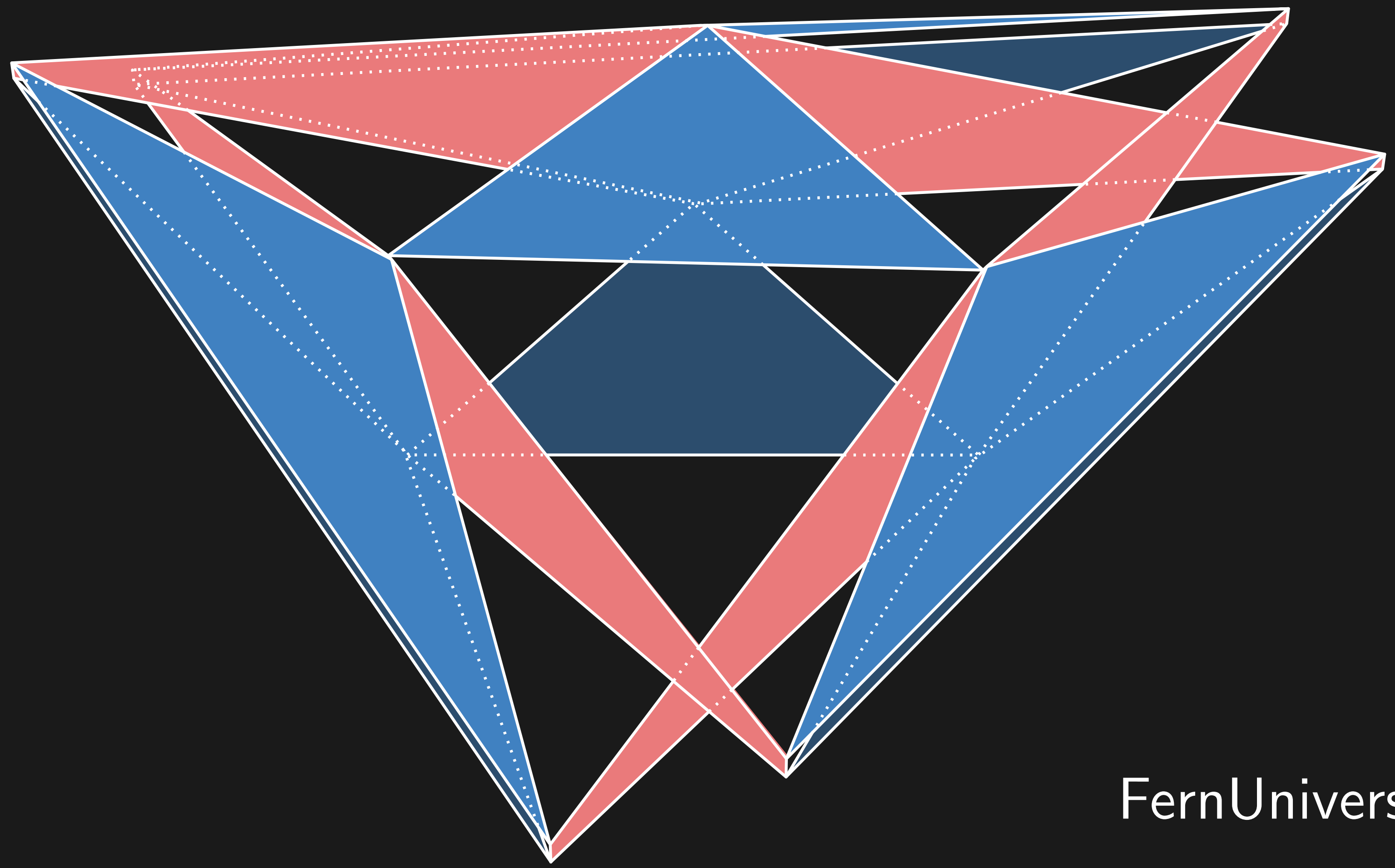
Side-Contact Representations with Convex Polygons in 3D: New Results for Complete Bipartite Graphs



GD 2023

André Schulz
FernUniversität in Hagen

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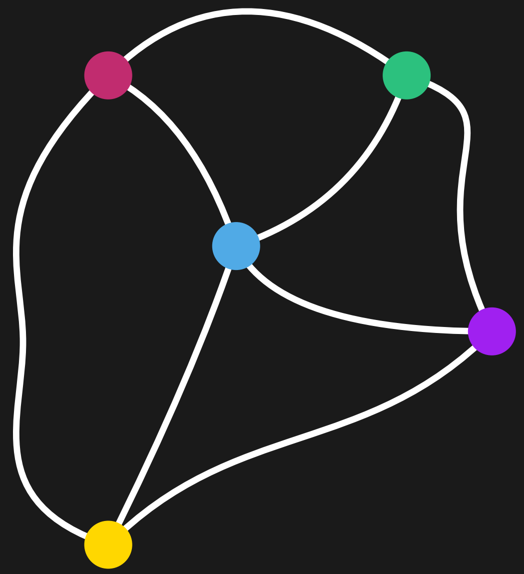

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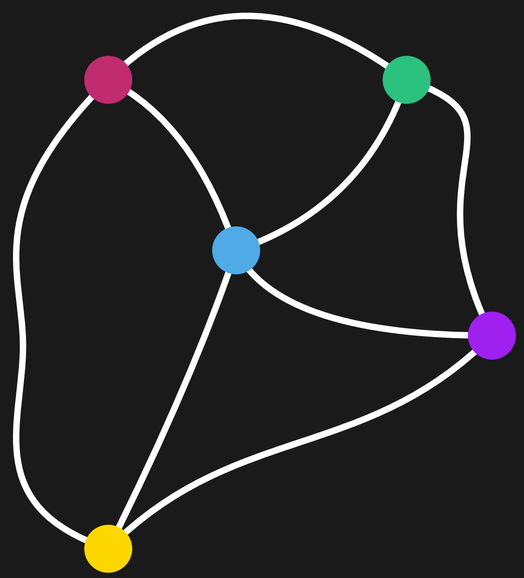

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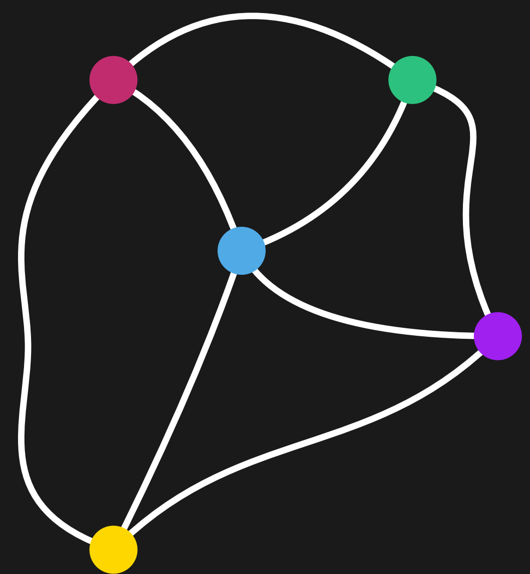

objects: squares

contact: intersecting boundary

Our model

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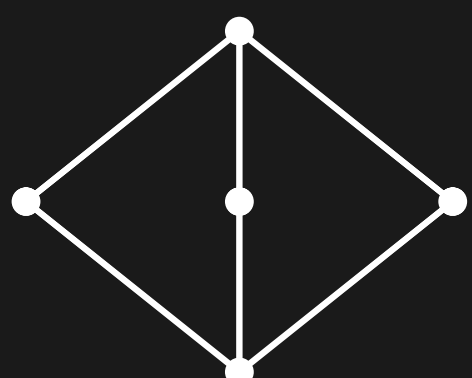

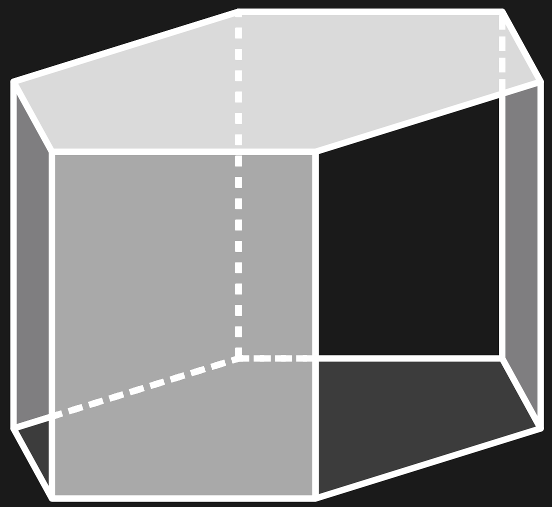

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- the $K_{4,4}$ and the $K_{3,5}$ have a representation in this model
- all planar graphs can be represented in this model, even in 2d
- (related) if we consider nonconvex polygons or corner-corner contacts [Evans et al. '19], all graphs can be represented

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convex set of segments in 2d

