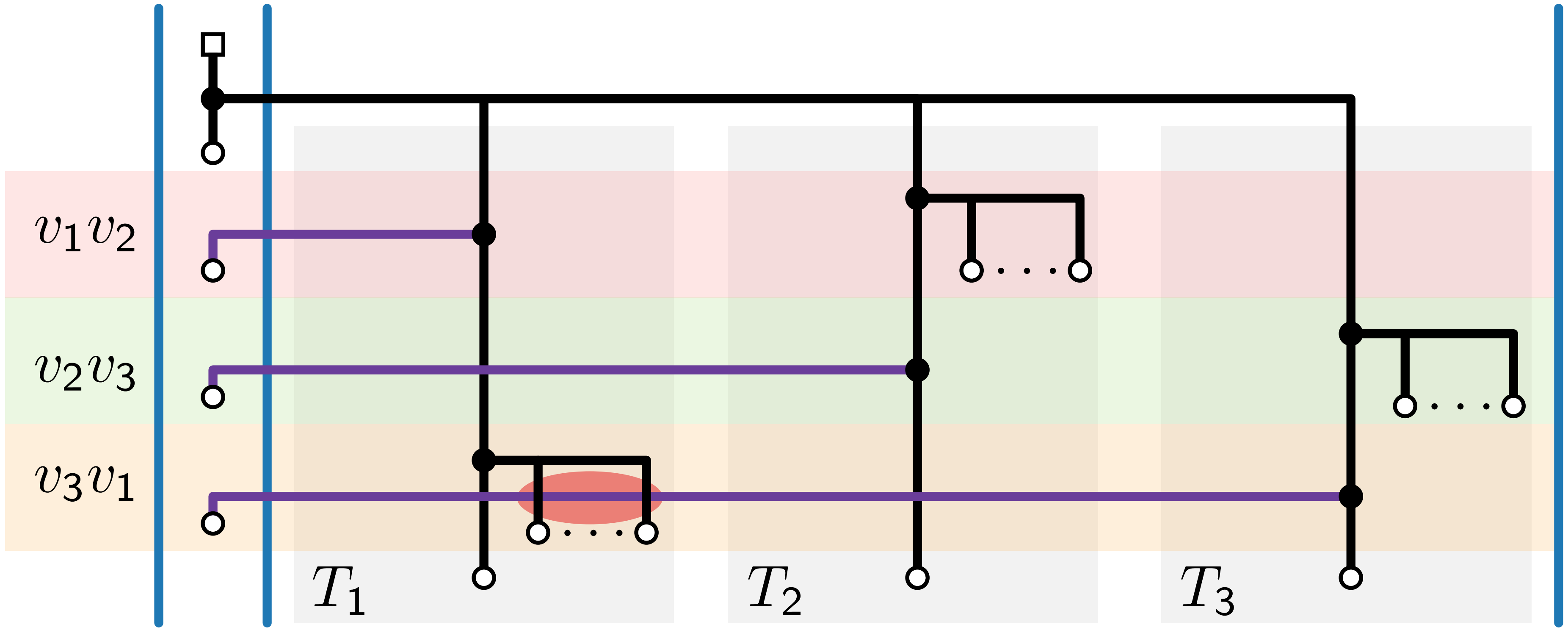

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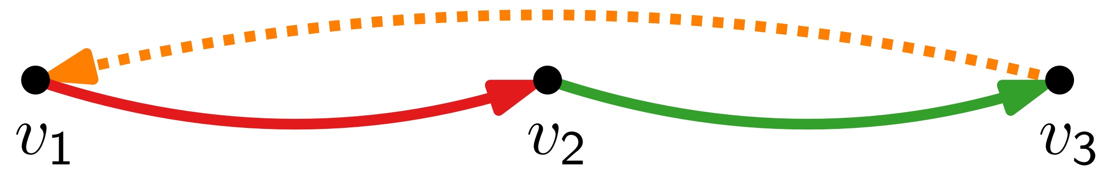

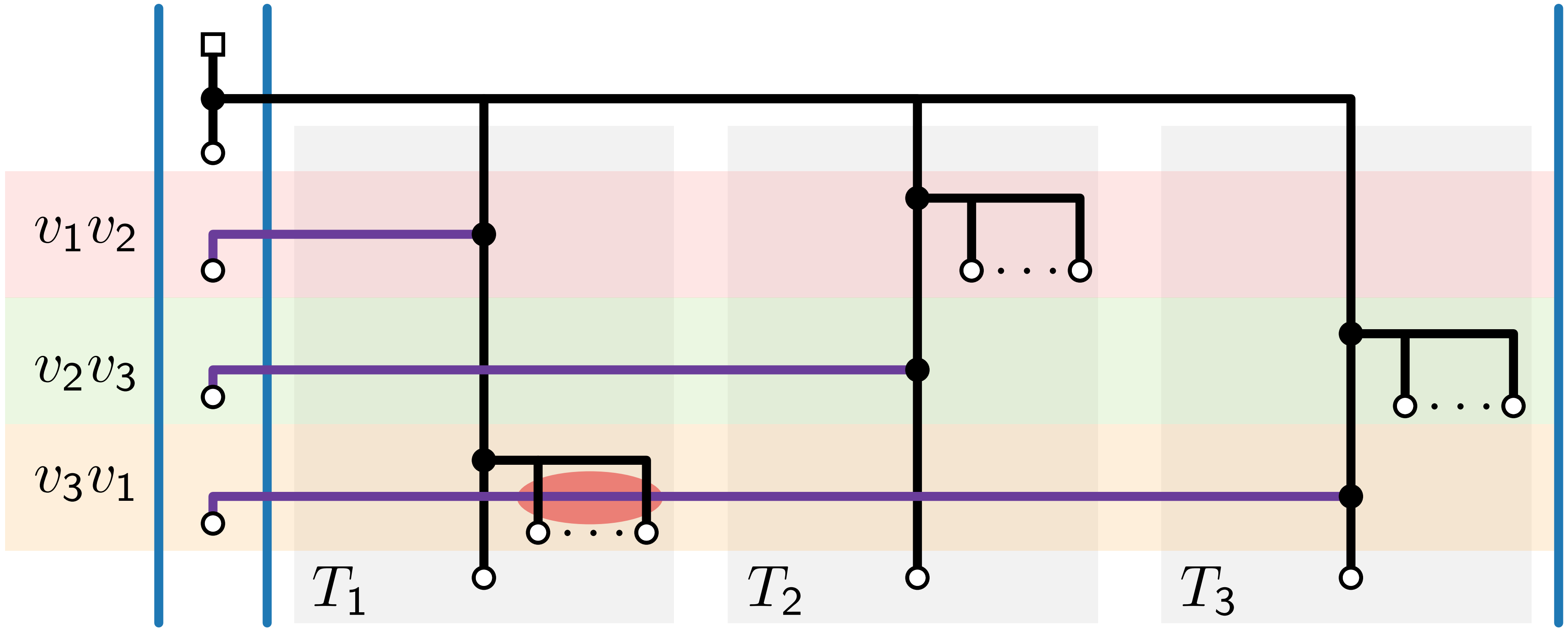

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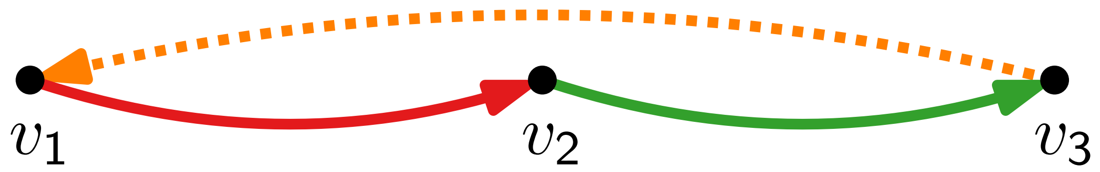

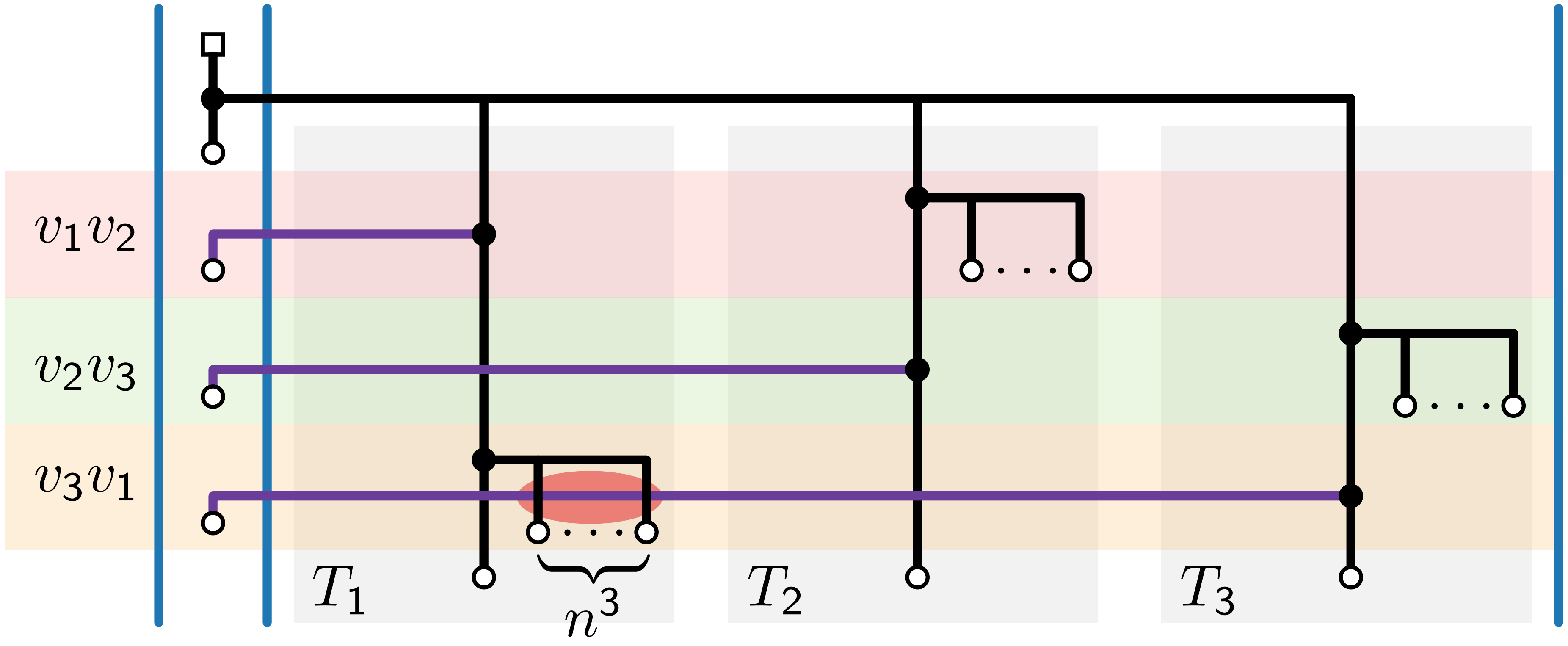

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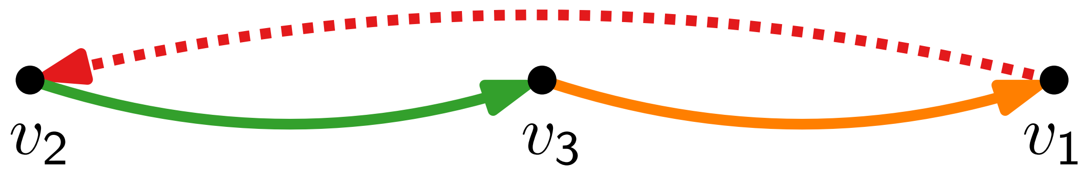

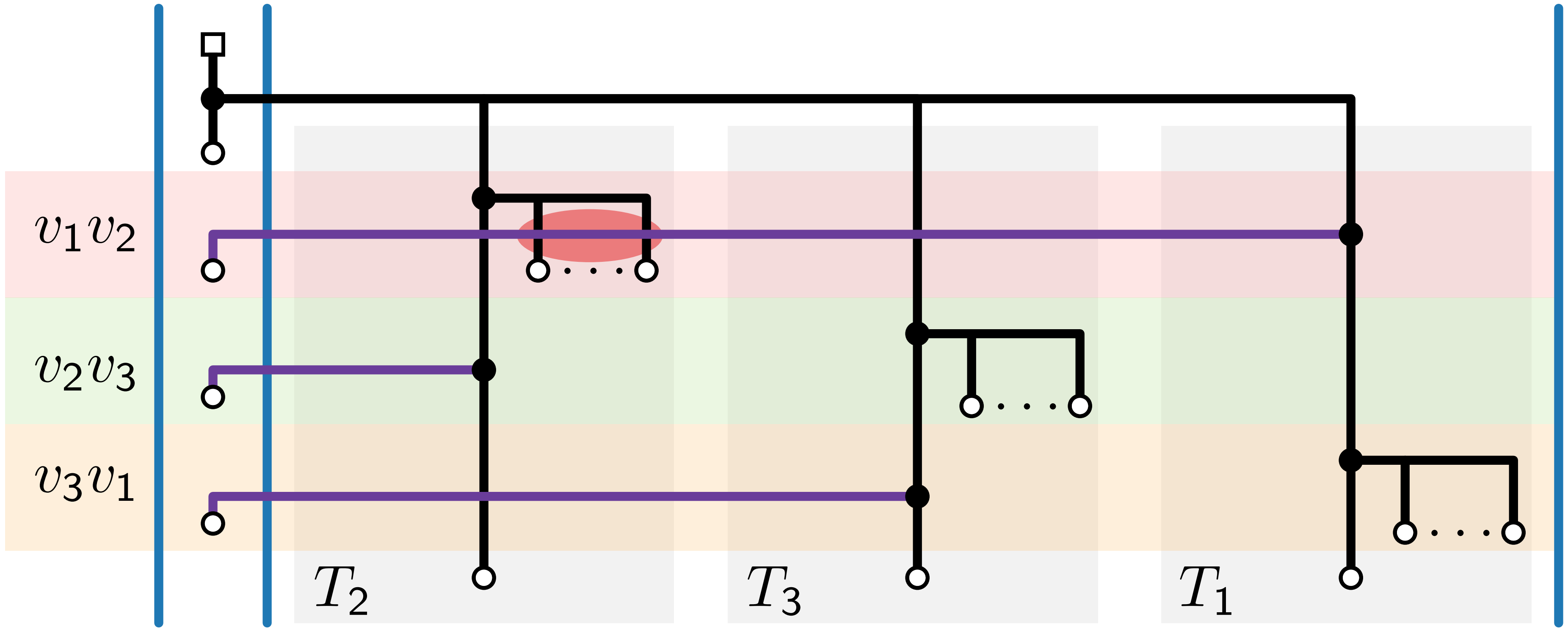

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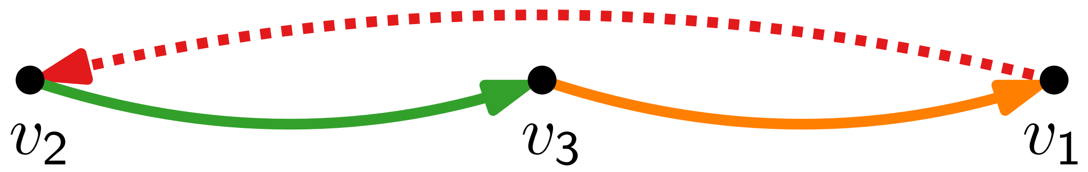

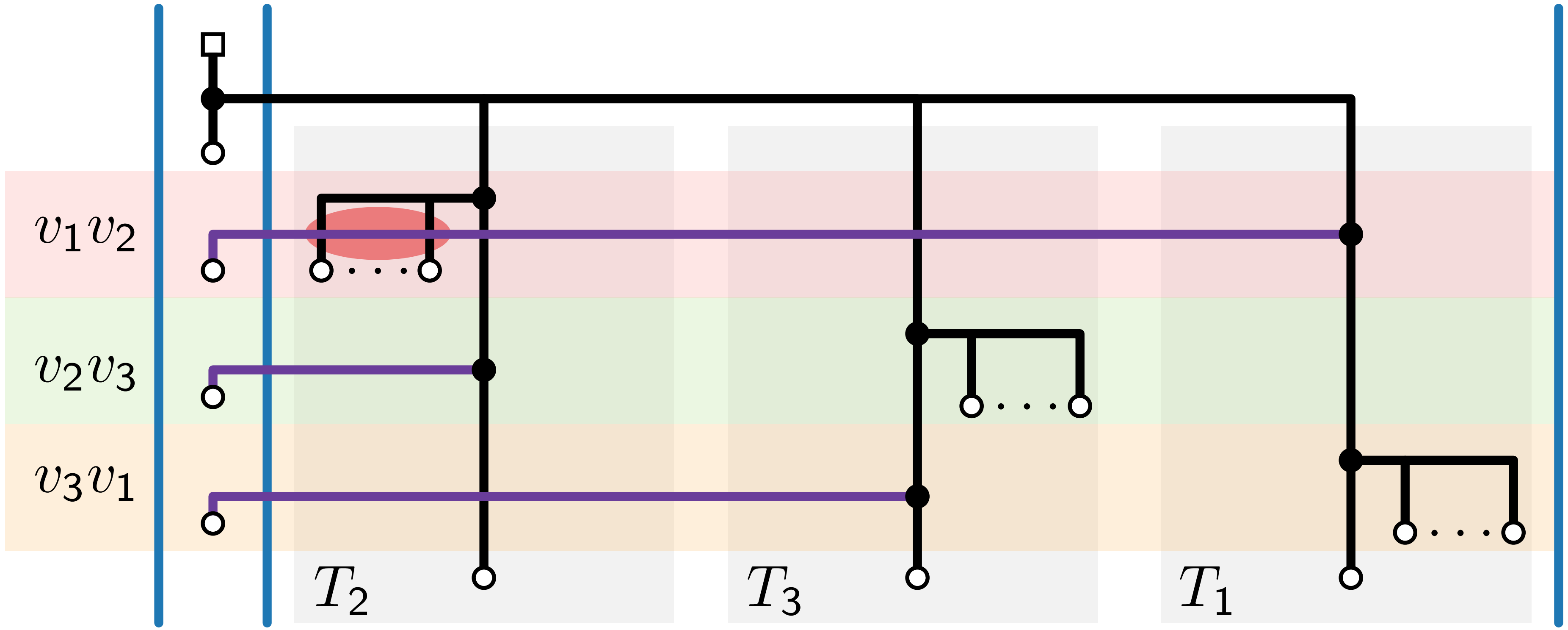

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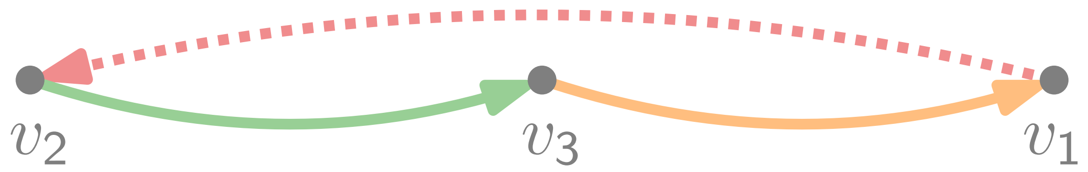

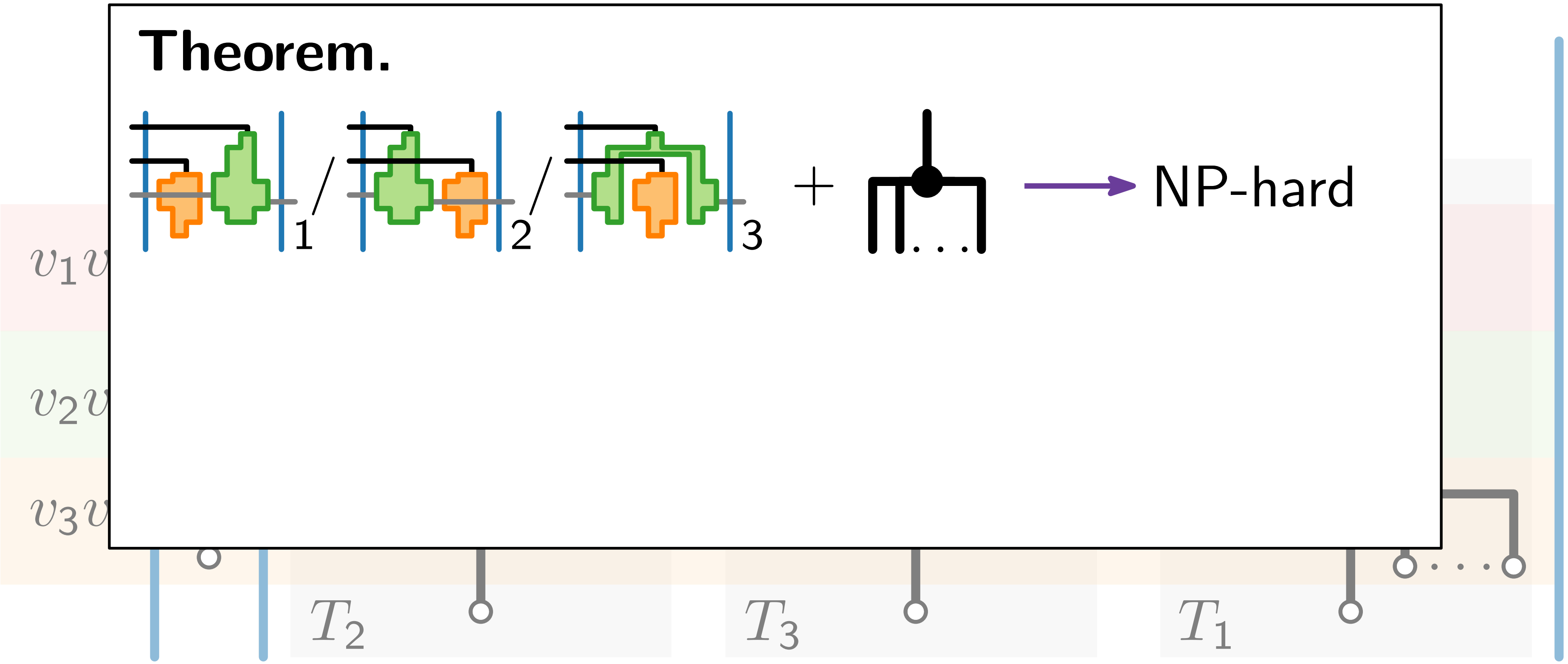

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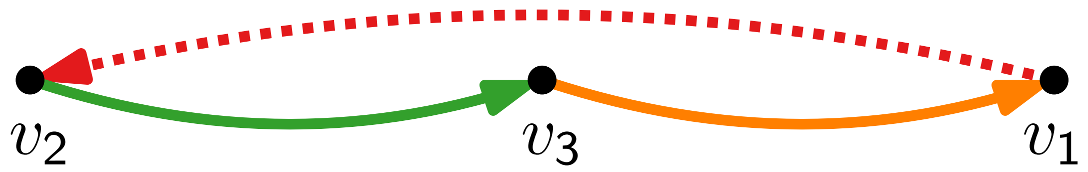