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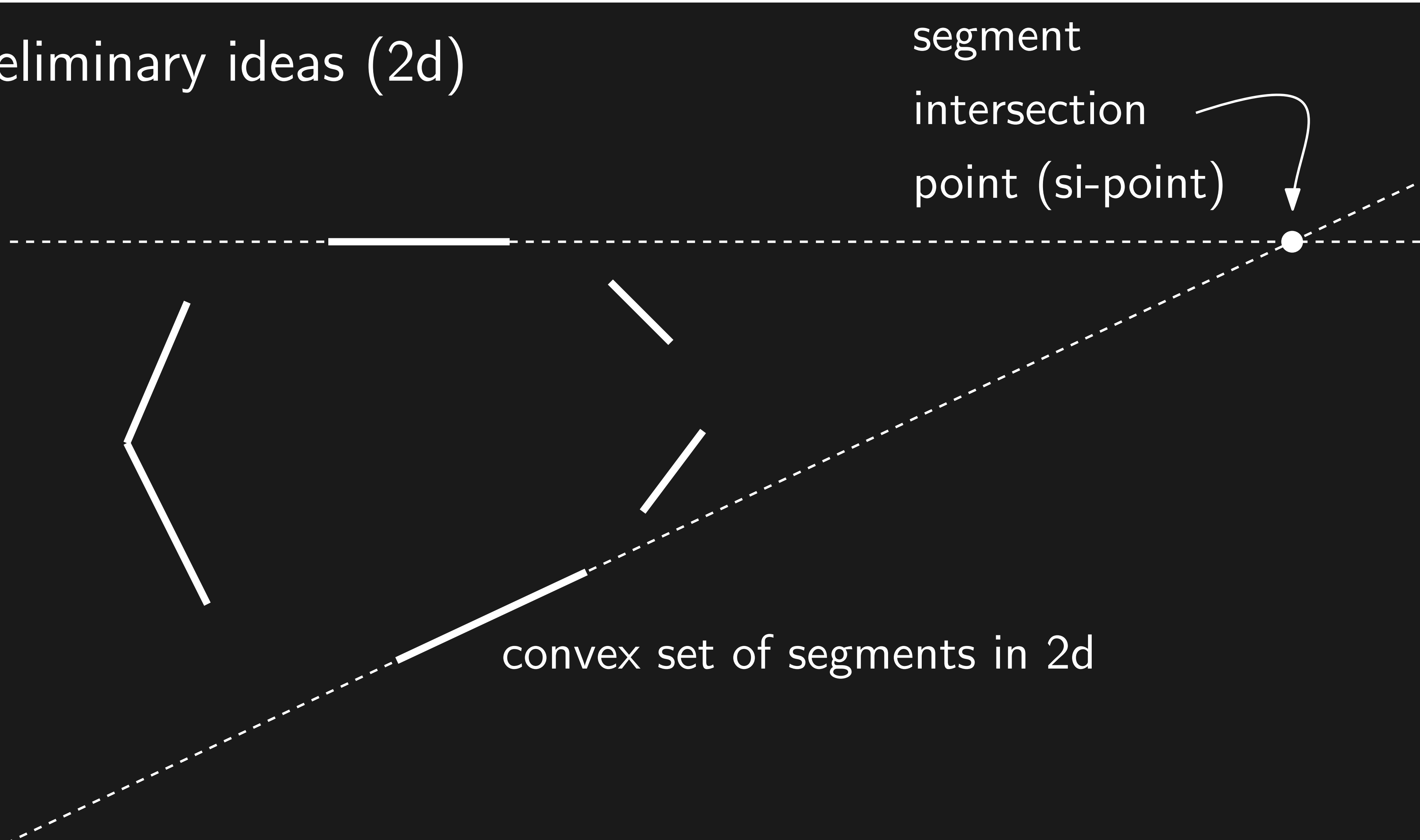

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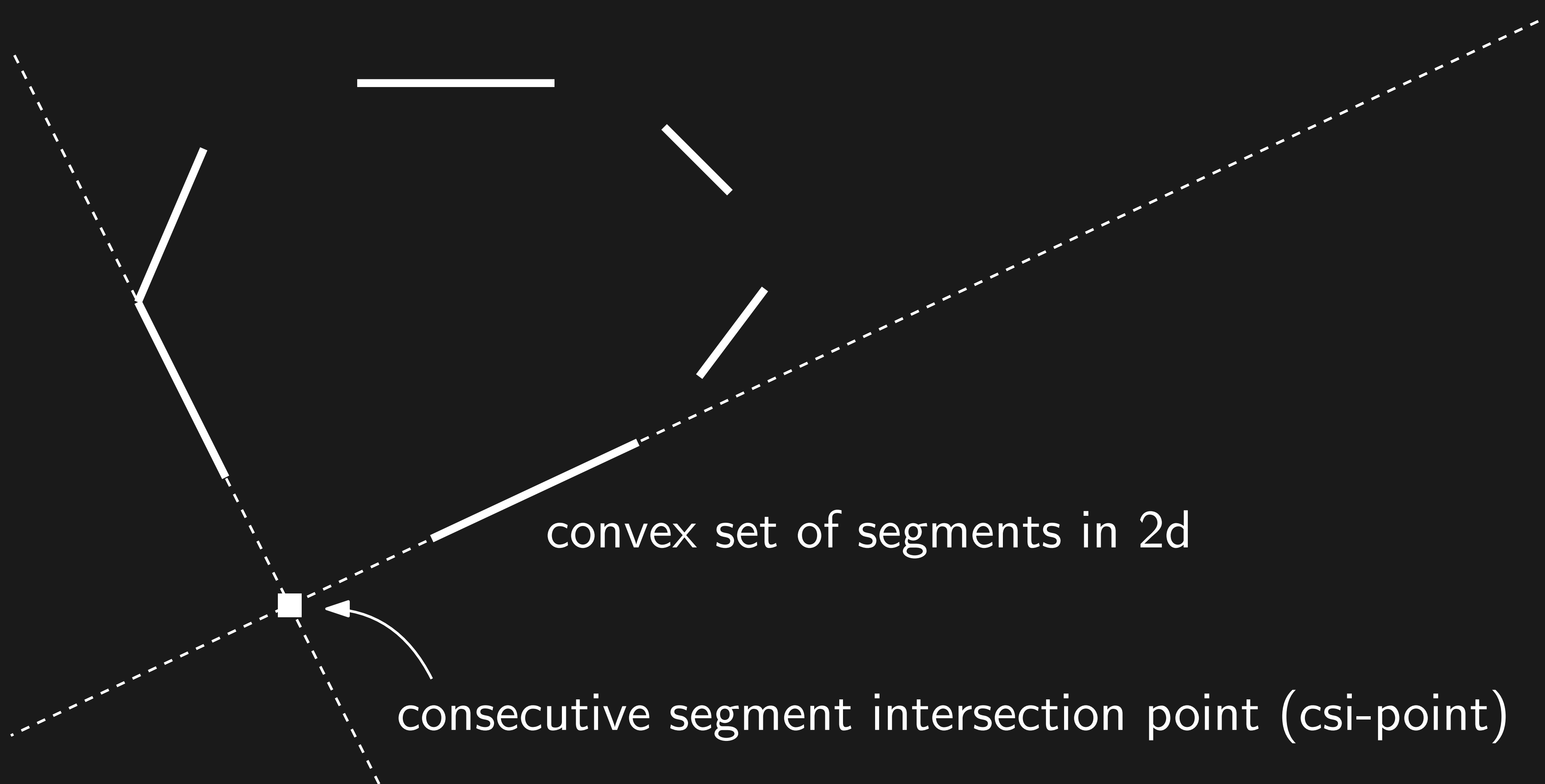

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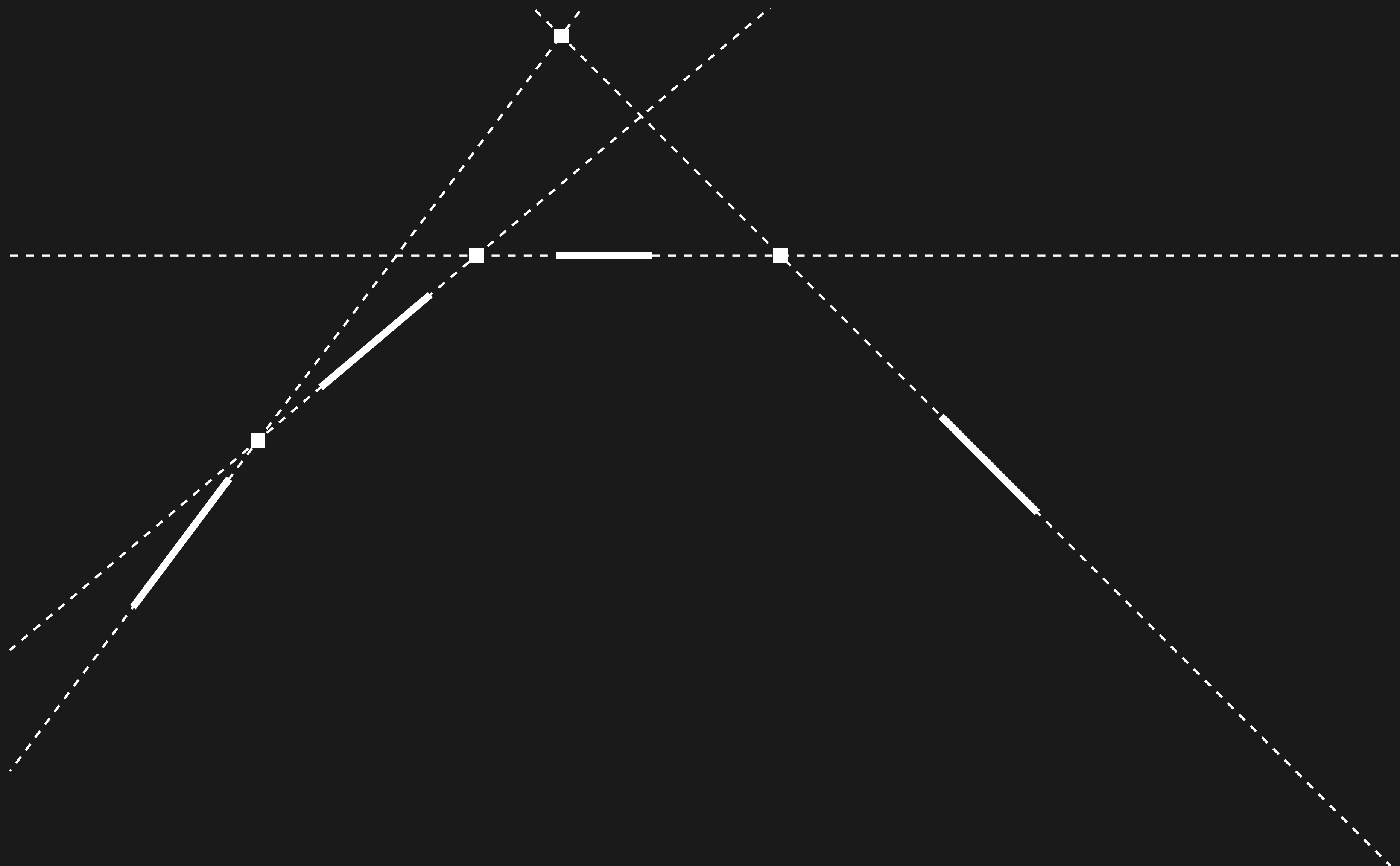