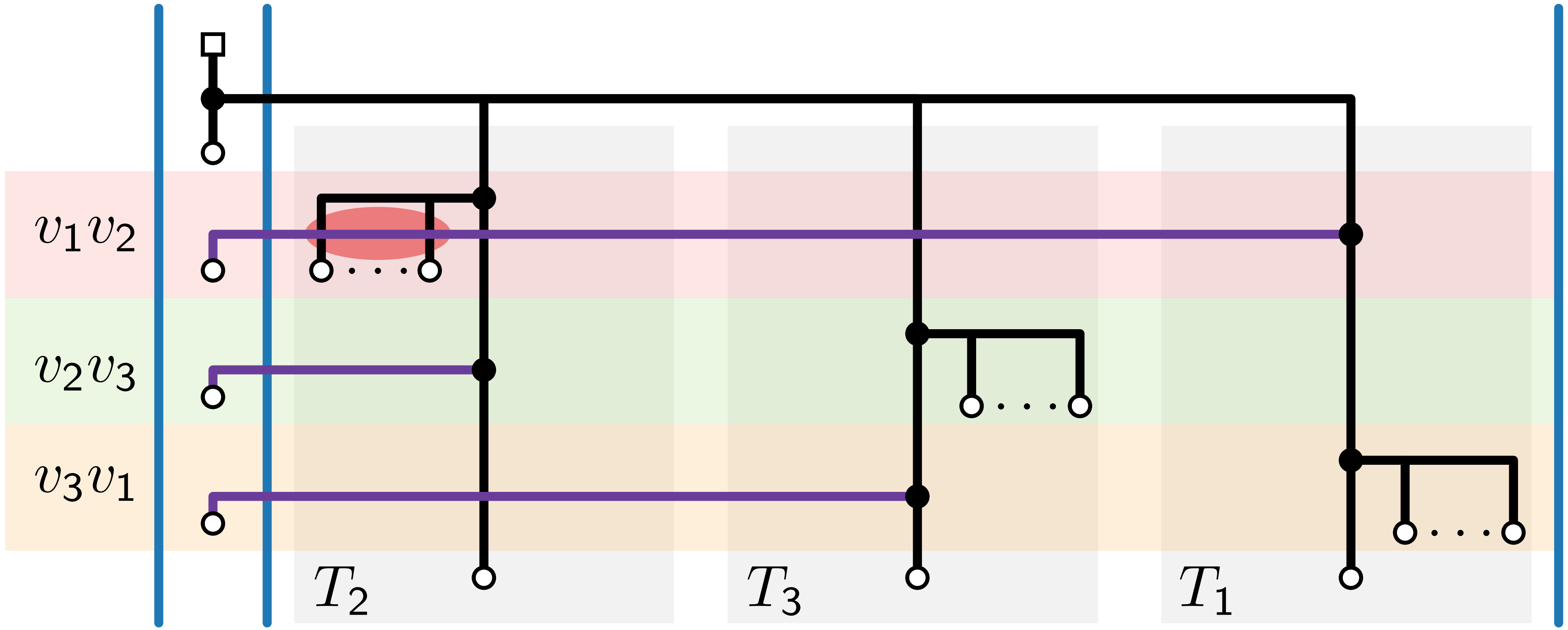

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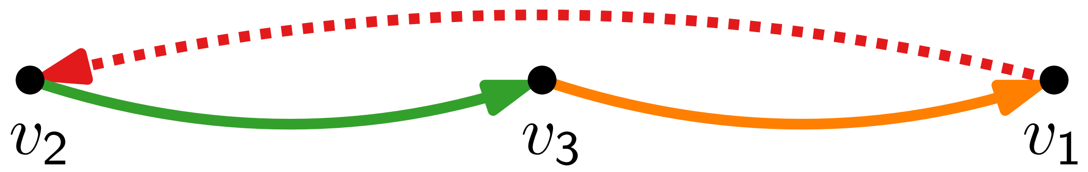

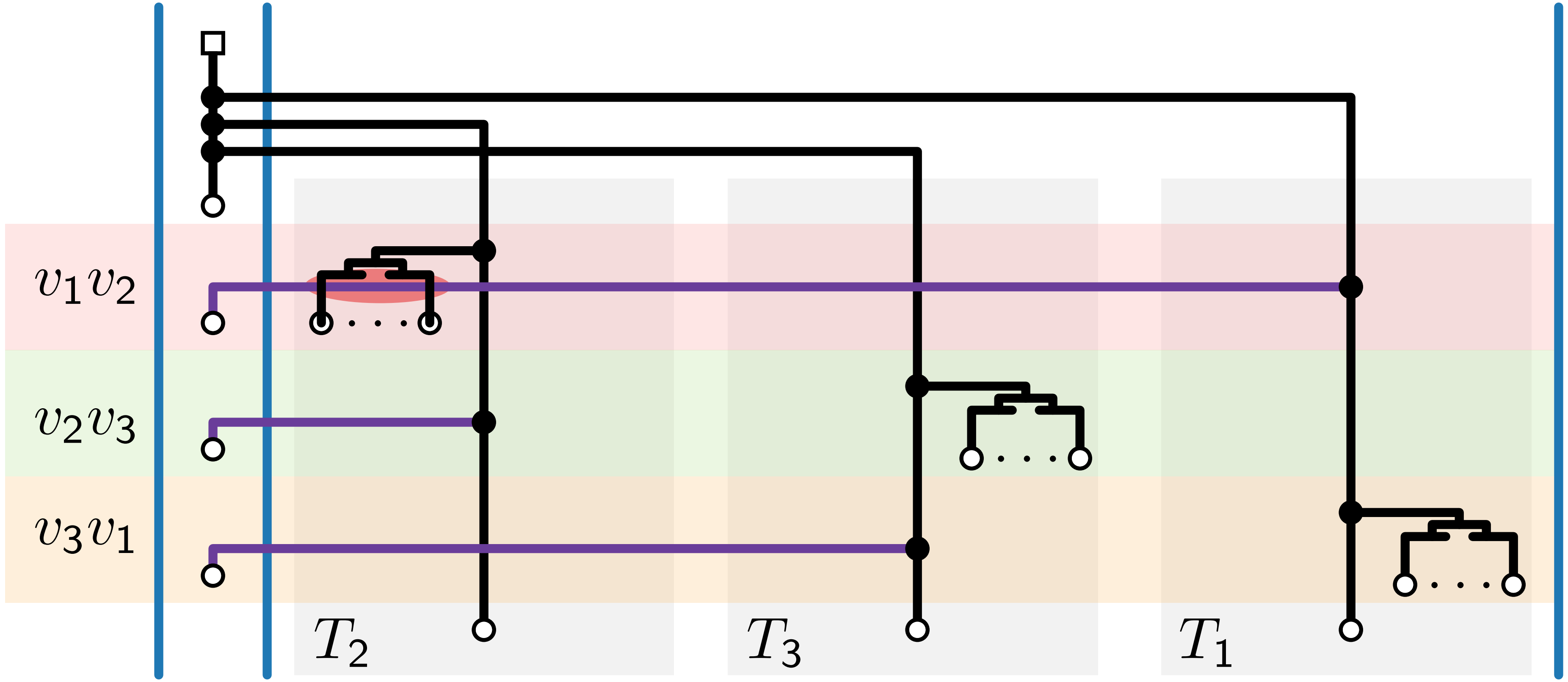

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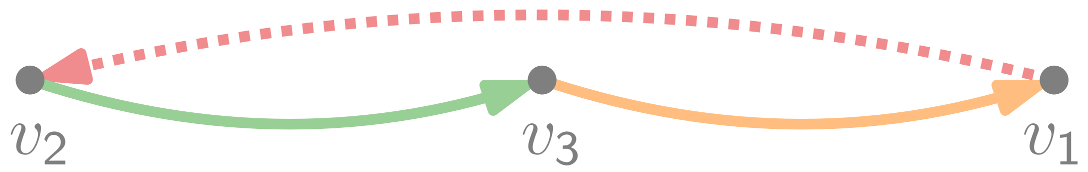

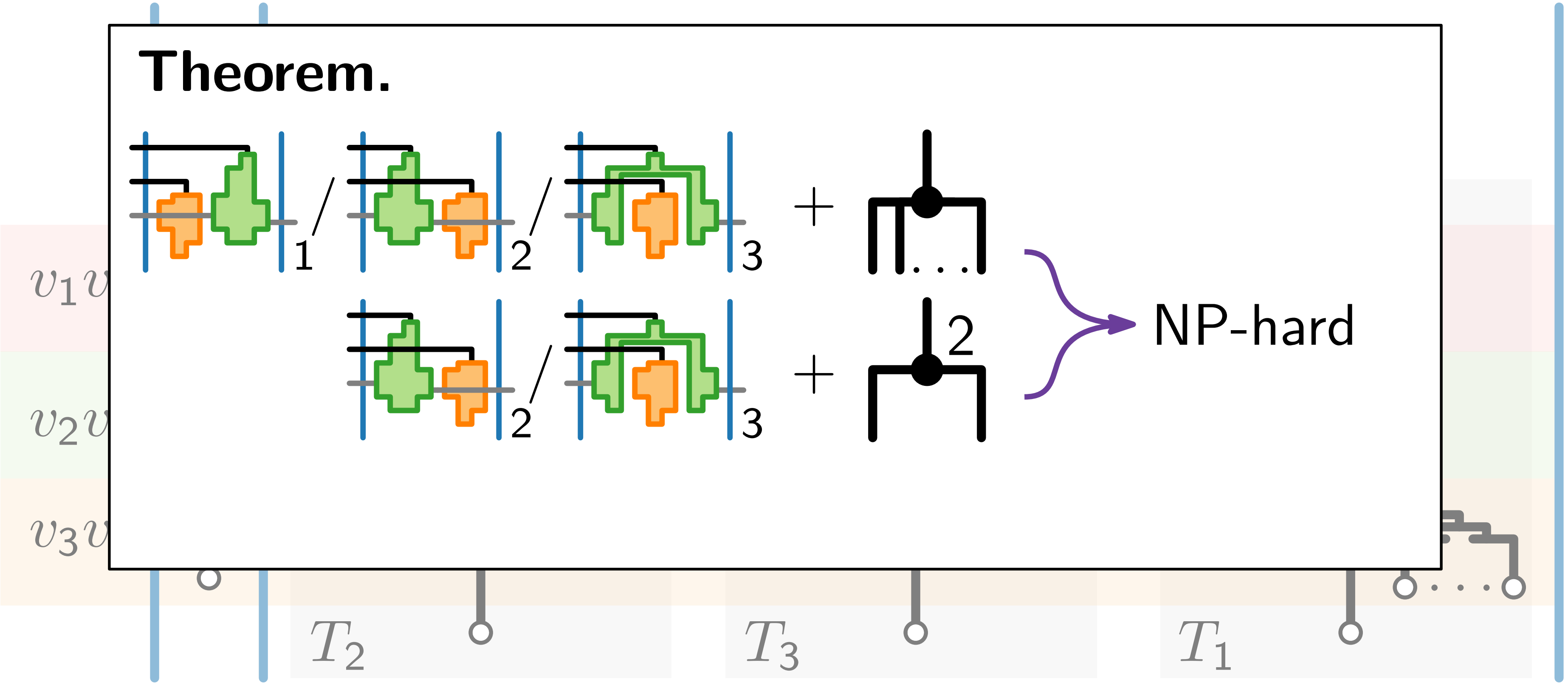

Theorem.

NP-hardness

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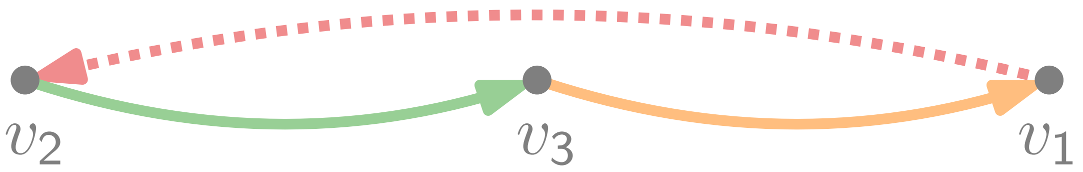

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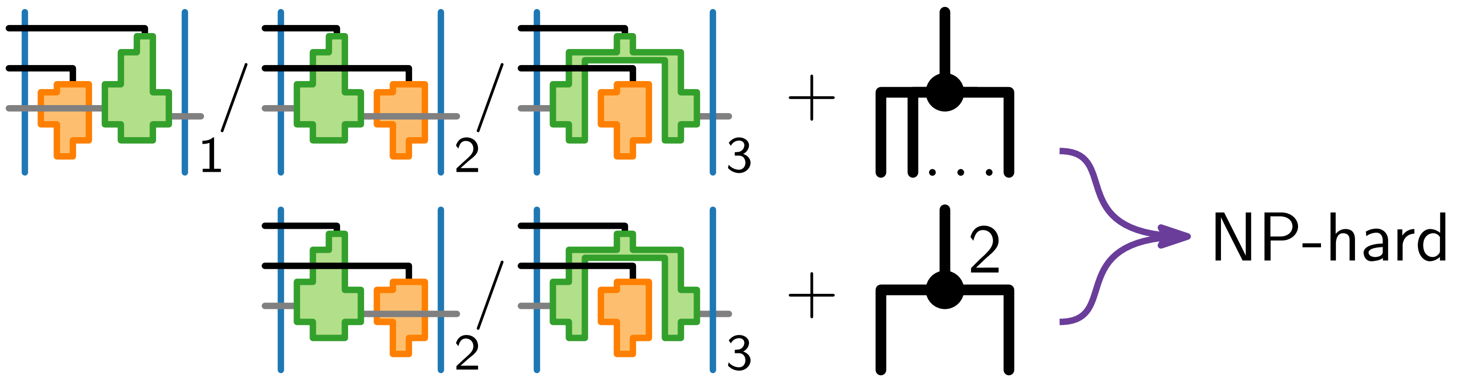

. . . also hard to approximate.

Overview

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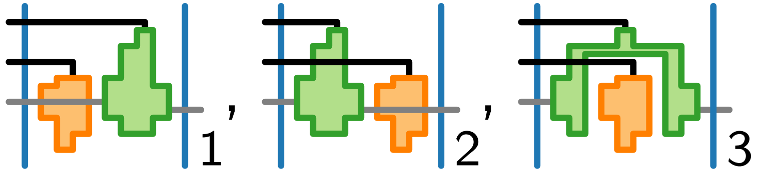

Crossing Minimisation

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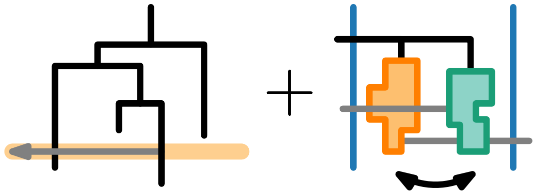
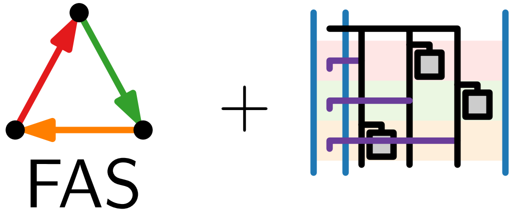


NP



FPT

Subtree Arrangement Algorithm

- **subtree arrangement** $\leftrightarrow$ Integer-Weighted FAS (IFAS) $\leftrightarrow$ FAS

Subtree Arrangement Algorithm

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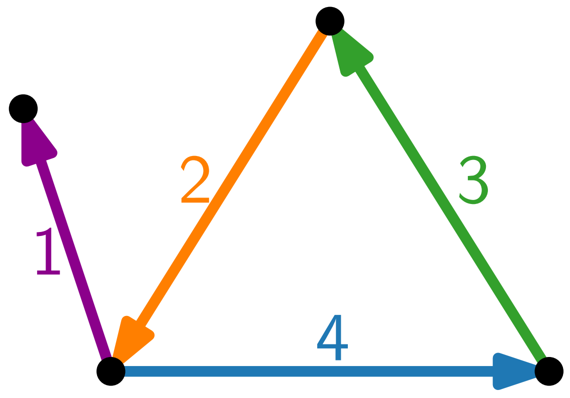
IFAS



Subtree Arrangement Algorithm

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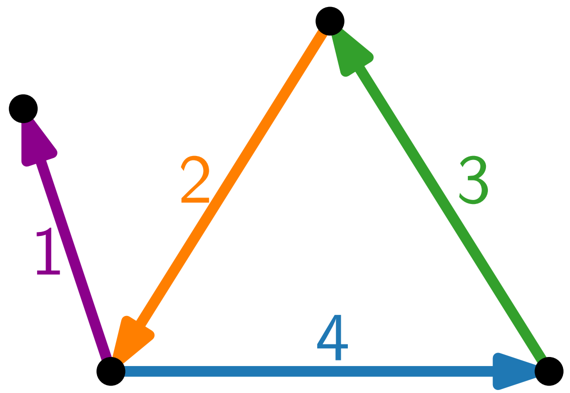
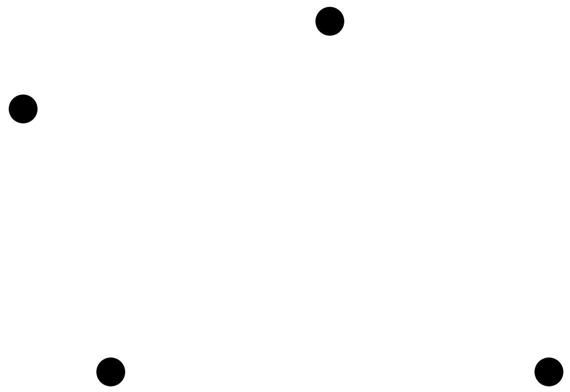
IFAS





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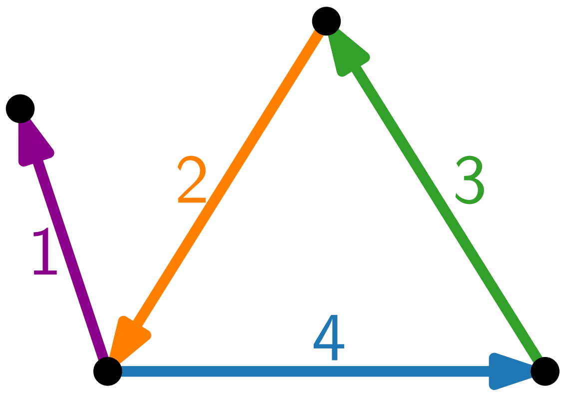
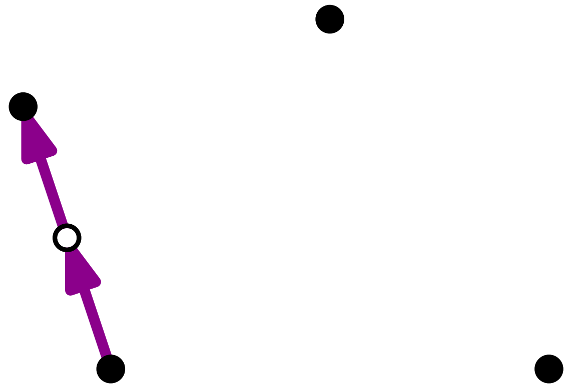
IFAS





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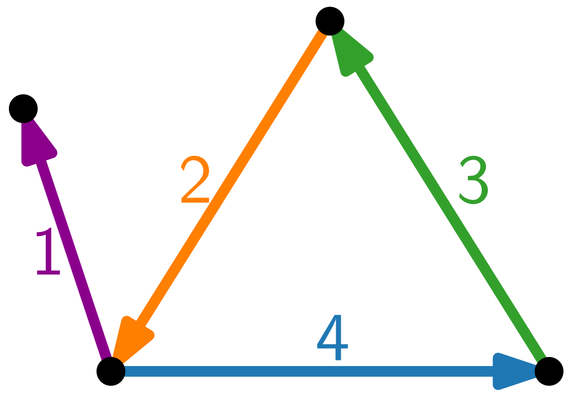

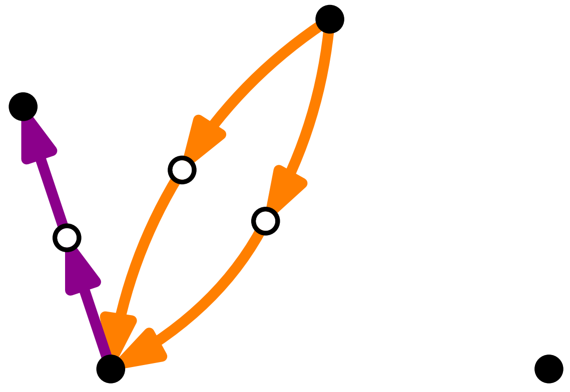

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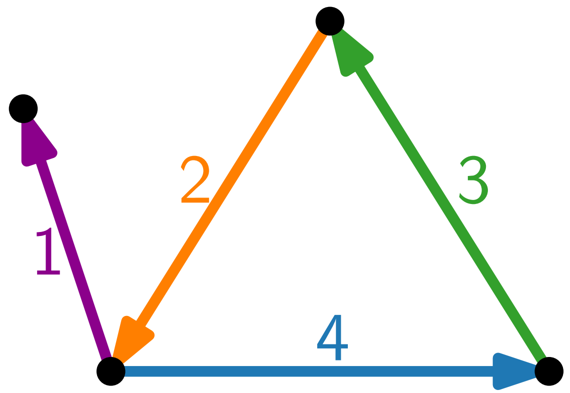
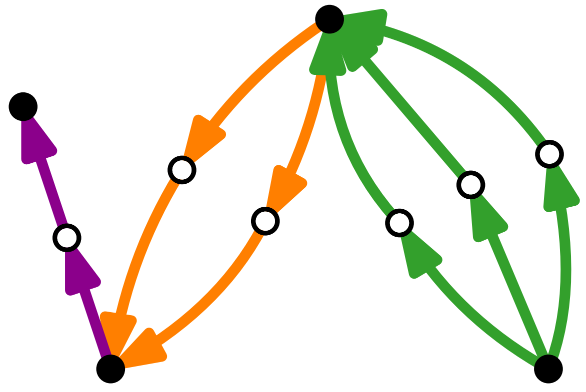
IFAS