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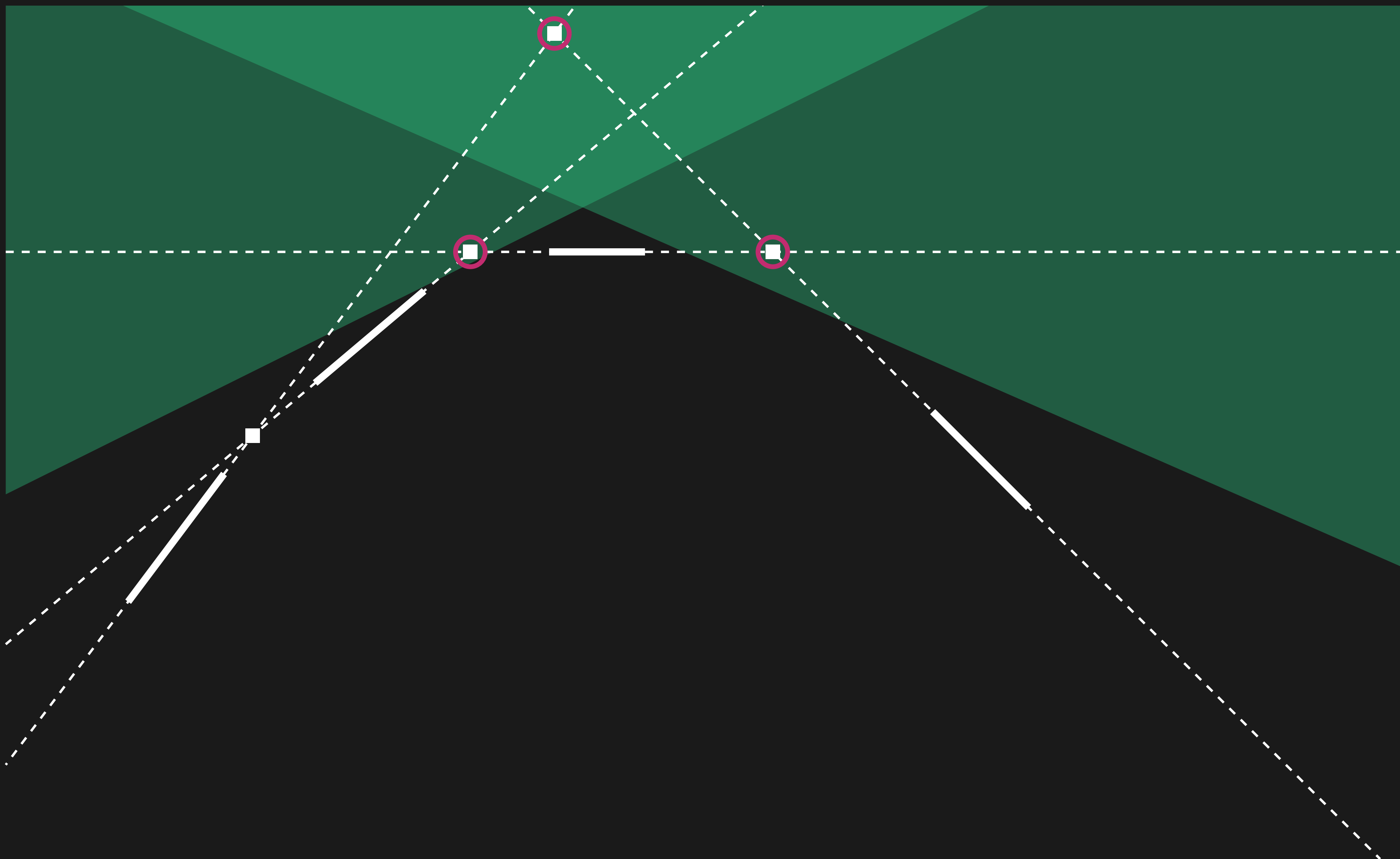

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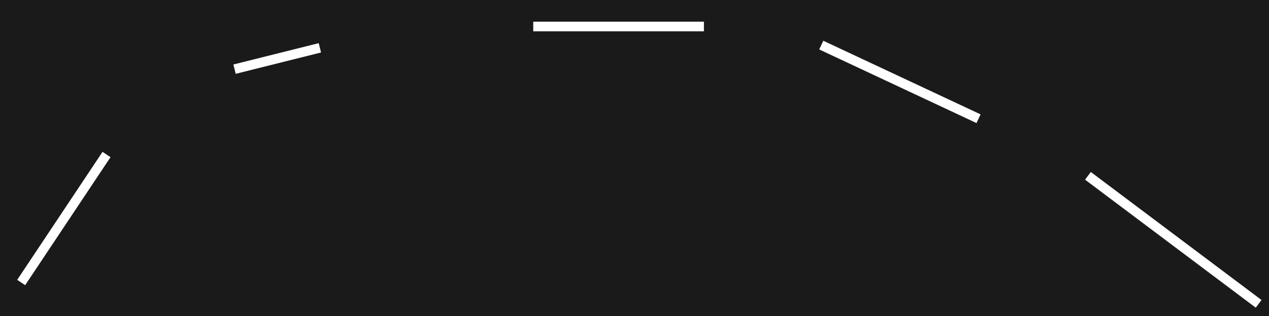

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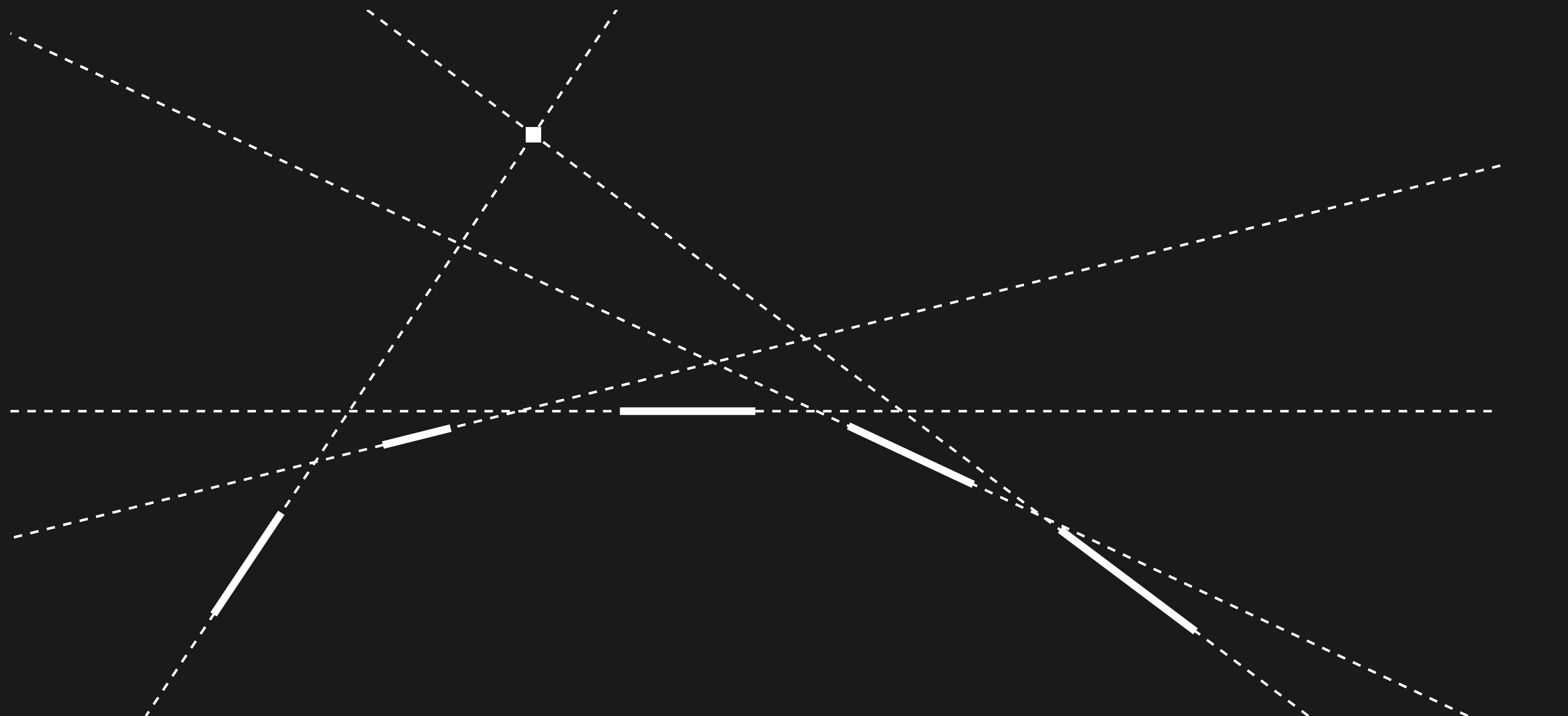