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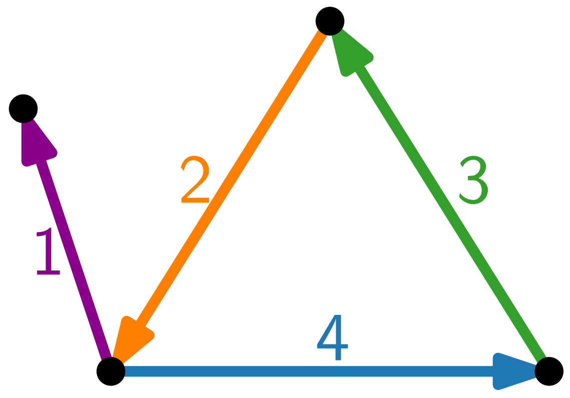

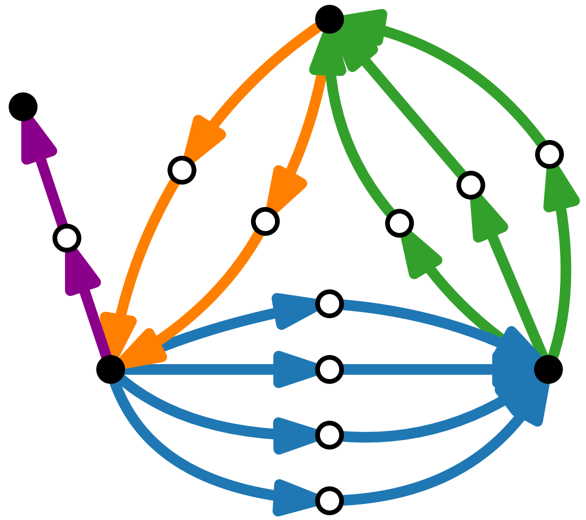

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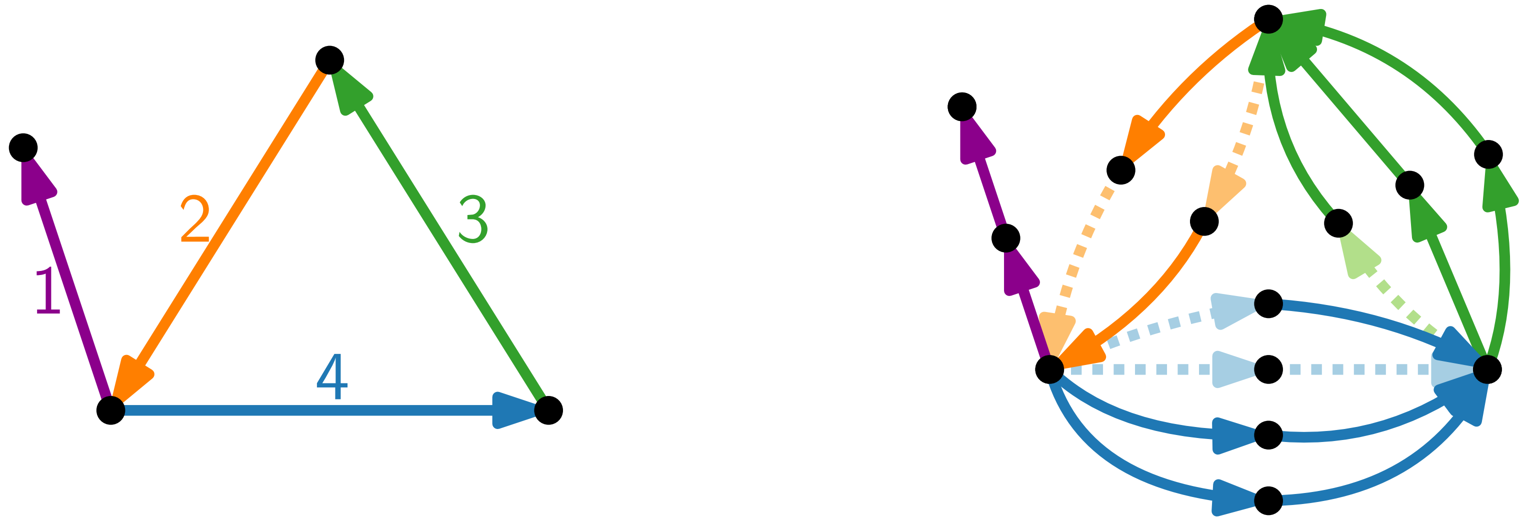

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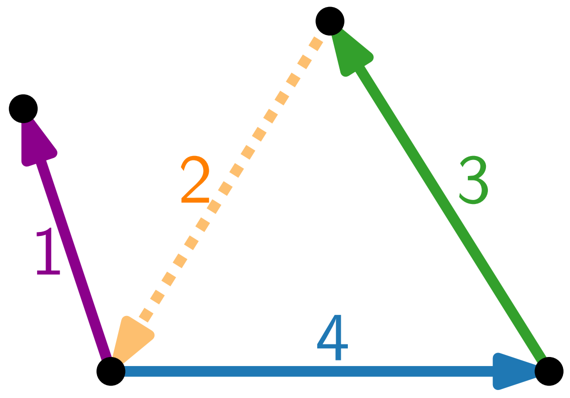
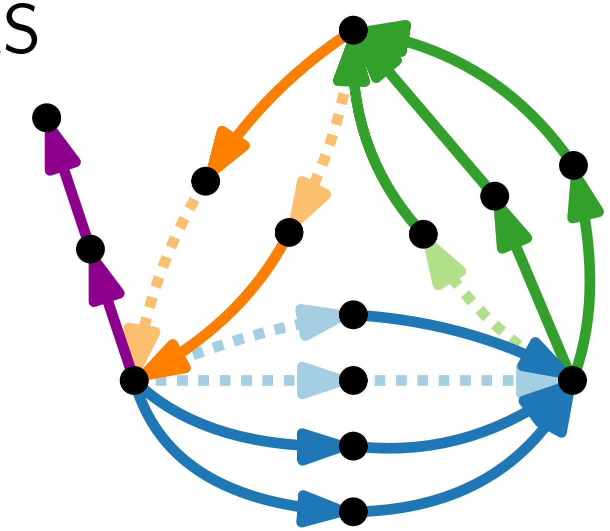
- **subtree arrangement** ↔ Integer-Weighted FAS (IFAS) ↔ FAS

IFAS



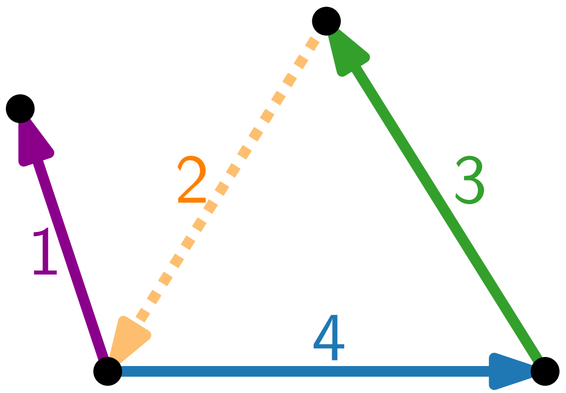
FAS



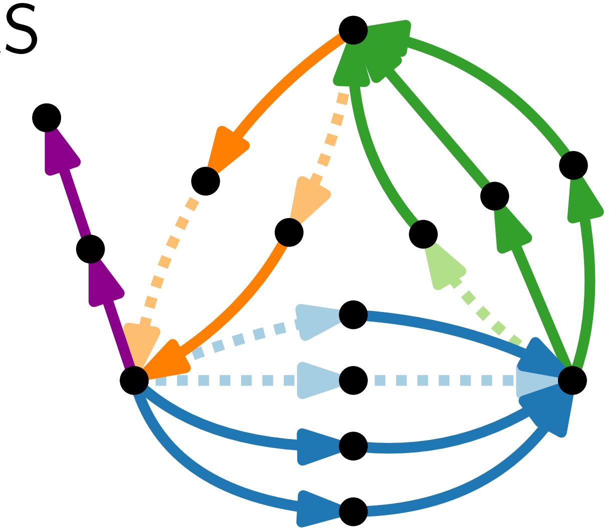
Subtree Arrangement Algorithm

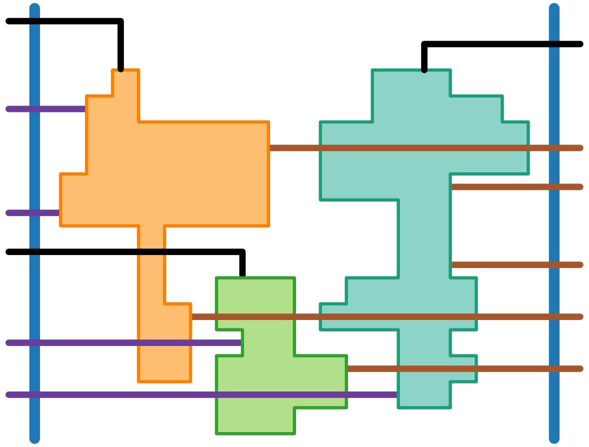
- **subtree arrangement** ↔ Integer-Weighted FAS (IFAS) ↔ FAS