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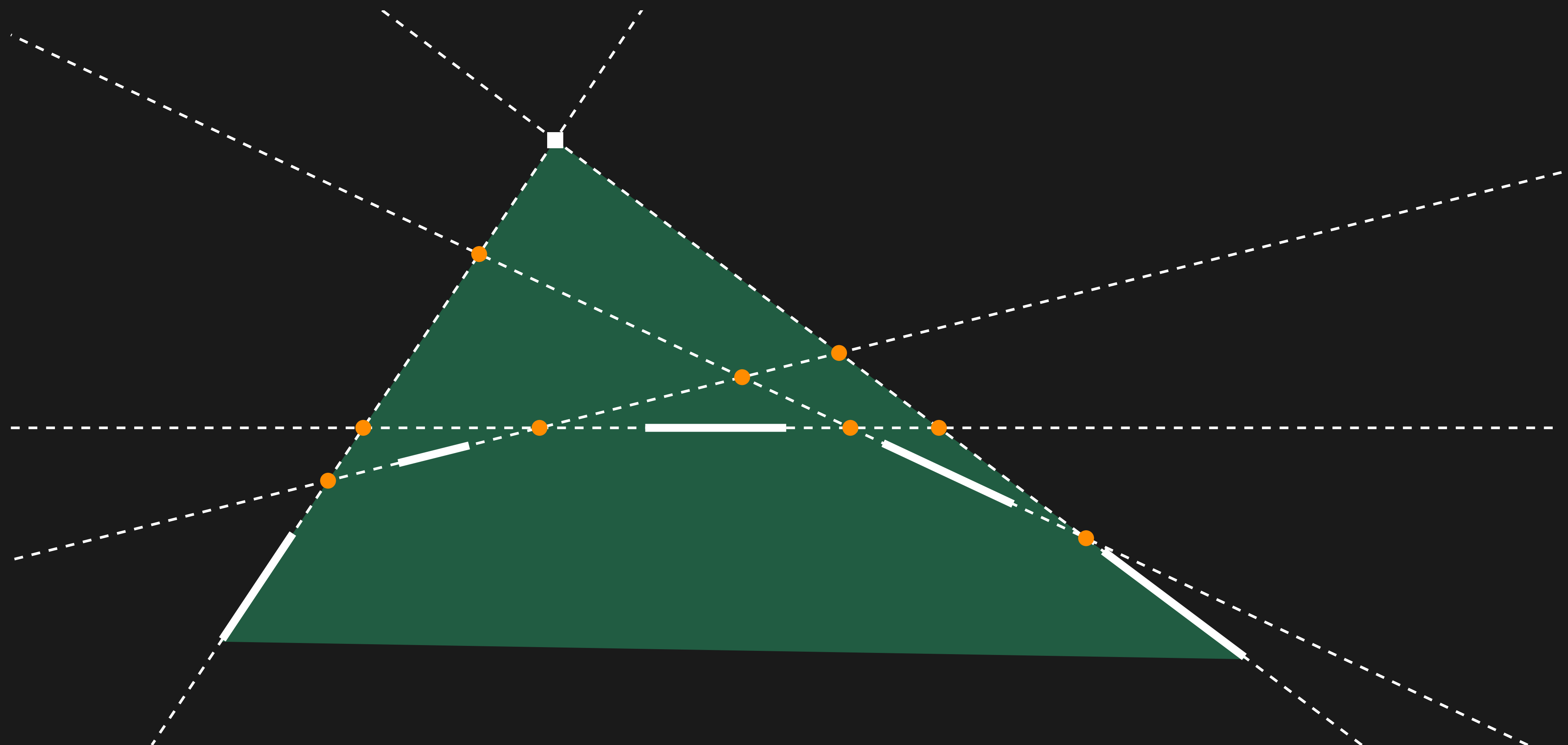

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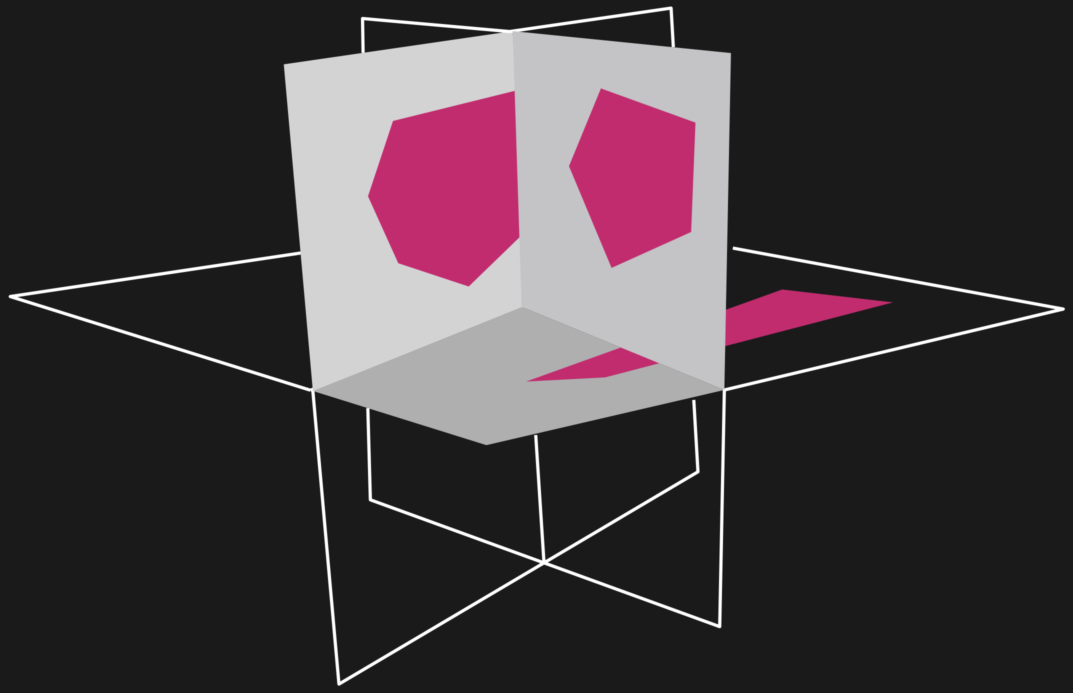

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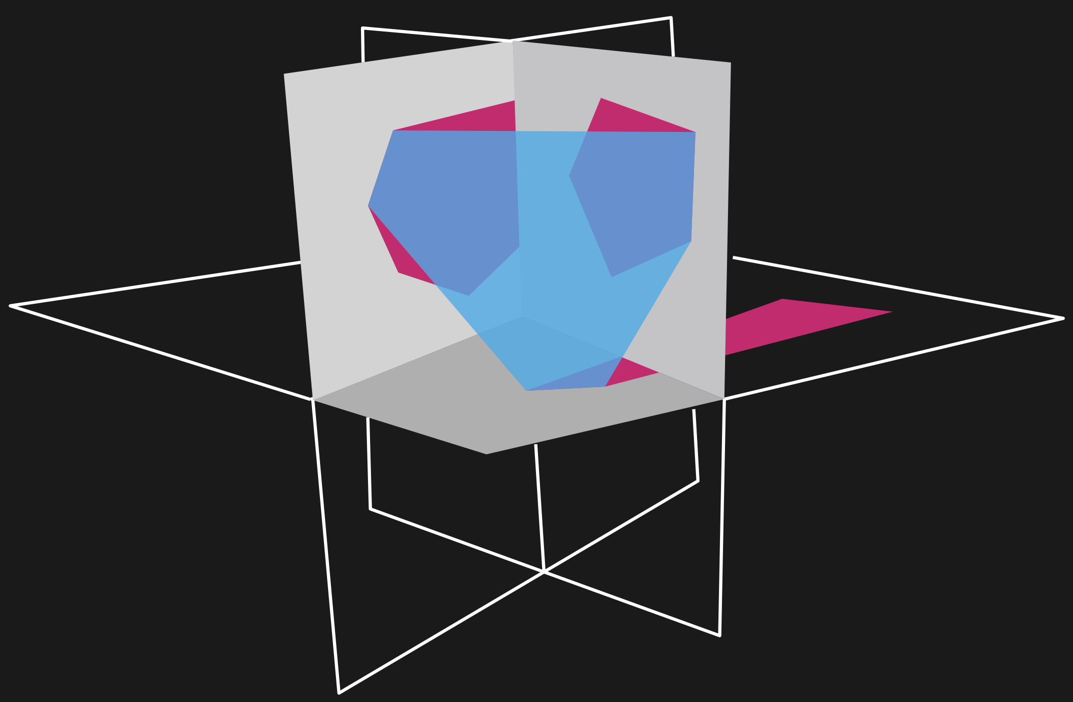

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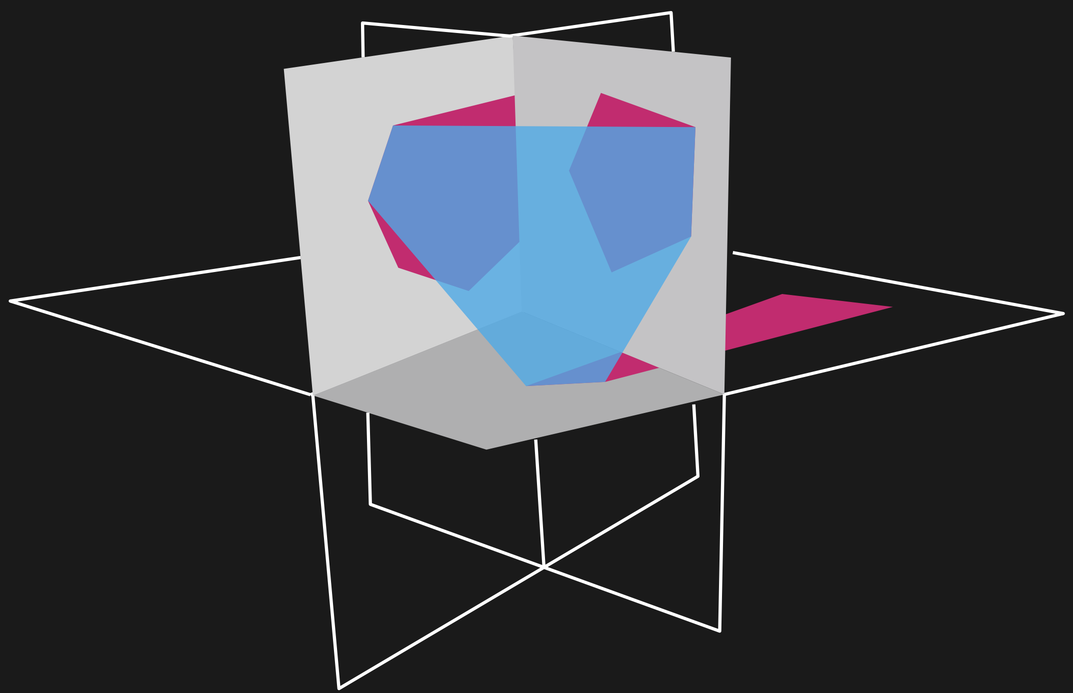

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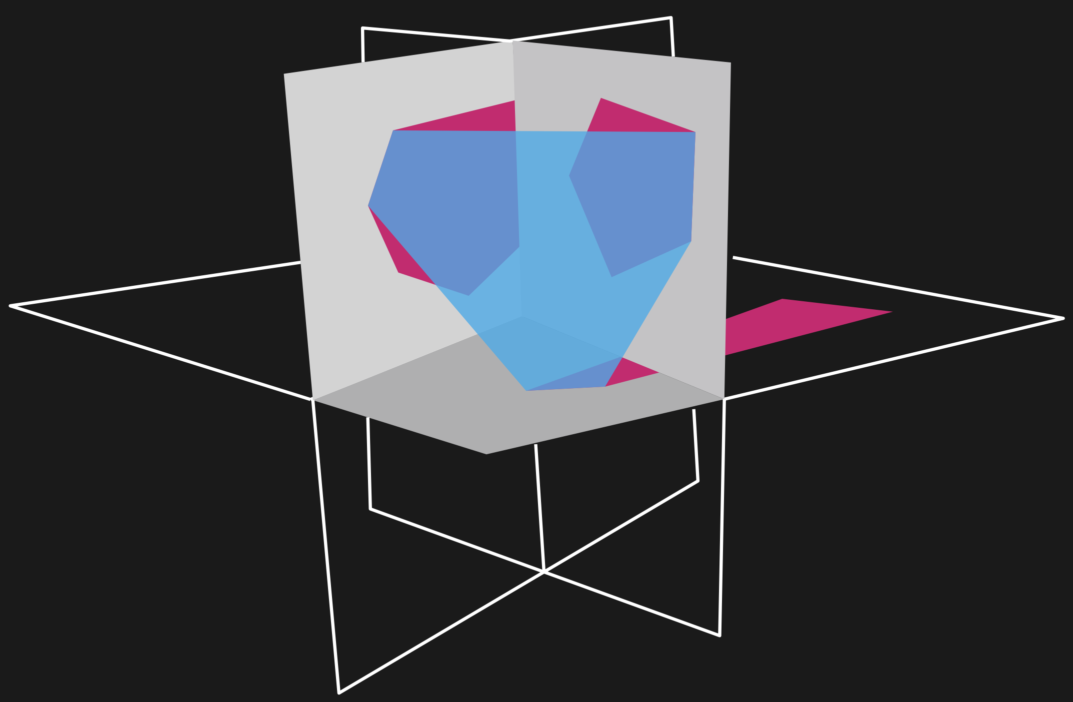

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there is one octant $\mathcal{C}$ with ≥ 50 blue polygons

Applying the 2d Ideas

supporting plane of 1st red polygon

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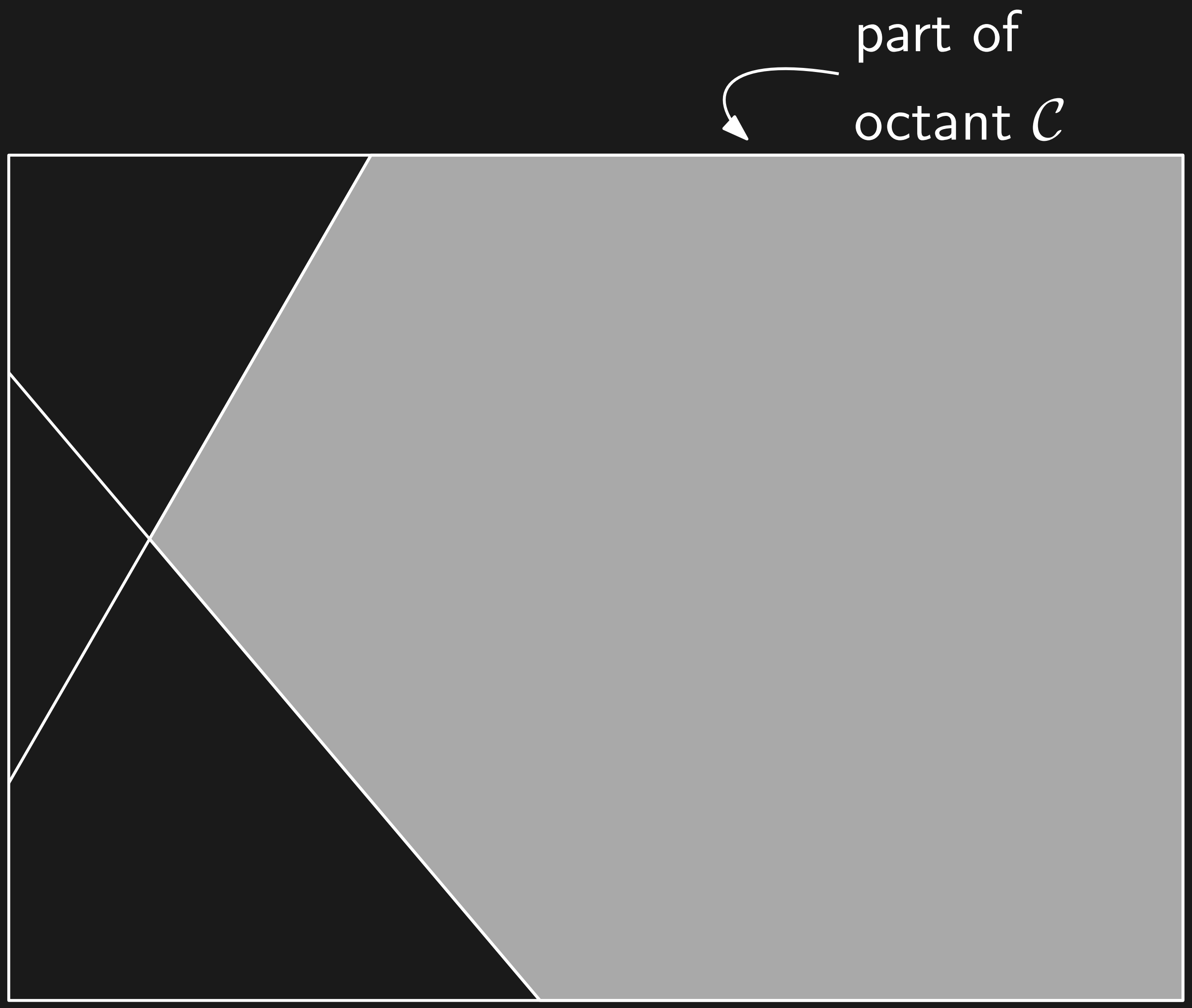

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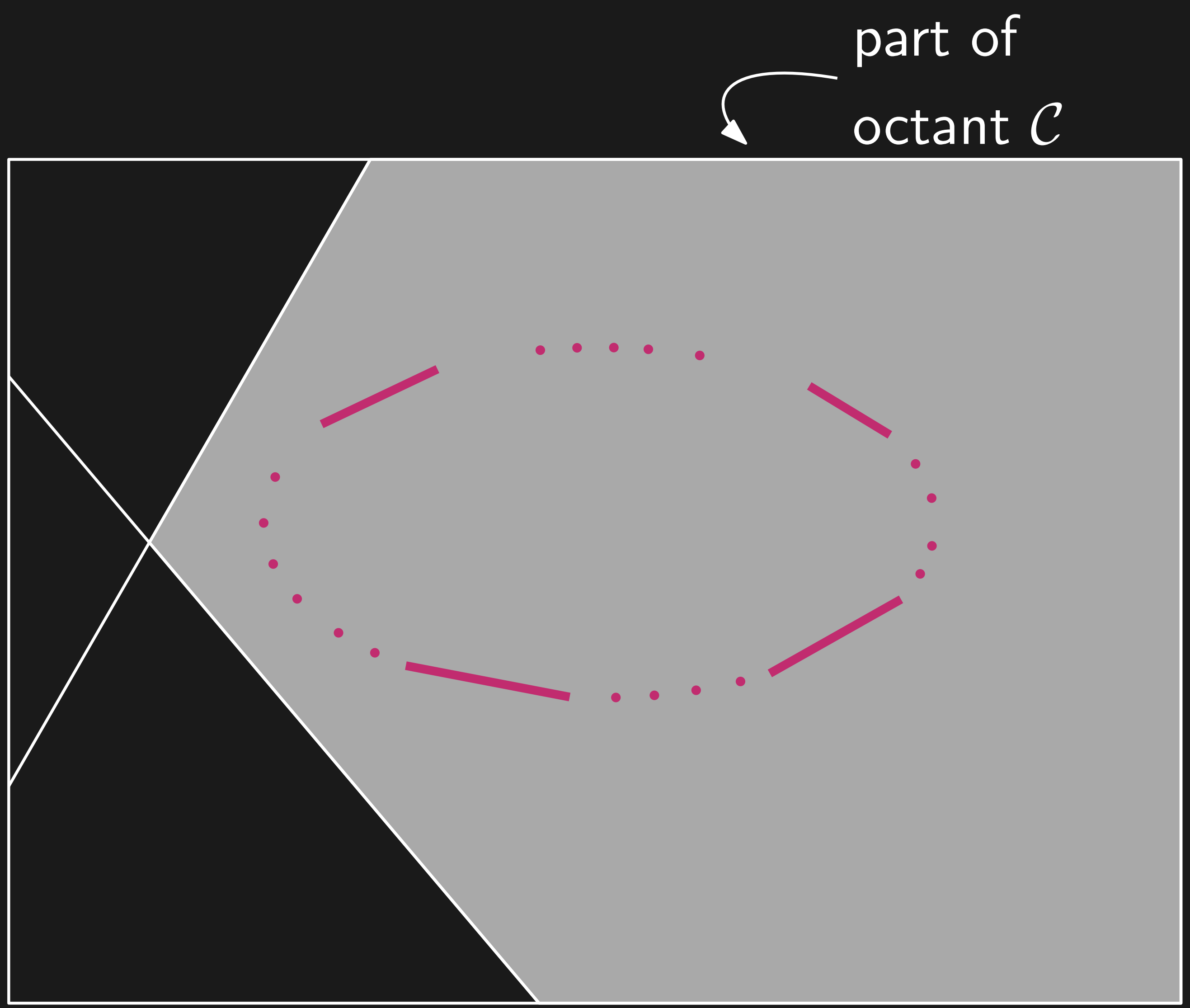