IFAS



FAS

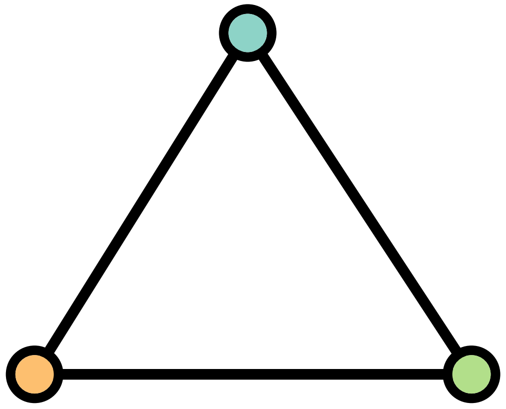




right

left

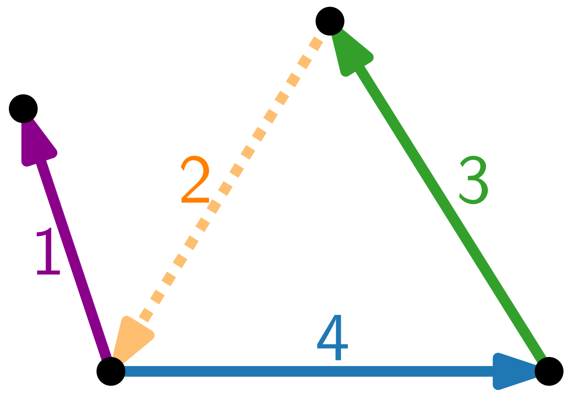
	1	2	3
1	—		
2		—	
3			—



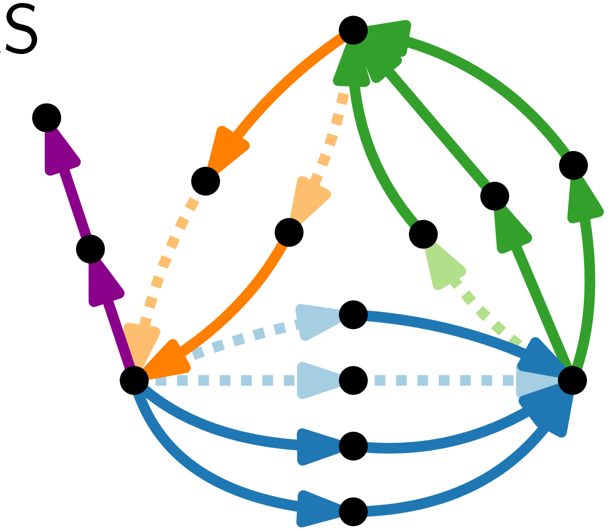
Subtree Arrangement Algorithm

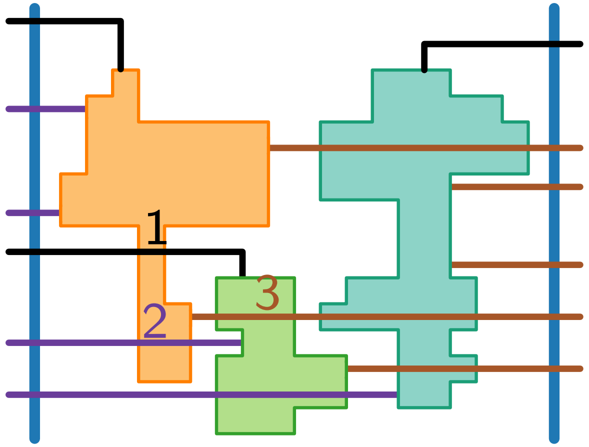
- **subtree arrangement** ↔ Integer-Weighted FAS (IFAS) ↔ FAS

IFAS



FAS

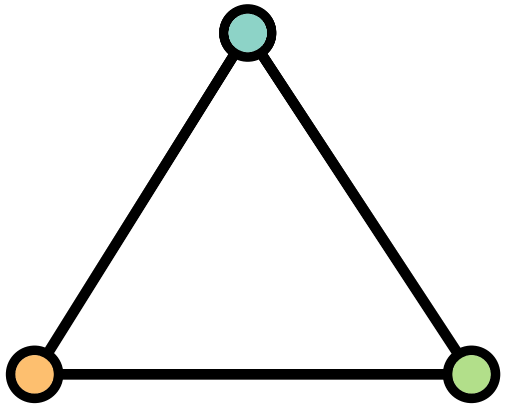




right

left

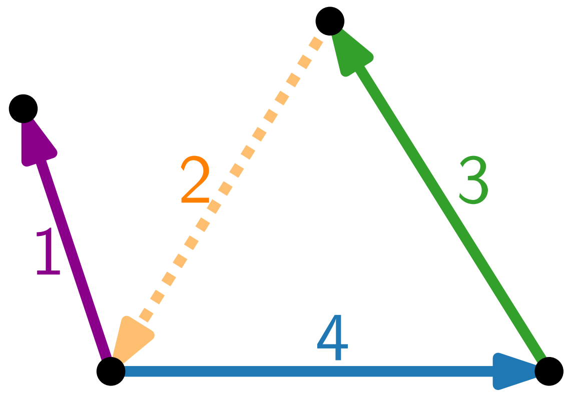
	1	2	3
1	—	6	
2		—	
3			—



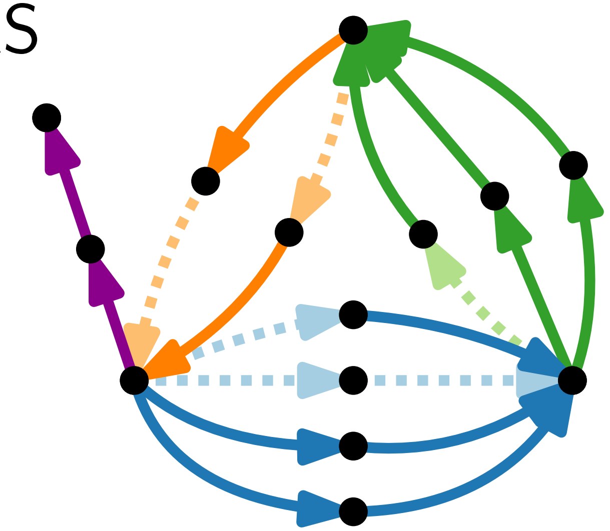
Subtree Arrangement Algorithm

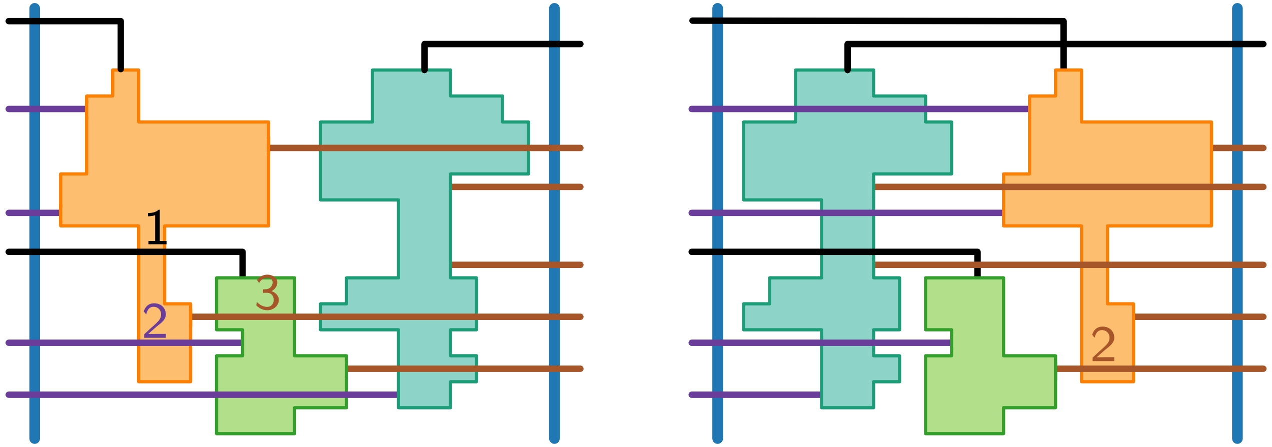
- **subtree arrangement** $\leftrightarrow$ Integer-Weighted FAS (IFAS) $\leftrightarrow$ FAS

IFAS



FAS

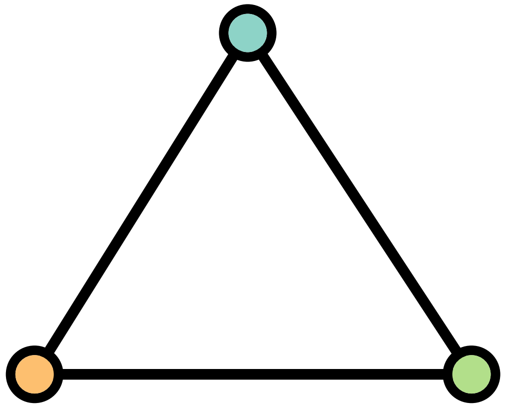




right

left

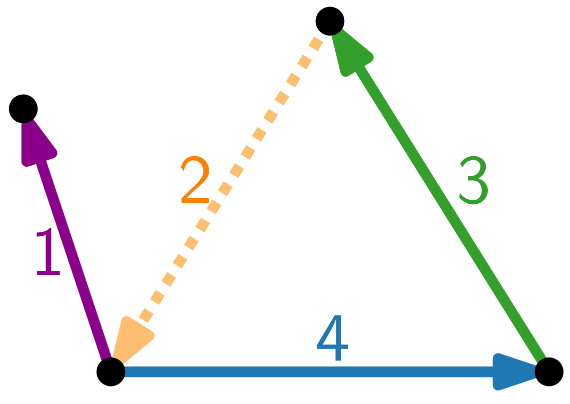
	1	2	3
1	—	6	
2	2	—	
3			—



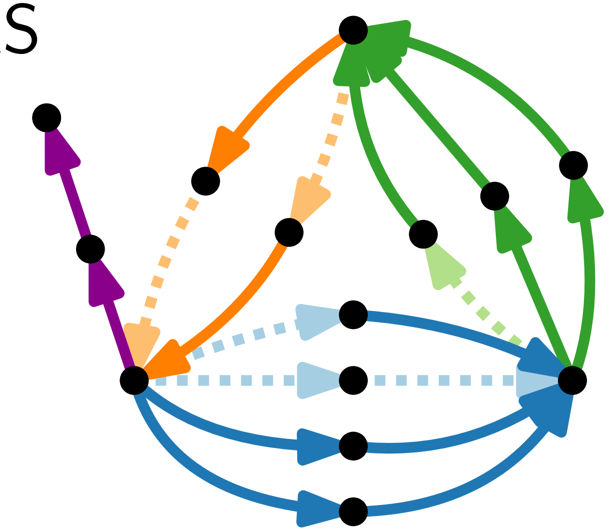
Subtree Arrangement Algorithm

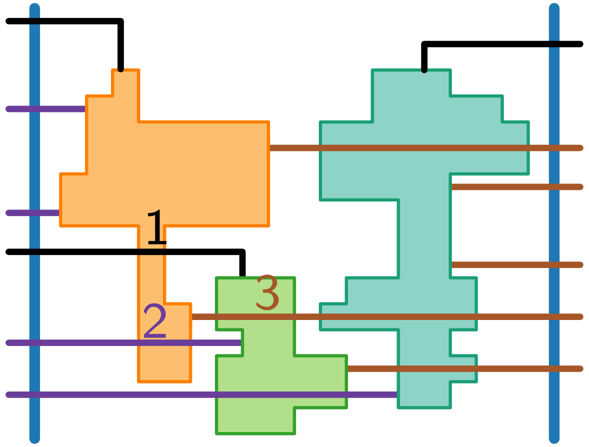
- **subtree arrangement** ↔ Integer-Weighted FAS (IFAS) ↔ FAS