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ignore some blue polygons that have a side on an edge of $\mathcal{C}$ $\rightarrow$ 44 left

consider every 11th segment where blue und red polygons touch

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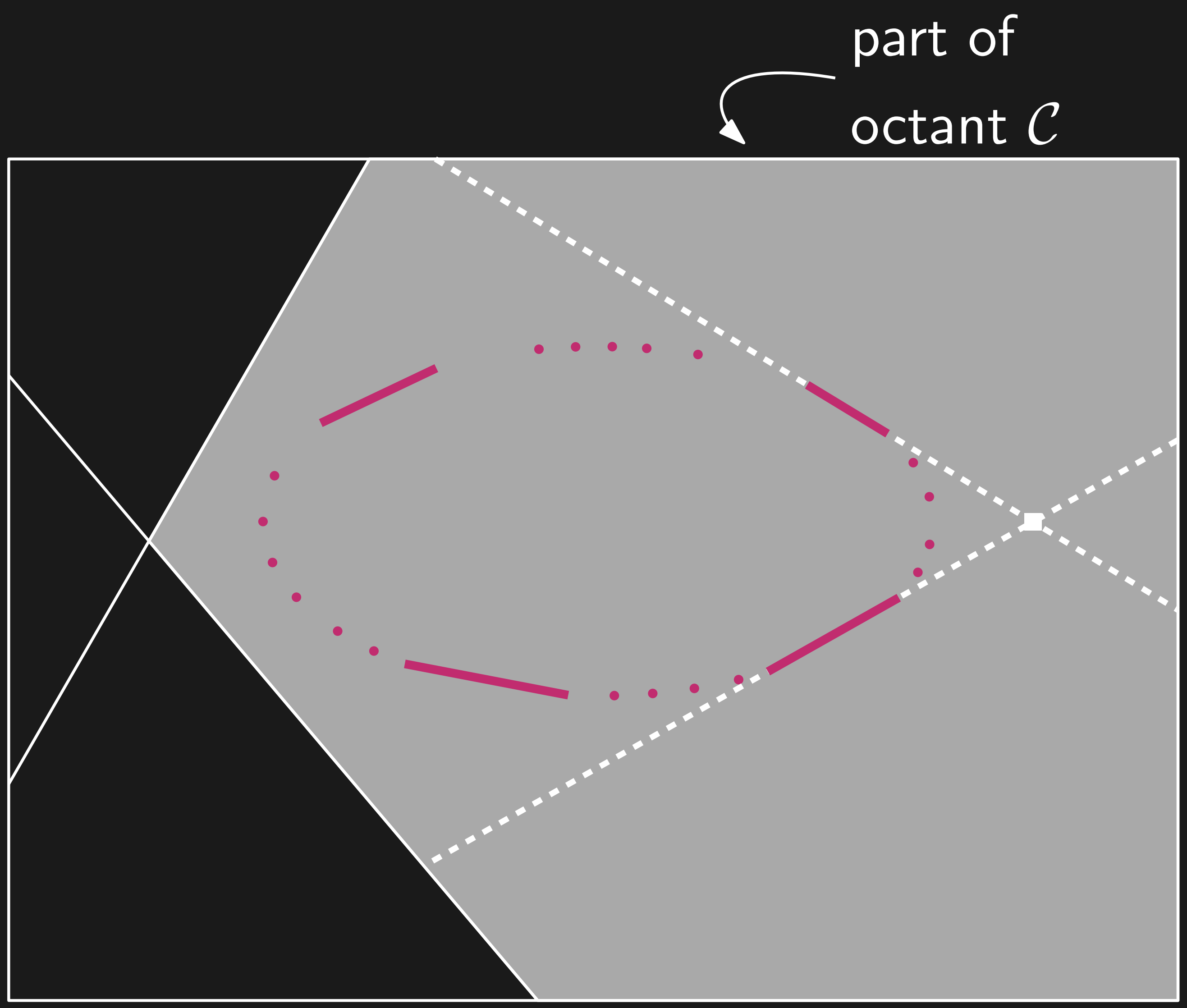

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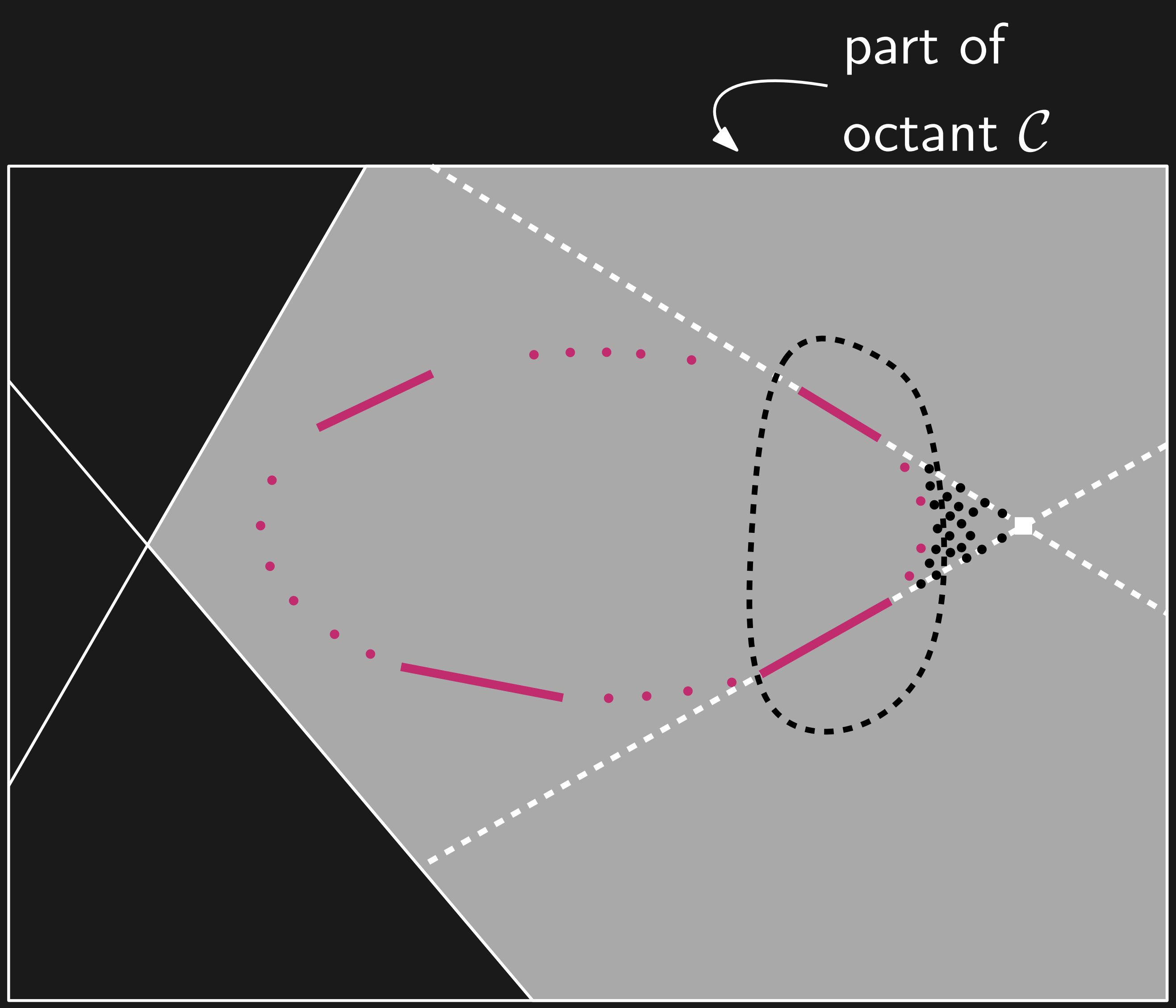

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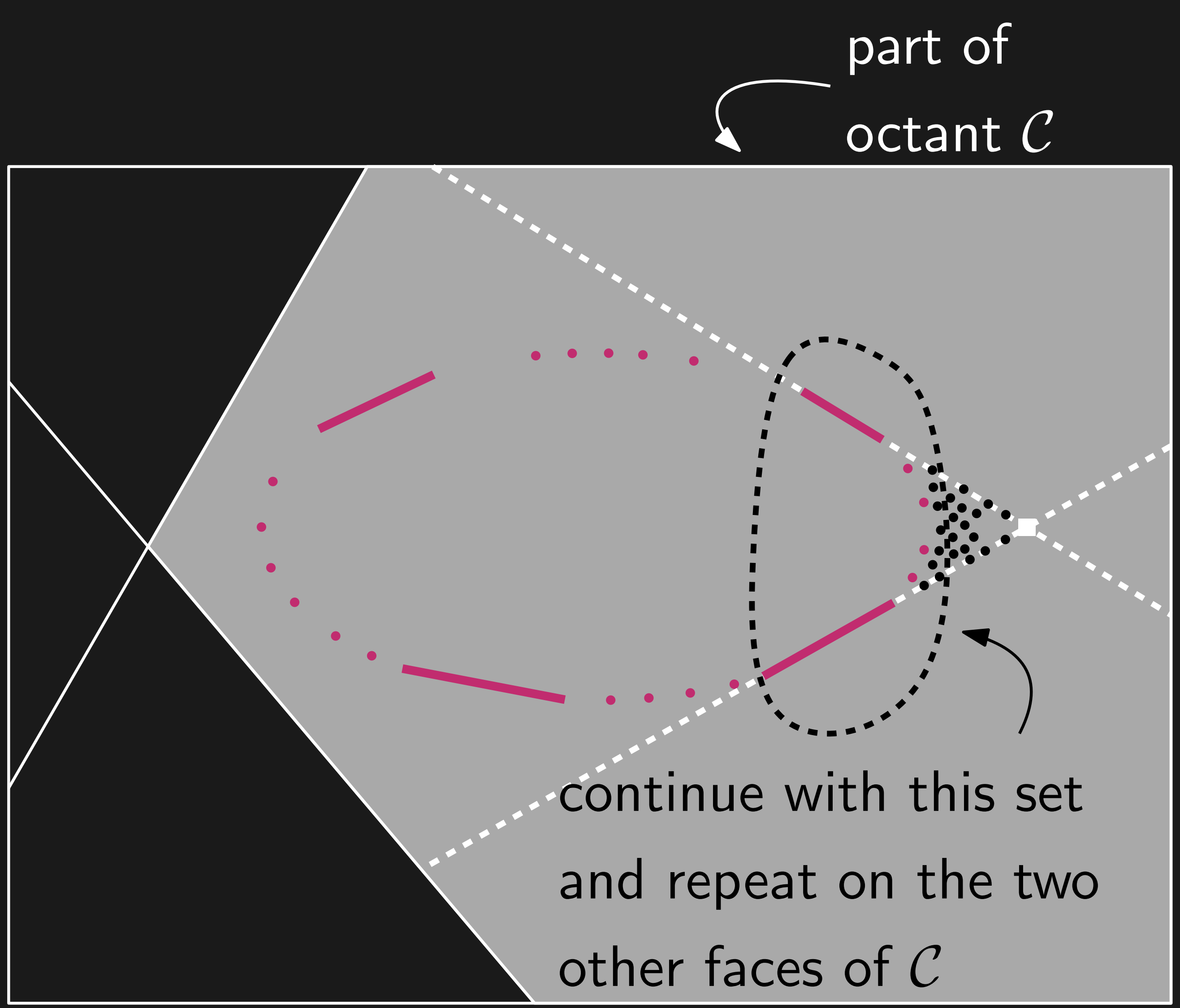

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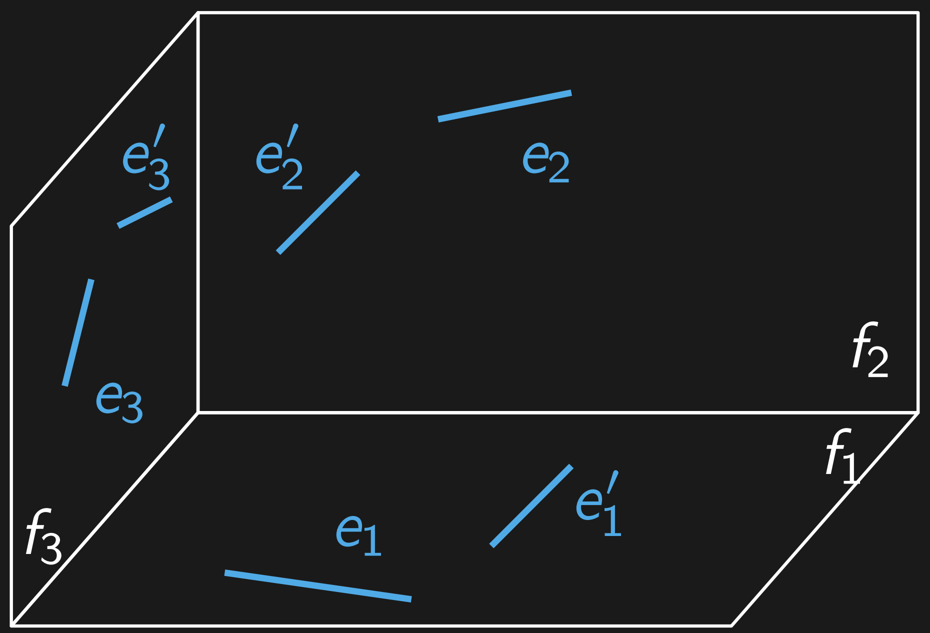

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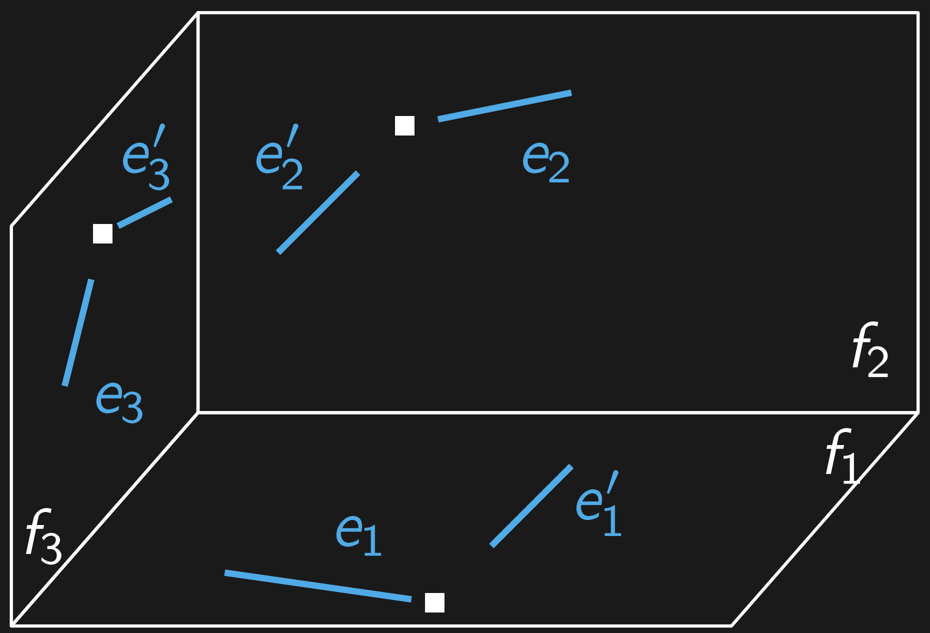

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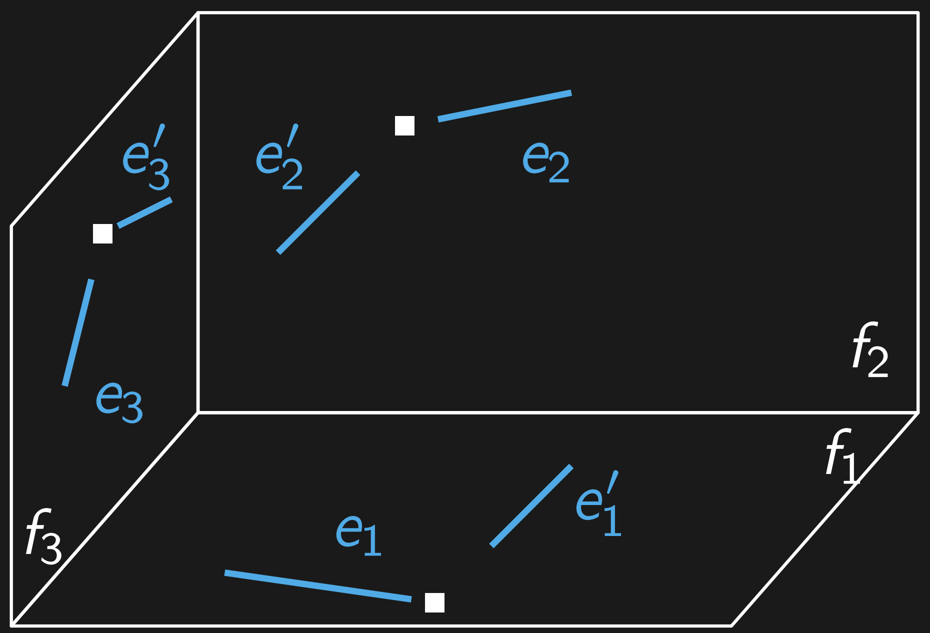