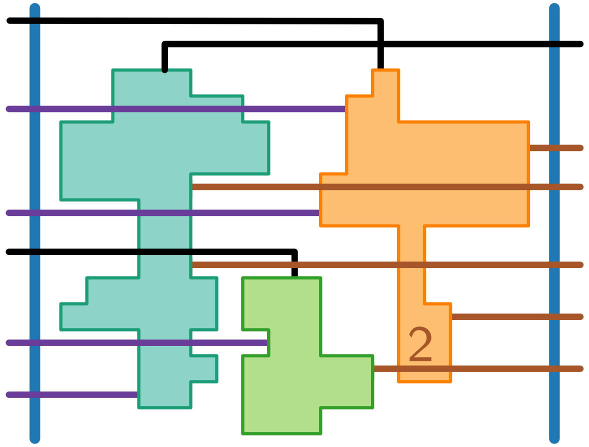
IFAS



FAS

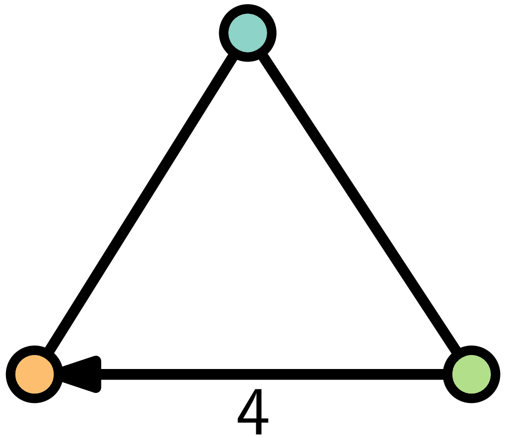






right

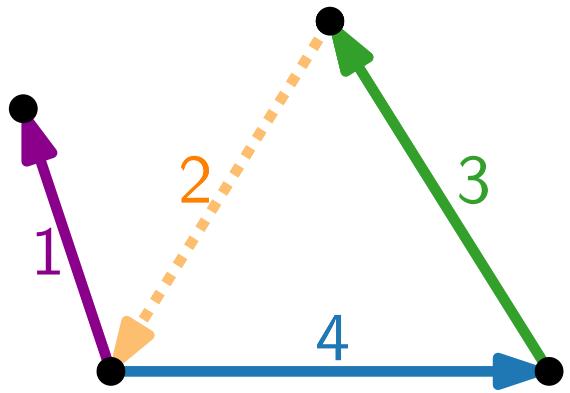
left	1	2	3
1	—	6	
2	2	—	
3			—



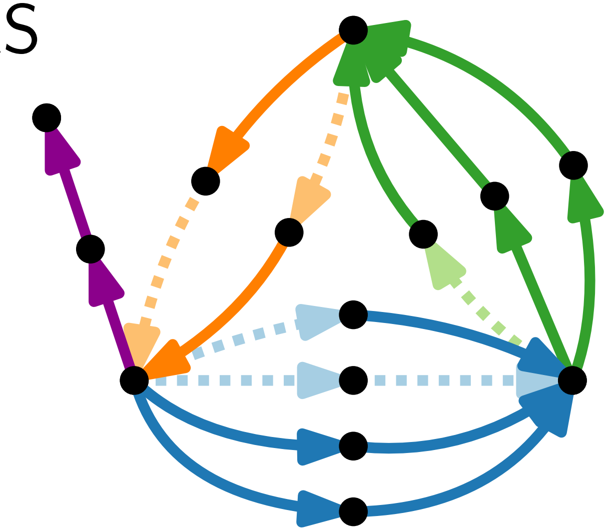
Subtree Arrangement Algorithm

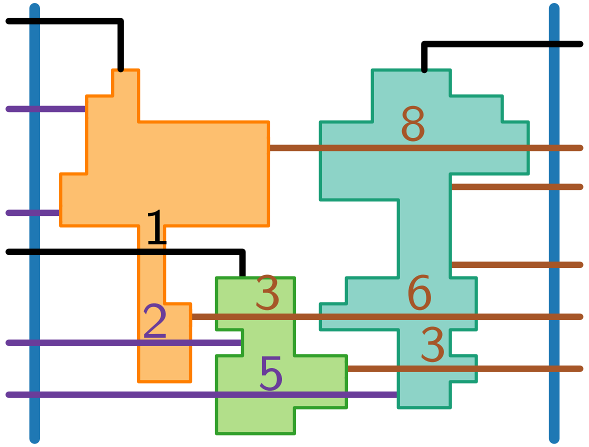
- **subtree arrangement** ↔ Integer-Weighted FAS (IFAS) ↔ FAS

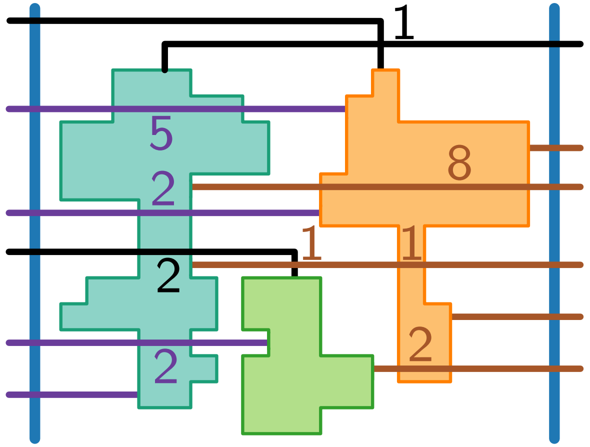
IFAS



FAS



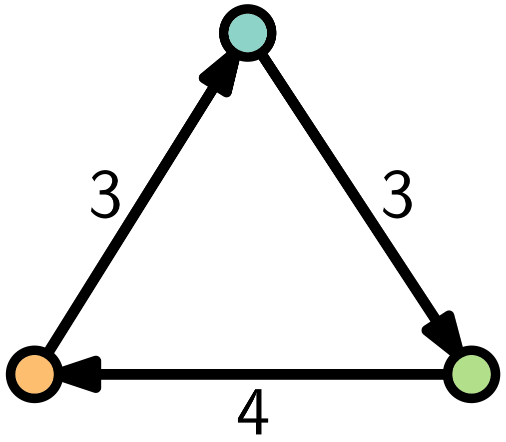




right

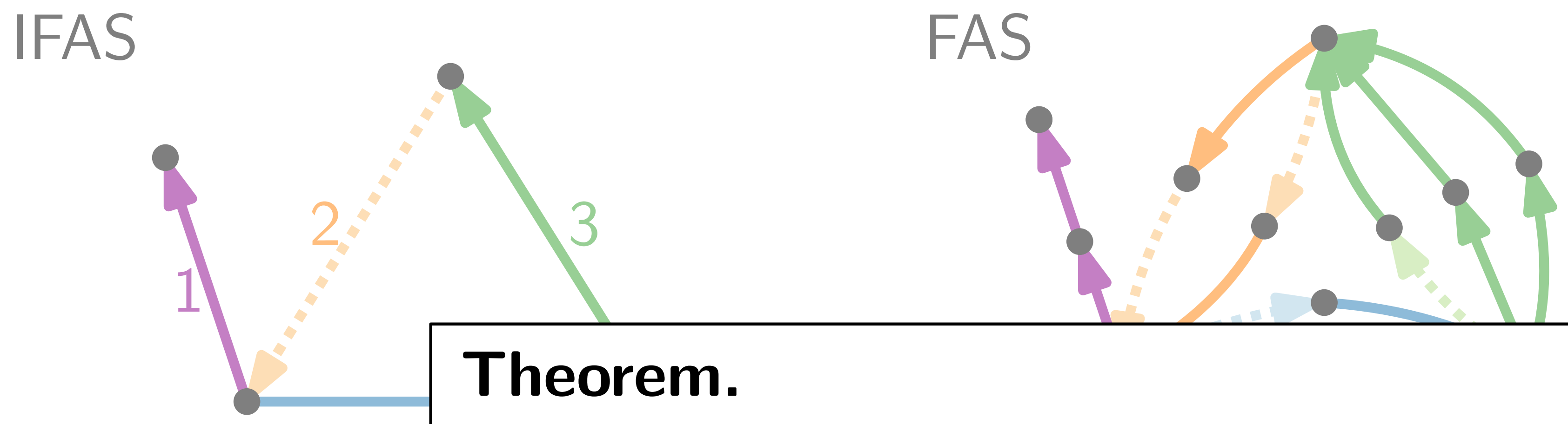
left

	1	2	3
1	—	6	14
2	2	—	8
3	17	5	—

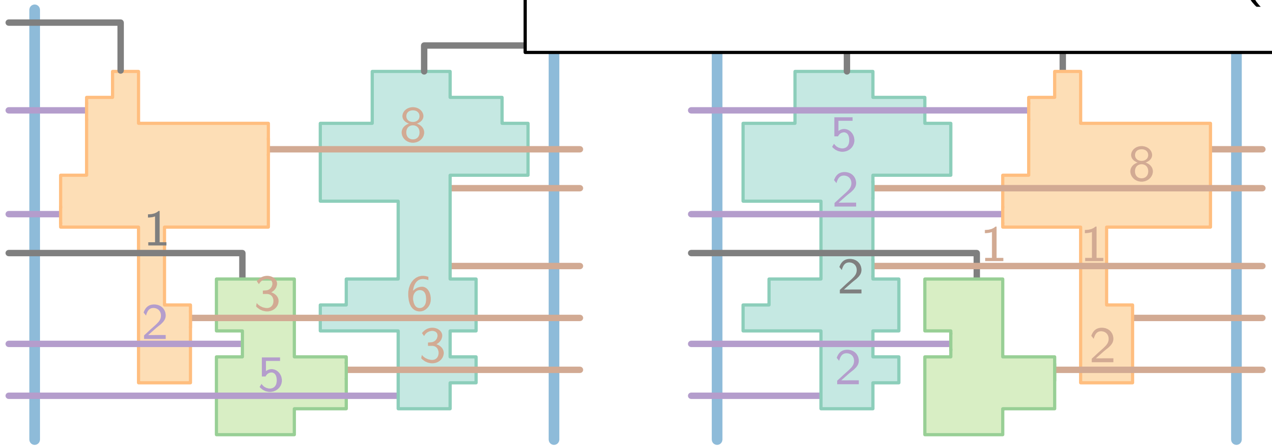


Subtree Arrangement Algorithm

- **subtree arrangement** $\leftrightarrow$ Integer-Weighted FAS (IFAS) $\leftrightarrow$ FAS

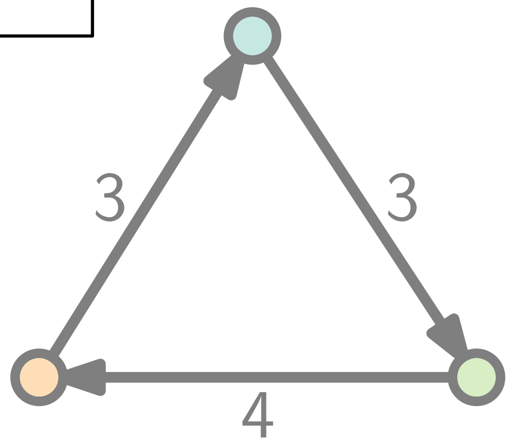


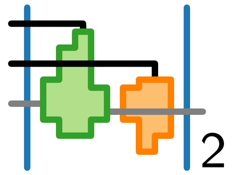
> **Theorem.**
>
> Can reduce our problem in $\mathcal{O}(n^4)$ time to an instance of FAS with $\mathcal{O}(n^4)$ size, ...



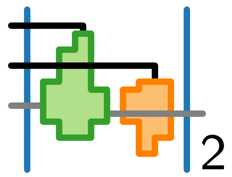
left

	1	2	3
1	–	6	14
2	2	–	8
3	17	5	–

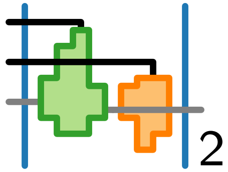


FPT and Approx Algorithms for 

- use **subtree embedding** algo for each maximal subtree in a column
- reduce the **subtree arrangement** problem of each column to a FAS instance

FPT and Approx Algorithms for 

- use **subtree embedding** algo for each maximal subtree in a column
- reduce the **subtree arrangement** problem of each column to a FAS instance
- use FPT and approx algorithms for FAS

FPT and Approx Algorithms for 

- use **subtree embedding** algo for each maximal subtree in a column
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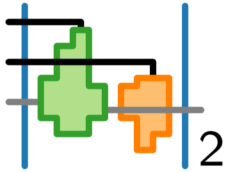
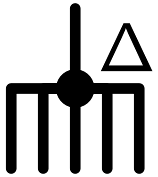
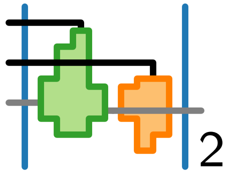
Theorem.



$+$ 

$+$ k crossings

→ FPT algo with running time $\mathcal{O}(\Delta!\Delta n^2 + n^{16} 4^k k^3 k!)$

FPT and Approx Algorithms for 

- use **subtree embedding** algo for each maximal subtree in a column
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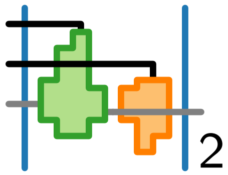
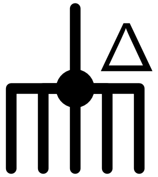
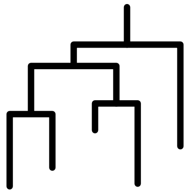
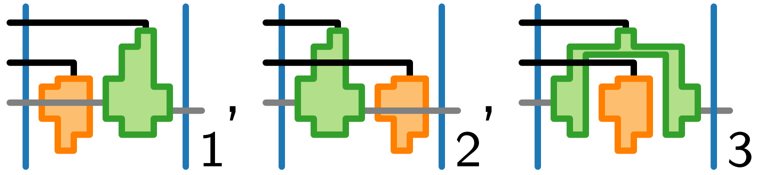
Theorem.

 +  + k crossings

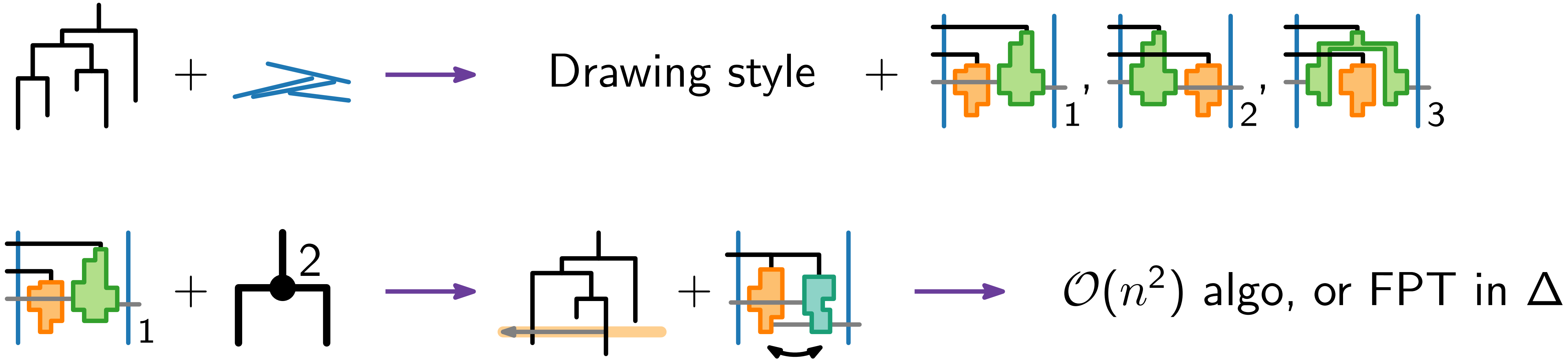
→ FPT algo with running time $\mathcal{O}(\Delta!\Delta n^2 + n^{16} 4^k k^3 k!)$

→ $\mathcal{O}(\text{poly}(n)\Delta!\Delta)$-time approx algo with approx factor in $\mathcal{O}(\log n \log\log n)$

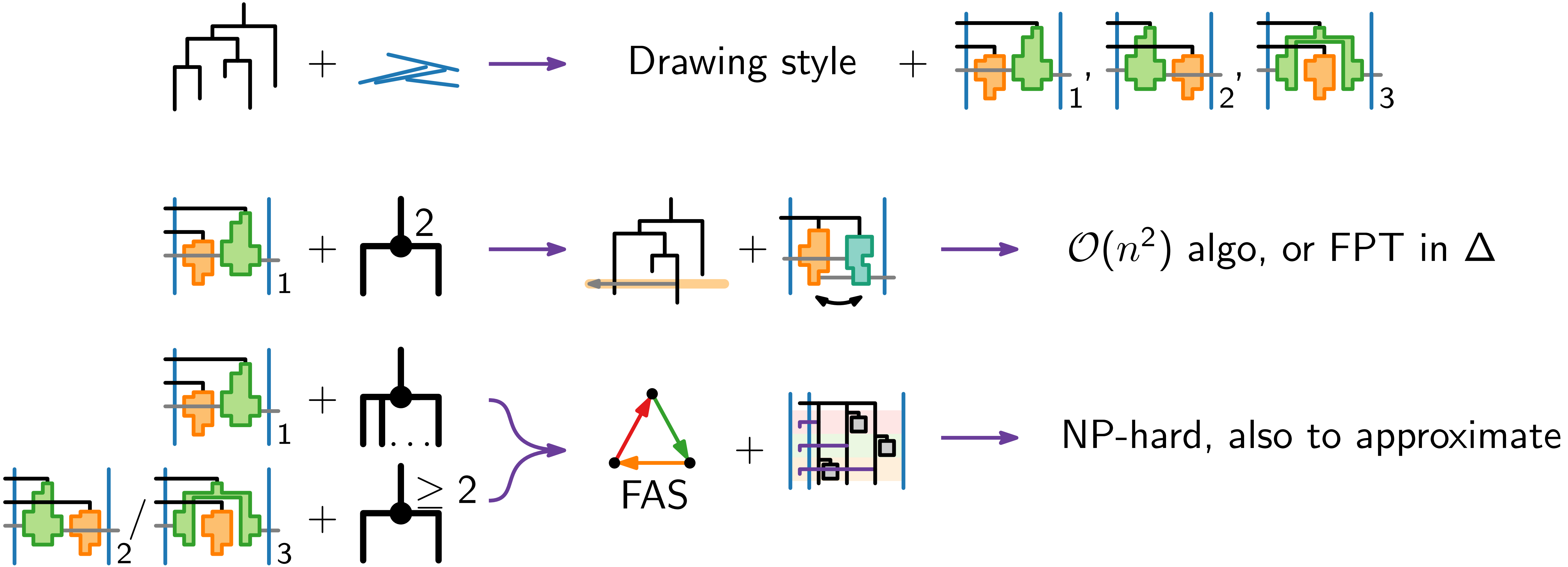
Summary

 +  → Drawing style + 

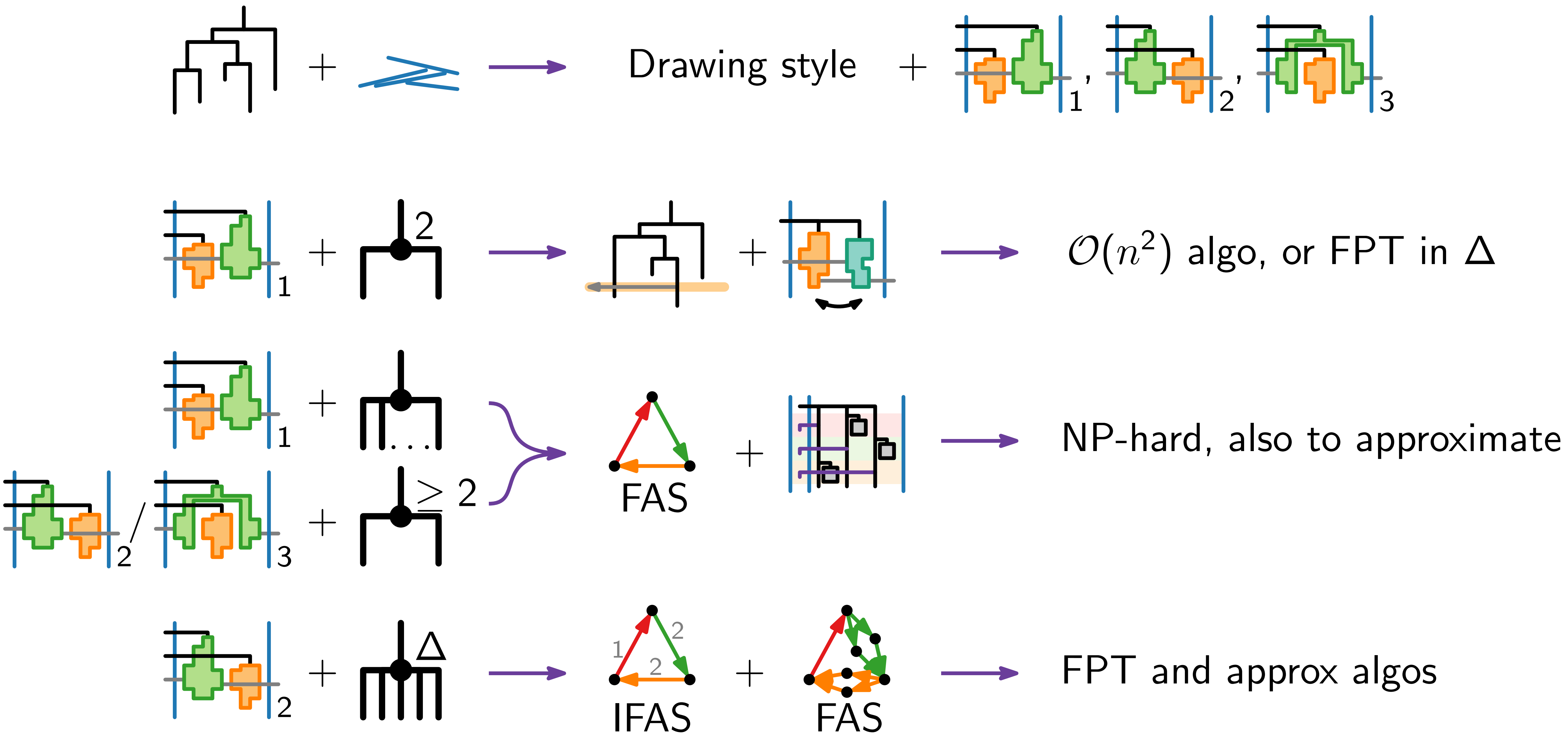
Summary



Summary


+
Drawing style
+
1, 2, 3
1 + 2
+
$\mathcal{O}(n^2)$ algo, or FPT in Δ
1 + . . .
2 / 3 + ≥ 2
FAS
+
NP-hard, also to approximate

Summary



Drawing style
1, 2, 3
2
O(n²) algo, or FPT in Δ
1
2 / 3
≥ 2
FAS
NP-hard, also to approximate
2
Δ
1 2 2
IFAS
FAS
FPT and approx algos