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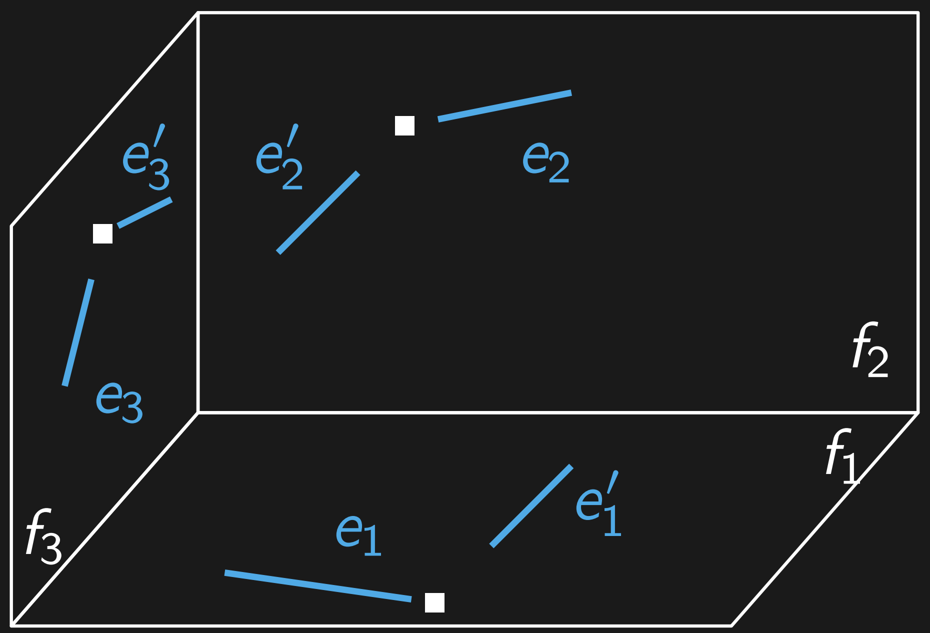

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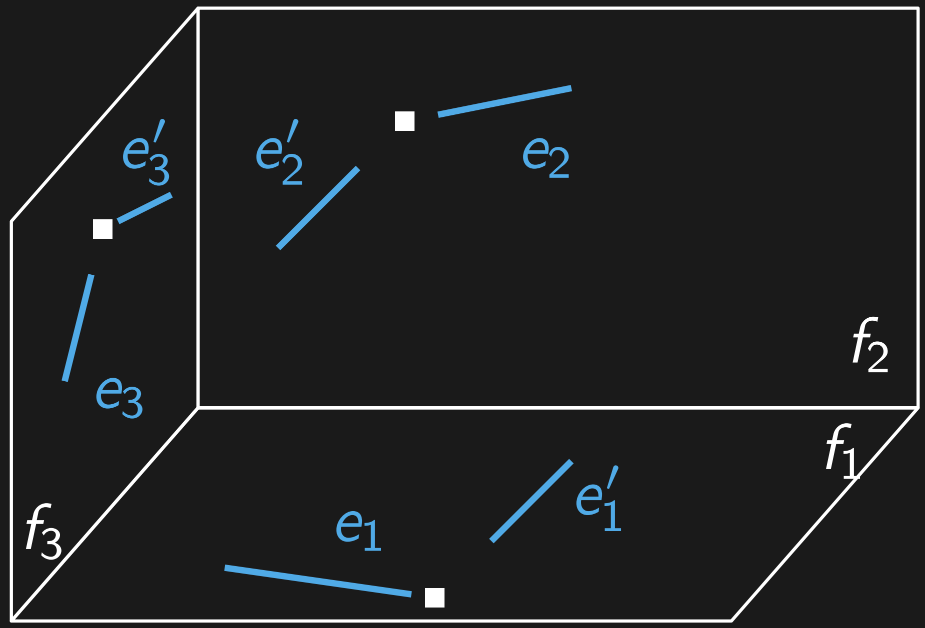

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$\rightarrow$ improved bound on the edge density: $O(n^{5/3})$

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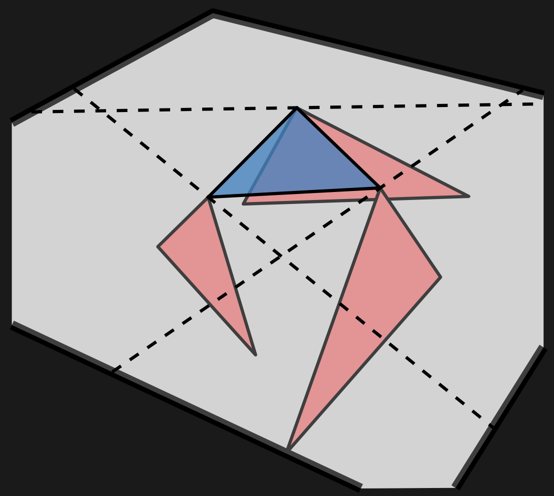

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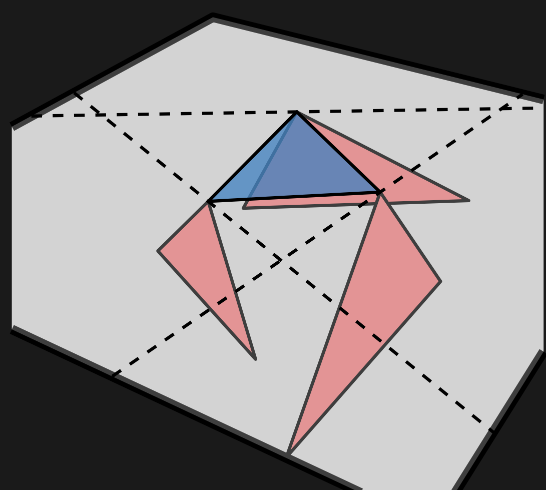
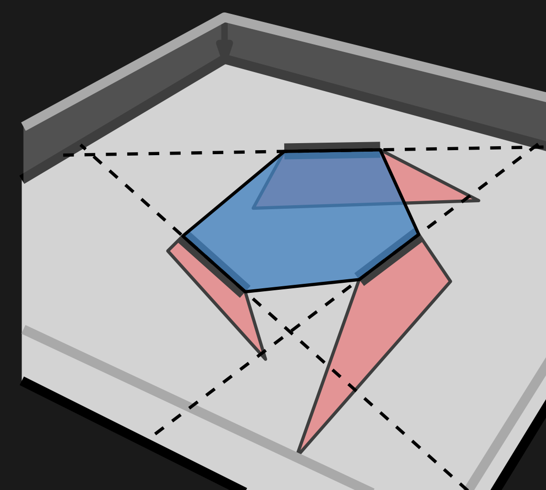


move the blue supporting plane down



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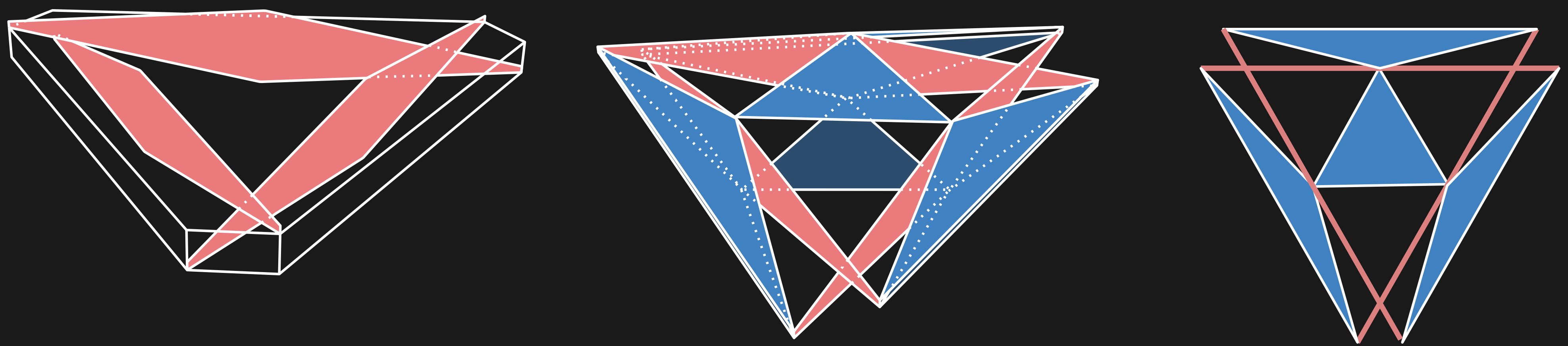

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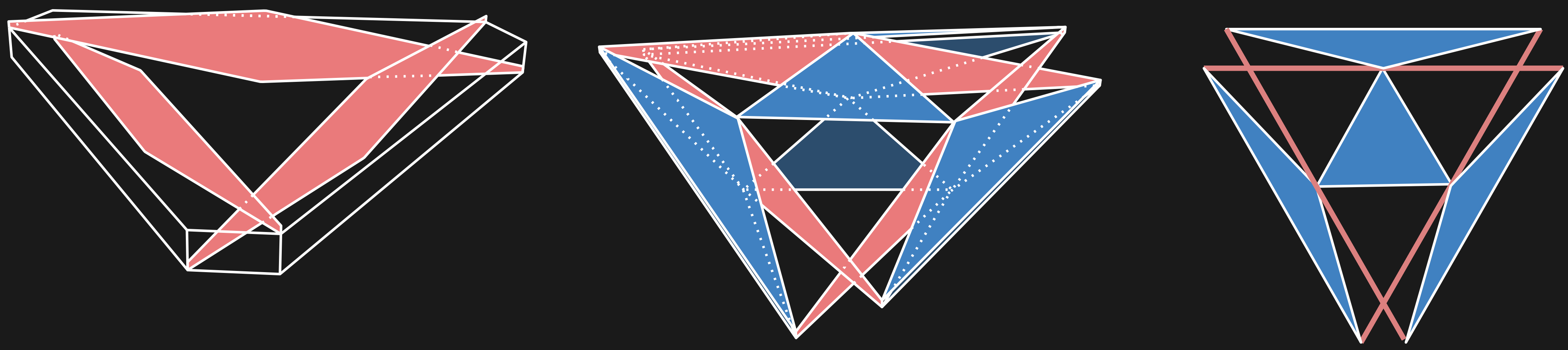

exact coordinates and script to check for the “good”-property are in the arxiv-version